WORCESTER POLYTECHNIC INSTITUTE

FOURTEENTH ANNUAL INVITATIONAL MATH MEET
OCTOBER 25, 2001
INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 45 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

[1] Simplify

$$\sin^3(\text{Arctan}(5/12))$$

$\boxed{\left(\frac{5}{13}\right)^3 \text{ or } \frac{124}{2197}}$

[2] Solve for y:

$$16\sin^4 y - 16\sin^2 y + 3 = 0$$

$\boxed{\pm 30°, \pm 60°, 180° \pm 30°, 180° \pm 60°}$

[3] Find a value of d so that the equation $9x^2 + dx + 4 = 0$ has one solution. $\boxed{d = +12 \text{ or } -12}$

[4] How many points are there with integer coordinates which satisfy $3x + 12y = 1$? $\boxed{\text{None}}$

[5] In the expansion of $(2x + y)^{30}$, what is the coefficient of x^2y^{28} ? $\boxed{1740}$

[6] If $i = \sqrt{-1}$, what is $\sqrt{i}$? (your answer must be in standard complex form, $a+bi$) $\boxed{\pm(1+i)/\sqrt{2}}$

[7] If the y axis is reflected about the $y = (1/\sqrt{3})x$ line, what is the equation of the resulting line? $\boxed{y = \sqrt{3}x}$

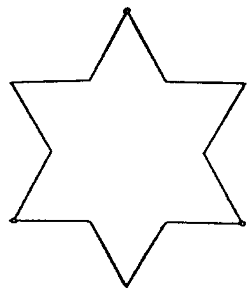
[8] Simplify the following:

$$\sum_{n=2}^{20}((n+2)^3 - (n+1)^3 + 1)$$

$\boxed{10640}$

[9] Find the point on the line $x + 2y = 20$ which is closest to the point $(1, -3)$. $\boxed{(6, 7)}$

10 A Koch Snowflake is constructed recursively by beginning with an equilateral triangle and, on each iteration, dividing each side in thirds and replacing the middle third by two sides of an equilateral triangle whose sides are the length of the removed segment. Please see the **drawing** which illustrates the first iteration.



Write an expression for the total **length** of the n^{th} iteration if the three sides of the original triangle are each 3 units long. $\boxed{9\left(\frac{4}{3}\right)^n}$

11 Consider an ellipse whose equation is $x^2/49 + y^2/13 = 1$. If P is a point on the ellipse, what is the sum of the distances from P to the two foci of the ellipse? $\boxed{14}$