ANOVA

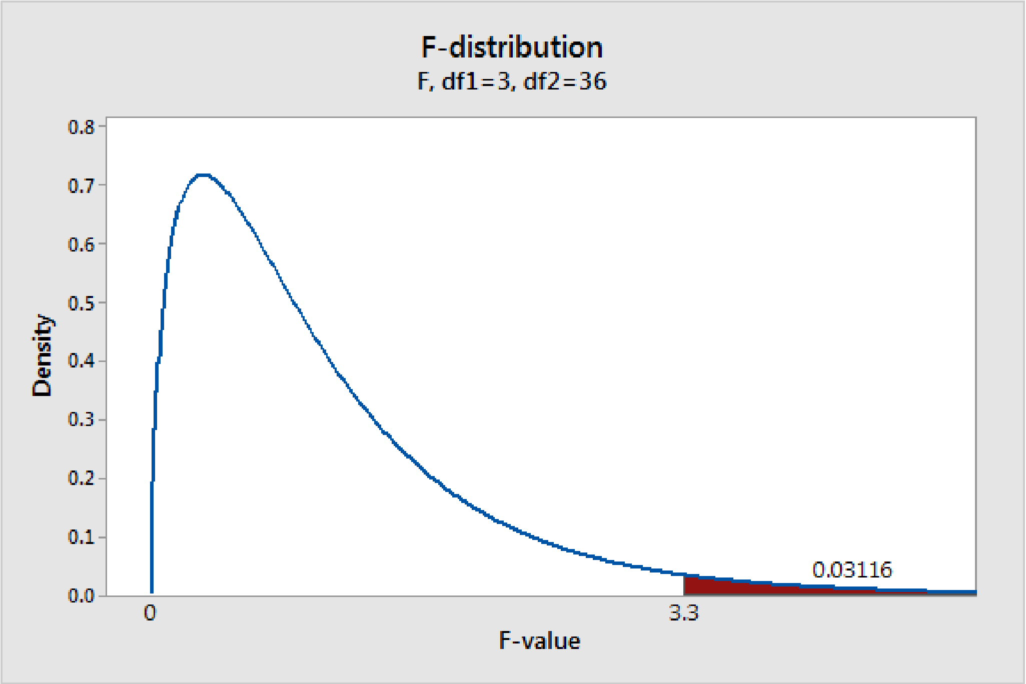
Analysis of Variance

Section E: Grace, Andrey, Danny, Ila

What is ANOVA?

The Analysis of Variance

- Determines whether the differences of means between 3+ groups are statistically significant
- Mitigates the need for doing tons of t-tests



(Minitab Blog)

1-Way ANOVA Procedure

1. Establish hypotheses (see above)
2. State α
    1. Usually 0.05

3. Calculate degrees of freedom

where:

N = the number of people in each group
n = the number of people in the experiment (total)
a = # of levels of the factor

$df_{Between} = a - 1$
$df_{Within} = N - a$
$df_{Total} = N - 1$

4. Critical Value

- Use $df_{Between}$ and df_{Within} to find the critical value.

(a – 1, N – a)

- Use the f-table (next slide), where the x-axis is the "between" and the y-axis is the "within" and input your coordinates to find the critical value

F-table of Critical Values of α = 0.01 for F(df1, df2)

	DF1=1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
DF2=1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6157.29	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.86
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.51	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.67	3.59	3.51	3.43	3.34	3.26	3.17
14	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.85	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.84	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.93	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.70	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.59	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01	2.87	2.73	2.57	2.50	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.67	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.81
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.04	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.19	2.04	1.88	1.79	1.70	1.59	1.47	1.33	1.00

5. Test Statistic

	SS (sum of squares)	df	MS	F
Between	$\frac{\Sigma(\Sigma a_i)^2}{n} - \frac{T^2}{N}$ *for the numerator in the first term, take the sum of each level, square it, then add them all together *T is just the sum of all of the values	a – 1	$\frac{SS_{between}}{df_{between}}$	$F = \frac{MS_{between}}{MS_{within}}$
Within	$\Sigma Y^2 - \frac{\Sigma(\Sigma a_i)^2}{n}$ *for ΣY^2, take the sum of the square of *every* value	N – a	$\frac{SS_{within}}{df_{within}}$	
Total	$\Sigma Y^2 - \frac{T^2}{N}$	N – 1		

6. Decide

- If F is greater than the critical value, reject the null.

7. Conclude

- The [three] conditions differed significantly on level $F(a - 1, N - a)$, $\alpha<0.05$

1-Way ANOVA Example

Jeff is trying to see if customers like the sandwich a different amount based on which sauce he puts on his special sandwich. He has random people at his shop try 3 different sandwiches: one with mayo, one with mustard, and one with ranch and then give them a score.

Here are the scores:

Mayo: 78, 84, 98, 76, 69

Mustard: 56, 67, 48, 87, 54

Ranch: 94, 87, 90, 86, 59

Hypothesis

μ1:The true average score of customers who ate a sandwich with mayo

μ2: The true average score of customers who ate a sandwich with mustard

μ3: The true average score of customers who ate a sandwich with ranch

Null Hypothesis: $\mu1=\mu2=\mu3$

Alternate Hypothesis: At least one true average score for the sandwich is different

Conditions

Independence: All the tests are independent of each other and random

Normal: Assume normality for the sauce groups.

Variance: The standard deviations of 10.86, 15.37, and 13.68 its around the same variance

Stuff To Calculate

- Total Mean of all Scores: 75.53
- Group Means
  - Mayo sample mean: 81
  - Mustard sample mean: 62.4
  - Ranch sample mean: 83.2
- Degrees of Freedom Between: Number of Groups – 1: 2
- Degrees of Freedom Within: Total Number of scores - Number of Groups : 12
- Sum of Squares Between Groups: ∑ (n1 * (Mean_1- Grand Mean)^2:1305.73
- Sum of Squares Within Groups: ∑ (∑ (X1 – Mgroup1)2):2192
- Mean Squares Between: Sum of Squares Between Groups/Degrees of Freedom Between: 652.87
- Mean Squares within: Sum of Squares Within Groups/ Degrees of Freedom Within: 182.67
- F-ratio (test statistic): Mean Squares Between/Mean Squares Within: 3.57

Answer

Since the F test statistic is less than the critical value found on the F statistic critical value table for the degrees of freedom seen within the problem, we are unable to reject the null hypothesis. We do not have statistically significant evidence to say that at least one true average score for the sandwich is different from another for different sauce types.

2-Way ANOVA: What it does and what it can tell you

- Compare populations with different subgroups
- Two types: with replication (default) and without replication
- Focuses on three main tests
- The effect of each of the variables and the interaction
- Classify data by two independent factors

2-Way ANOVA: Procedure (general)

- Define the Factors and Levels

2-Way: Factors

- Fixed factors
  - Chosen by the researcher & are the only levels of interest in the study
  - Interested in comparing the factors
  - Not trying to generalize beyond the levels in your study
  - Fertilizers, drugs, brands etc.
- Random factors
  - Randomly selected from a large population (does that sound familiar?)
  - No interest in the specific factors
  - Interested by the variability caused by the factor, not comparing factors
  - Want to generalize to a larger set of levels beyond the experiment
    - Schools, animals, locations
- Mixed factors (Mixed ANOVA)
  - Can compare specific levels of some factors
  - Generalize to broader populations for other factors

2-Way ANOVA: Procedure (general)

- Define the Factors and Levels
- Set up data table

2-Way: Data table set up

Factor 1: Drug A, B, C (rows)

Factor 2: Symptoms: X, Y (columns)

- With Replication (3 replications)

Drug	Symptom X (Rep 1)	Symptom X (Rep 2)	Symptom X (Rep 3)	Symptom Y (Rep 1)	Symptom Y (Rep 2)	Symptom Y (Rep 3)
A						
B						
C						

- Without Replication (no replication)

Drug	Symptom X	Symptom Y
A		
B		
C		

2-Way ANOVA: Procedure - The Grand Mean

- Define the Factors and Levels
- Set up data table
- Calculate "grand mean" (the overall average of all data points across all levels of both factors)

$$\frac{\text{Average of all your data points}}{\text{Total number of measurements}}$$

Drug	Symptom X	Symptom Y
A	4.2	2.7
B	1.5	2.3
C	2.6	3.9

$$\frac{4.2 + 1.5 + 2.6 + 2.7 + 2.4 + 3.9}{2 \times 3} \approx 2.88$$

2-Way ANOVA: Procedure (general)

- Define the Factors and Levels
- Set up data table
- Calculate “grand mean” (the overall average of all data points across all levels of both factors)
- Calculate the Sum of Squares (SS) for all your factors (eg 2 factors here)
    - Sum of Squares for Factor 1 (SS_Factor 1)
    - Sum of Squares for Factor 2 (SS_Factor 2)
    - Sum of Squares for the Interaction (SS_Interaction)
    - Sum of Squares for Error (SS_Error)

2-Way ANOVA: Calculating the Sum of Squares

Drug	Symptom X	Symptom Y
A	4.2	2.7
B	1.5	2.3
C	2.6	3.9

Factor 1: Drug A, B, C (rows)
Factor 2: Symptoms: X, Y (columns)

- For SS Factors 1 and 2 check how much different levels of a factor affect the results
- Calculate the average of every level in a factor
- Find the difference between each level's avg and the Grand mean
- Square the differences and multiply by the number of levels of factor 2
  - Levels of factor 2 is two because of the two symptoms
- SS Interaction
  - The combination of Factor 1 and Factor 2
  - Calculate the difference between the observed value and the average for that combination then square etc.
- SS Error
  - Calculated as a residual variation because of no replication

2-Way ANOVA: Procedure – Degrees of Freedom

- Define the Factors and Levels
- Set up data table
- Calculate "grand mean" (the overall average of all data points across all levels of both factors)
- Calculate the Sum of Squares (SS) for all your factors (eg 2 factors here)
  - Sum of Squares for Factor 1 (SS_Factor 1)
  - Sum of Squares for Factor 2 (SS_Factor 2)
  - Sum of Squares for the Interaction (SS_Interaction)
  - Sum of Squares for Error (SS_Error)
- Calculate Degrees of Freedom (DF)

Factor 1 DF = (# of levels of Factor 1) – 1
Factor 2 DF = (# of levels of Factor 2) – 1
Interaction DF = (# of levels of Factor 1 - 1) × (# of levels of Factor 2 - 1)
Error DF = (total number of measurements - 1) - (DF for Factor 1 + DF for Factor 2 + DF for Interaction).

2-Way ANOVA: Procedure – Mean Squared

- Define the Factors and Levels
- Set up data table
- Calculate "grand mean" (the overall average of all data points across all levels of both factors)
- Calculate the Sum of Squares (SS) for all your factors (eg 2 factors here)
  - Sum of Squares for Factor 1 (SS_Factor 1)
  - Sum of Squares for Factor 2 (SS_Factor 2)
  - Sum of Squares for the Interaction (SS_Interaction)
  - Sum of Squares for Error (SS_Error)
- Calculate Degrees of Freedom (DF)
- Calculate Mean Squares (MS)

$$\frac{SS}{\mathrm{DF}} = MS$$

2-Way ANOVA: Procedure (general)

- Define the Factors and Levels
- Set up data table
- Calculate "grand mean" (the overall average of all data points across all levels of both factors)
- Calculate the Sum of Squares (SS) for all your factors (eg 2 factors here)
  - Sum of Squares for Factor 1 (SS_Factor 1)
  - Sum of Squares for Factor 2 (SS_Factor 2)
  - Sum of Squares for the Interaction (SS_Interaction)
  - Sum of Squares for Error (SS_Error)
- Calculate Degrees of Freedom (DF)
- Calculate Mean Squares (MS)
- Calculate F-ratios, look at F-distribution tables
- Compare it with the critical value
- Draw conclusions

If F-ratio > critical value; the factor/interaction has a significant effect

2-Way: Assumptions

- All samples are drawn from normally distributed populations
- These samples were drawn independently from each other
- All populations have a common variance
- Within each sample, the observations were sampled randomly and independently of each other

Anova 2-way Example

Bread Type	Sauce Type	Scores
Italian	Mayo	78,93,39,83,44
Italian	Mustard	89,98,99,78,69
Italian	Ranch	92,89,69,87,76
White	Mayo	56,79,86,56,89
White	Mustard	56,78,89,68,90
White	Ranch	67,89,67,76,89
French	Mayo	52,78,63,78,93
French	Mustard	97,96,95,94,61
French	Ranch	75,89,93,45,32

- Jeff now wants to see how the type bread affects how much customers enjoy the sandwich and how the type of sauce on the sandwich affects the customers enjoyment of the sandwich. He also would like to see how these two variables combined affect the customers enjoyment. He has random customers try the sandwiches bread types of Italian, white, and French bread and different types of sauce mayo, mustard, and ranch.

Hypothesis

Null Hypothesis: The type of bread doesn't affect the score the sandwich gets.

Null Hypothesis: The type of sauce doesn't affect the score the sandwich gets.

Null Hypothesis: The type of bread and sauce combo doesn't affect the score the sandwich gets.

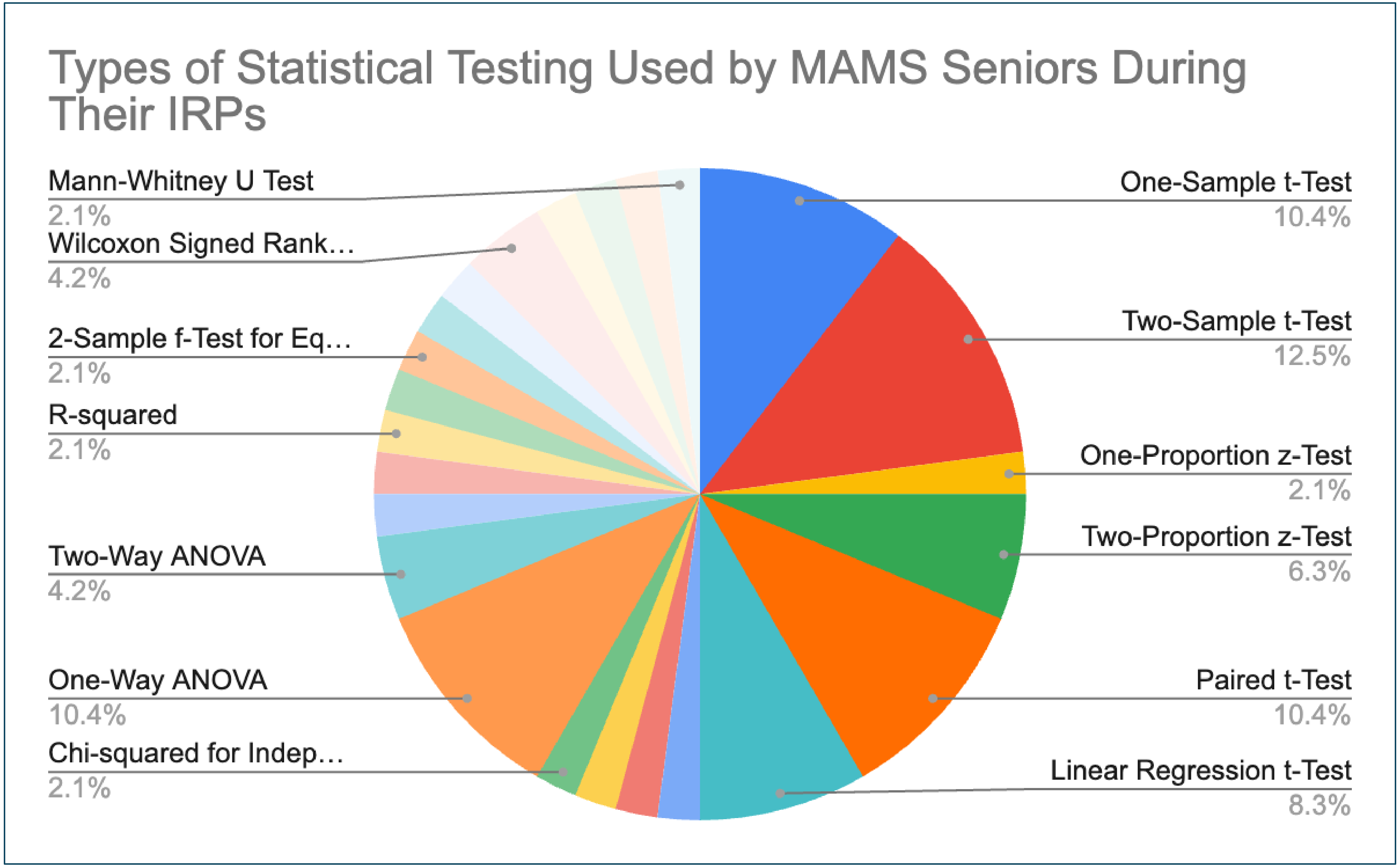
Stuff To Calculate

- Italian Bread means:
- Mayo: 67.4
- Mustard: 86.6
- Ranch: 82.6
- White Bread:
- Mayo: 73.2
- Mustard:76.2
- Ranch: 77.6
- French Bread:
- Mayo: 72.8
- Mustard: 88.6
- Ranch: 66.8
- Grand Mean = 75.58
- Total Sum of Square
- SSTotal = Σ(X - Grand Mean)^2: 20,244.98
- Square Sum for Bread
- SSBread = n * Σ(Bread Mean - Grand Mean)^2:15 * (10.82 + 0.01 + 0.24) = 165.95
- Square Sum for Sauce
- SSSauce = n * Σ(Sauce Mean - Grand Mean)^2:1,310.70
- Square Sum for Interaction
- SSInteraction = n * Σ(Cell Mean - Row Mean - Column Mean + Grand Mean)^2: 2,814.53
- Sum of Square Error
- SSError = SSTotal - SSBread - SSSauce - SSInteraction:15,953.80
- Df bread = 2
- Df sauce = 2
- Df interaction = df_bread * df_sauce = 4
- Df error = 36
- Df total = 44
- Mean Square Bread = SSBread / df bread = 82.98
- Mean Square Sauce = SSSauce / df sauce = 655.35
- Mean Square Interaction = SSInteraction / df_interaction = 703.63
- Mean Square Error = SSError / df error = 443.16
- F bread = Mean Square Bread / Mean Square Error = 0.19
- F sauce = Mean Square Sauce / Mean Square Error = 1.48
- F interaction = Mean Square Interaction / Mean Square Error = 1.59

Conclusion

Since the F test statistic is less than the critical value found on the F statistic critical value table for the degrees of freedom seen within the problem, we are unable to reject the null hypothesis for any of the given hypothesis. We do not have statistically significant evidence to say that at least one bread type, one sauce type, or the combination of a bread and a sauce type can cause different satisfaction within the sandwich.

Some Interesting Data from the seniors



Potential Tests for You to Use!

- One & Two-Way ANOVA
- Linear Regression t-Test
  - Determines whether the independent variable has a significant relationship with the dependent variable, controlling for the other predictors
- Wilcoxon Signed-Rank Test
  - Kind of like a Paired t-Test
  - Does not require Normality
- Mann-Whitney U Test
  - A.K.A. Wilcoxon Rank-Sum Test
  - Helps determine whether two samples are likely from the same population
  - Use when assumptions for a regular t-Test (like Normality or Large Counts) are violated

Thank you!

Sources

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