Question: When a block on a flat, rough surface is connected by pulley to a cart on a decline, does the relationship between the acceleration of the cart and the angle of the incline follow Newton's second law?

Hypothesis: The relationship between acceleration and cosine of the decline angle will be linear.

Strategy:

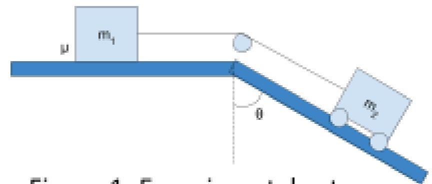

Figure 1: Experimental setup

- The block was dragged at constant velocity with a force sensor on the surface, and then, using the measured value with Newton's Second Law, the kinetic coefficient of friction was calculated. The angle in the experimental setup was varied by stacking textbooks under the declined cart track, and measured using the iPhone Measure app. Acceleration was calculated by taking the slope of the cart's velocity, which was measured by its Vernier system.
- The measured acceleration was graphed vs. $\cos\theta$ to verify a linear relationship with slope $\frac{m_2 g}{m_1+m_2}$ and y-intercept $\frac{\mu m_1 g}{m_1+m_2}$.

Data:

Block mass (m_1): 0.1247 kg

Block kinetic coefficient of friction (μ): 0.770

Cart mass (m_2): 0.3035 kg

Theta (degrees)	Acceleration (m/s^2)
30	3.687
53	1.115
60	0.732
66	0.500
69	0.320

The acceleration is an average of five trials

Analysis:

The free body diagrams in Figure 2 show the forces on the masses in the experimental setup.

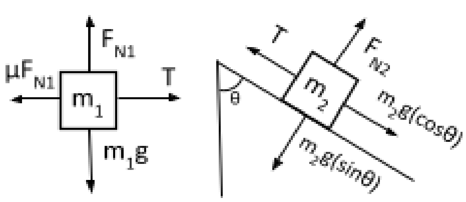

Figure 2: Free Body Diagrams

Friction between the cart and the track is negligible because the cart's wheels spin freely. Positive motion is defined as to the right, so:

$$T - \mu m_1 g = m_1 a \rightarrow T = m_1 a + \mu m_1 g$$

$$m_2 g\cos\theta - T = m_2 a \rightarrow T = m_2 g \cos\theta - m_2 a$$

T can be substituted to get the equation

$$m_1 a + \mu m_1 g = m_2 g\cos\theta - m_2 a$$

$$a(m_1 + m_2) = m_2 g\cos\theta - \mu m_1 g$$

$$a = \frac{m_2 g\cos\theta - \mu m_1 g}{m_1+m_2} = \frac{m_2 g}{m_1+m_2}\cos\theta - \frac{\mu m_1 g}{m_1+m_2}$$

This equation indicates that there should be a linear relationship between the acceleration and the cosine of the angle of decline.

A graph of the data shows it is indeed linear, with a slope of 6.63 and a y-intercept of -2.35.

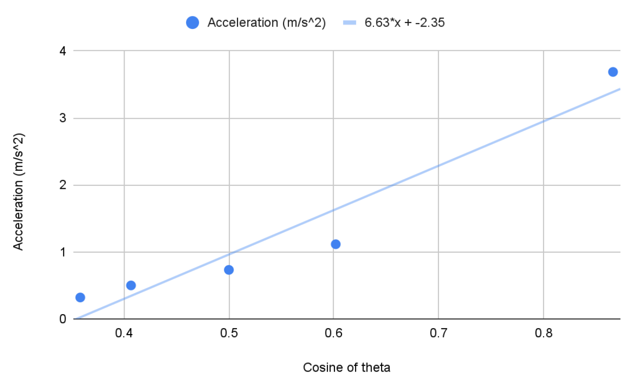

Figure 3: Acceleration vs. cos θ graph

$\frac{m_2 g}{m_1+m_2}$ is actually 6.95, so the slope is 4.55% smaller than expected. $-\frac{\mu m_1 g}{m_1+m_2}$ is -2.20, so the y-intercept is 6.94% smaller than expected. This indicates that acceleration is smaller than expected, which may be due to the unaccounted-for friction on the cart.