

# WORCESTER POLYTECHNIC INSTITUTE

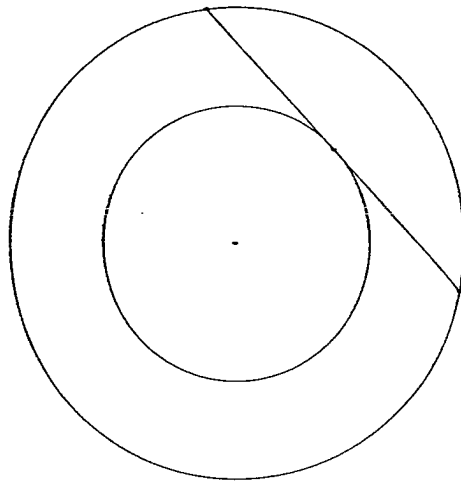
FIFTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 17, 2002

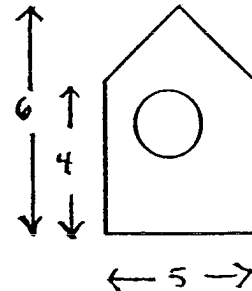
TEAM EXAM QUESTION SHEET

**DIRECTIONS:** Please write your answers on the Team Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-14 is worth 3 points. Calculators **MAY NOT** be used.

- 1 Find the area of the annulus between the two concentric circles given that the length of any cord of the outer circle, tangent to the inner circle, is 40 cm.



- 2 Gamma rays strike a pentagonal flat surface with uniform probability. There is a disk on the surface. What should its diameter be so that the probability of striking it is  $1/3$ ?



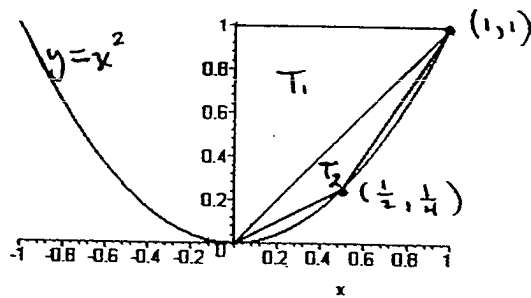
- 3 How many roots in  $[0, 4]$  does the following function have?

$$f(t) = (t^3 - 7t^2 - 34t + 40) \sin^2\left(\frac{7t}{2}\right)$$

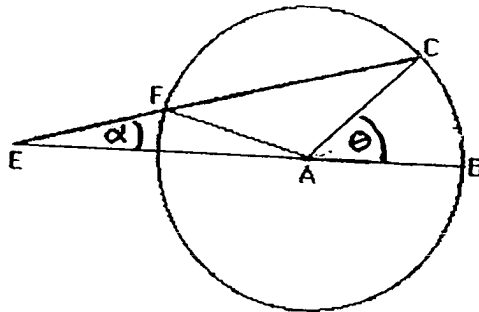
- 4 What is  $i^i$  in standard complex form? ( $i = \sqrt{-1}$ )
- 5 Assuming the radius of the Earth to be 1 unit, what are the Cartesian Coordinates of a location whose longitude is  $30^\circ w$  and whose latitude is  $60^\circ$  north? Assume that the equator is in the  $x - y$  plane, the  $z$  axis goes through the north and south poles, and the  $x - z$  plane passes through  $0^\circ$  longitude.
- 6 For the function  $f(x) = x^2 + 1$ , find  $f(f(\frac{1}{2}))$ .
- 7 Find a 3<sup>rd</sup> point so an equilateral triangle is formed where two of the points are  $(2, 1)$  and  $(5\sqrt{3} + 2, 6)$

8 Find all cube roots of  $-27$ .

9 In the following graph, what is the ratio of the area of triangle  $T_1$  to the area of triangle  $T_2$ ?



10 In the following drawing, what is the relationship between angle  $\alpha$  and angle  $\theta$ ?



11 What is the sum of the following series?

$$41 + 45 + 49 + 53 + \dots + 437$$

12 If  $f(x) = \log\left(\frac{1+x}{1-x}\right)$  for  $-1 < x < +1$  then  $f\left(\frac{3x+x^3}{1+3x^2}\right)$  in terms of  $f(x)$  is what?

13 The area of a rectangle remains unchanged when it is made  $2\frac{1}{2}$  inches longer and  $\frac{2}{3}$  inch narrower, or when it is made  $2\frac{1}{2}$  inches shorter and  $\frac{4}{3}$  inch wider. What is its area in square inches?

14 A triangle has sides with lengths of 25, 15 and 20. It is inscribed in a circle. What is the radius of the circle?