

WORCESTER POLYTECHNIC INSTITUTE

SIXTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 23, 2003

INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

1] Where does the parabola $y = 6x^2 + 13x - 63$ cross the x -axis?

2] Find an expression for the angle made by the line $y = \frac{1}{9}x - 22$ and the x -axis.

3] Factor $x^{16} - 1$

4] For the function

$$f(x) = 8x^7 + 6x^5 + x^4 + 9x^3 + 6x^2 + 5x + 1$$

how many positive roots does it have?

5] If $f(x) = \frac{2x - 1}{3x - 5}$ find a formula for f^{-1} .

6] Simplify $10^{-3\log_{10}(5)}$.

7] Find the coordinates of the center of the conic section

$$2x^2 - 12x + 72y + 4y^2 = 329$$

8] If $x = 231$ in base 10, what is it in binary?

9] If one selects a random point from a 2' x 2' square, what is the probability that such a point is within one foot of the center of the square?

10] Find all roots of $x^3 + 4x^2 + x - 6 = 0$

11] If $x = .783478347834\dots$ is a repeating decimal, express it as a rational number.