

WORCESTER POLYTECHNIC INSTITUTE

THIRTEENTH ANNUAL INVITATIONAL MATH MEET

OCTOBER 26, 2000

INDIVIDUAL EXAM QUESTION SHEET

DIRECTIONS: Please write your answers on the Individual Answer Sheet provided. This part of the contest is 30 minutes. Each correct answer to questions 1-4 is worth 1 point, to questions 5-8 is worth 2 points and to questions 9-11 is worth 3 points. Calculators MAY NOT be used.

- 1] A ball is dropped from 300 feet and bounces off level ground. The ball rebounds to $\frac{2}{3}$ of its previous height after each bounce. Find the total distance traveled by the ball when it hits the ground the 3rd time.

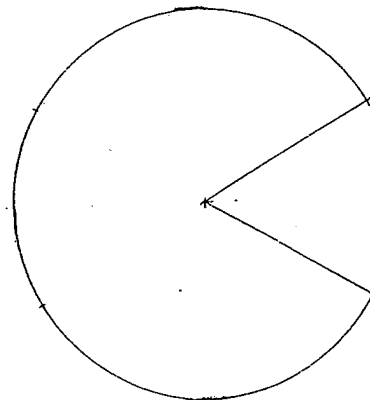
- 2] What is the slope of the line perpendicular to the line satisfying the following equation?

$$\frac{x - 2}{-y + 3} = -\frac{7}{8}$$

- 3] It is known that the perimeter of a semi circular region, measured in centimeters, is numerically equal to its area, measured in cm^2 . What is the radius of the semicircle?

- 4] Given: $f(0) = 3$; $f(n + 1) = 2f(n) + 3$, what is $f(3)$?

- 5] At a given moment, Pac-Man's mouth is open in such a way that the angle formed between his lips is 60 degrees. If one of his lips is 2 cm long, what is his total perimeter in cm? Assume he is circular.



- 6] What is the probability that at least two students in a class of size n , where $0 < n \leq 366$ have the same birthday? Assume 365 days in a year.

- 7] What is

$$\sum_{j=0}^{30} \sum_{k=1}^{100} \left(\frac{1}{k} - \frac{1}{k+1} \right)?$$

- 8] If $\log_9(\log_3(\log_2(x))) = 0$, then $x^{-1/2}$ is equal to what?

- 9] How many different letter arrangements can be made from the word:

COMPETITION ?

- 10] In triangle ABC , point F divides side AC in the ratio 1:2. Let E be the point of intersection of side BC and AG where G is the midpoint of BF . Point E now divides side BC in what ratio? (Please express as $B : C$.)

- 11] In the following picture, BO bisects $\angle CBA$, CO bisects $\angle ACB$, and MN is parallel to BC . If $AB = 12$, $BC = 24$, and $AC = 18$, what is the perimeter of triangle AMN ?

