

# WORCESTER POLYTECHNIC INSTITUTE

SIXTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 23, 2003

TEAM EXAM QUESTION SHEET

**DIRECTIONS:** Please write your answers on the Team Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators **MAY NOT** be used.

1 In the expression  $\frac{(x+p)^7 - x^7}{p}$  What is the coefficient of  $x^4p^2$ ?

2 What is the period of the function

$$f(t) = 10 \sin(312t + 64) + 5 ?$$

3 If a sequence is defined recursively by

$$x_0 = 0 \quad x_{n+1} = f(x_n)$$

where  $f(x) = ax + b$ , find an explicit formula for the  $n$ th term,  $x_n$ .

4 A triangle is bounded by 3 lines:

$$y = x + 8 \quad y = -7x + 32 \quad y = -x - 28$$

what is its area?

5 Suppose  $z$  and  $w$  are constants, with  $z > 2w$ . A triangle has sides of length  $z^2 - 4w^2$ ,  $4zw$ , and  $z^2 + w^2$ . Find a value for  $v$  so it is a right triangle.

6 Simplify  $(a + b)^5 \pmod{5}$  where  $a$  and  $b$  are integers.

7 A parabola is described by  $x = \frac{y^2}{4} + 1$ . What are the coordinates of its focus?

8 Simplify

$$\sin^8 \theta + 4 \sin^6 \theta \cos^2 \theta + 6 \sin^4 \theta \cos^4 \theta + 4 \sin^2 \theta \cos^6 \theta + \cos^8 \theta.$$

9 Consider the region in the  $x - y$  plane bounded by  $y = 0$ ,  $y = x$ ,  $x^2 + y^2 = 4$  and  $x^2 + y^2 = 9$ . What is its area?

10 A long cylinder has elliptical cross section; major axis 6 and minor axis 4 units. Two parallel slices at  $45^\circ$  to the axes of the cylinder are made resulting in an object 30 units long. What is its volume?

11 Evaluate  $8 + 11 + 14 + 17 + \dots + 305$  as a single integer.

- 12] An object follows a 2 dimensional path described by

$$x(t) = 4t^2 + t$$

$$y(t) = 3t^4 + 2t^2 - 21t^3 - 14t \quad t > 0$$

How far is the object from the origin when it strikes the ground?

- 13] A light bulb is hanging from the ceiling of the second floor of a building, 10 feet above the floor. Directly below it is a circular hole in the floor of diameter 6 feet. The first floor has ceiling height of 12 feet. If the first floor has no light of its own, what will the area of the floor with light on it be?

- 14] There are two parallel lines,  $L1$  and  $L2$  lying in the plane. There are two points,  $B$  and  $C$  on the lower line,  $L1$ . There are also two distinct points  $A1$  and  $A2$  on the upper line. Two triangles are formed: one by  $A1, B$  and  $C$  and a second by  $A2, B$  and  $C$ . What is the ratio of the area of the first triangle to the area of the second triangle?