Evaluating WIMP Dark Matter Candidacy with Distant Astrophysical Phenomena

Methodology

To determine the annihilation cross section of dark matter, a binned likelihood analysis was performed on XXX extragalactic sources, a minimum of XXX away from the Earth using the FermiPy python package. A procedure similar to that of Ackermann et al. in 2015 was used to extrapolate the 95% upper confidence interval of the annihilation cross-section. The FermiPy package is integrated with the Fermi LAT's ScienceTools package to enable the analysis of photon data pulled from NASA's Fermi database. The data was filtered to photon events collected by the Fermi LAT collected from 2008-08-04 to 2023-01-01 within the energy range of 500 MeV to 500 GeV to limit potential noise or uncertainty in the data due to Earth-based or minute background radiation. This study utilized the P8R3_SOURCE_V3 instrument response functions (IRFs) to analyze the data and filter out low-quality or faulty photon event data. To construct a model for the likelihood analysis, both isotropic and galactic diffuse sources of gamma rays were considered using Fermi LAT's *gll_iem_v06.fit* model, detailing diffuse gamma rays, and *iso_P8R3_SOURCE_V3_v1.txt* to describe the extragalactic gamma-ray point sources.

A binned likelihood analysis was performed on each source using FermiPy version 1.1.0. Photon event data with energies of 500 MeV to 500 GeV within a 10º degree radius of the source through the first two years of the Fermi LAT's operation. Zenith angles less than 90º were chosen to avoid data contamination with Earth or Sun-based gamma-ray events. Using FermiPy's *GTAnalysis* class, a likelihood analysis was performed to normalize the fit of the background gamma-ray model to the data to eliminate possible energy discrepancies that may result in increased uncertainty within the DM data. Photon data within a 10ºx10º region around the ROI with 0.1° spatial binning distribution was separated into 24 logarithmically spaced bins between 500 MeV and 500 GeV. The energy dispersion of gamma-ray sources was also considered within this likelihood analysis. The dark matter source located at the center of the ROI was modeled using a point source following a power law. Studies have shown that galactic dark matter sources typically have a spectral index Γ=2. A second likelihood analysis was conducted to normalize this source.

Likelihood Analysis

The putative contribution of Dark Matter to the total photon flux of a galactic source is

$$\phi_s(\Delta\Omega) = \frac{1}{4\pi}\frac{<\sigma v>}{2m_{DM}^2}\int_{E_{min}}^{E_{max}}\frac{dN_\gamma}{dE_\gamma}dE_\gamma \cdot J,$$

where $< \sigma v >$ is the thermally averaged annihilation cross-section (often referred to simply as the annihilation cross-section), m_{DM} is the mass of the dark matter particle in GEV, $\int_{E_{min}}^{E_{max}} \frac{dN_\gamma}{dE_\gamma}$ is the sum of the differential energy yield within the energy range within the examined WIMP annihilation channels, and J is the J-factor, or the astrophysics factor affecting the distribution of dark matter throughout the source. This study examined the b$\overline{b}$ and $\tau^+\tau^-$ WIMP annihilation channels. This equation was used to construct a likelihood function as a function of $< \sigma v >$ and m_{DM}.

Once the background and unlisted point sources within the ROI have been normalized, a second likelihood analysis is conducted on the dark matter point source. The general LAT likelihood function for the photonic flux within a region given the true photon data, D, is $L(\mu, \theta \mid D)$, and is provided by FermiPy's *SED* class. This function contains the where the signal parameters, μ={$< \sigma v >, m_{DM}$}, and the set of nuisance parameters, θ={α, J}, where α is the fit parameters for the normalized Dark Matter source and J is the J-factor for the calculated J-factor. Using a binned likelihood analysis, the likelihood can be described as a function of the energy bin, j:

$$L(\{\mu_j\}, \{\theta_j\} \mid D) = \prod_j L_j(\mu_j, \theta_j \mid D_j).$$

This likelihood function is repeated for all regions of interest, i, and is subsequently iteratively combined:

$$L(\{\mu_{i,j}\}, \{\theta_{i,j}\} \mid D) = \prod_i L_i(\{\mu_{j_i}\}, \{\theta_{j_i}\} \mid D_i).$$

Following this, the delta log-likelihood technique is utilized to determine the 95% upper confidence interval on $< \sigma v >$. We construct the profile likelihood for these sources as a function of $< \sigma v >$. Research by Rolke et al. indicates that the log-likelihood corresponding to the 95% upper limit of $< \sigma v >$ is 2.71/2 less than that of the maximum log-likelihood (Rolke et al., 2005). Completely simplified, this relationship returns

$$\log\left(\frac{L(\{< \sigma v >_{max}, m_{DM}\}, \{\alpha, J\} \mid D)}{L(\{< \sigma v >_{95\%}, m_{DM}\}, \{\alpha, J\} \mid D)}\right) = 2.71/2.$$

Using Python Jupyter Notebook, the expected binned flux for a single source given set m_{DM} and $< \sigma v >$ values and the Dark Matter Flux equation are compared to that of the Dark Matter point source to yield the source likelihood. This process was repeated for all sources, whose log-likelihoods were added together to achieve the profile likelihood for the given m_{DM} and $< \sigma v >$. Through testing a large array of values for $< \sigma v >$, the delta log-likelihood technique was used to extrapolate the upper limit on the dark matter annihilation cross-section. This process was iterated for all observed masses within the chosen range, returning the relationship between the mass of dark matter and its annihilation cross-section.

A test statistic to evaluate the significance of the dark matter signal within a source was defined as a function of likelihood along the premises of the delta-log likelihood technique:

$$T = 2\log\left(\frac{L(\mu_0, \hat{\theta} \mid D)}{L(\hat{\mu}, \hat{\theta} \mid D)}\right),$$

where μ_0 is the DM-null signal parameters, and $\hat{\mu}$ and $\hat{\theta}$ are the signal and nuisance parameters that maximize the likelihood function.

To analyze the significance of the dark matter signal within the observed sources, the annihilation cross-section was compared to data collected from XXX background-only dark matter simulations. These simulations were modeled from high galactic latitudes ($|b| > 30°$) XXX° degrees away from inspected sources and XXX° away from cataloged gamma-ray point sources. Likelihood analysis was conducted on each of these sources, with the annihilation cross section obtained as the null hypothesis for $< \sigma v >$.

J-Factor Distribution

The mathematical formula for the J-factor for a given galactic source is the line-of-sight integral of the dark matter profile density, $\rho(r)$ squared over the compared to the solid-state angle, $\Delta\Omega$:

$$J = \int_{\Delta\Omega} \int_{l.o.s.} \rho^2(r) dl \, d\Omega'.$$

As stated previously, several dark matter profile equations have been defined, and are listed in Table 1, where ρ_0 and r_s are equivalent to the scale radius and characteristic density of the source. These quantities were derived using observations of the dark matter distribution within the Milky Way galaxy, with $\rho_0 = XXX$ and $r_s = XXX$. In this study, J-factors were calculated over an angular radius of 0.5° using the NFW and Burket profiles for analysis.