Question: Does the relationship between incline angle, force, and acceleration of a cart traveling upwards along a metal track of a modified Atwood's machine—with one side elevated with a hanging mass—have a sinusoidal incline angle-acceleration graph, and thus obey Newton's Second and Third Laws?

Hypothesis: Given constant object mass, as the incline of a modified Atwood's machine increases, the upwards acceleration of a cart on a metal track and tied to a hanging mass will decrease. The relationship between acceleration and incline angle will follow a sinusoidal function of the equation $a(\theta) = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$, where g is the gravitational constant, and m_1 and m_2 are the masses of the cart and hanging weight respectfully.

Strategy:

- Identical textbooks were used to elevate a side of the Atwood's machine. To increase or decrease the incline angle, textbooks were added or removed respectively.
- Incline angle was evaluated by taking the inverse sine of the height of the track's elevated side over the track's length ($\sin^{-1}(\frac{height}{length})$).
- One end of a string was attached to the cart and the other end was connected to a hanging weight via a paperclip. The mass of both objects remained consistent throughout testing.
- The hanging weight was supported from underneath to prevent any slack in the string and such that removing the support from the weight would result in the system's immediate acceleration governed by the force of gravity. The resulting acceleration was measured using a Vernier motion detector.
- The incline angle was graphed vs. the measured acceleration to verify that relationship followed the $a(\theta)$ equation described in the hypothesis section.

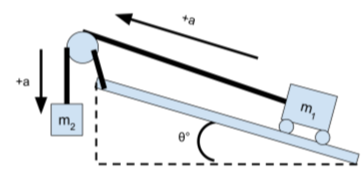

Fig 1: Inclined Modified Atwood's Machine

Data:

- Hanging mass: 0.1999 kg
- Cart mass: 0.5018 kg
- Track length: 2.28 m

# of Books	Incline Height (m)	Incline Angle (°)	Acceleration (m/s²)
0	0	0	2.729
3	0.14	3.528	2.341
6	0.273	6.886	1.956
9	0.389	9.834	1.624
12	0.514	13.032	1.194

The accelerations listed are an average of three trials

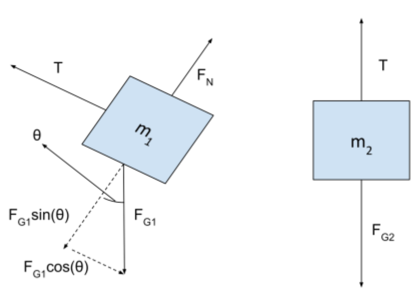

Fig 2: Free Body Diagrams

Analysis:

The free body diagrams in Figure 2 show the forces on the two masses in the modified Atwood's machine.

Friction between the cart and the track is negligible because the cart's wheels spin freely. The mass of the string is also considered negligible. The following equations are based on the free body diagrams. Note that ∥ denotes the direction parallel to the track's incline. Positive motion is defined as upwards along the incline for the cart, and directly downwards for the hanging mass.

$$F_{net_1} = F_{\parallel_1}, T - F_{g_1}\sin\theta = m_1 a$$
$$F_{net_2} = F_{y_2}, F_{g_2} - T = m_2 a$$

These equations can be combined to form the equation:

$$m_2 g - m_1 g \sin\theta = (m_1 + m_2)a$$
$$a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$$

This combined equation indicates that there is a relationship between the acceleration of the cart and the incline of the metal track.

A graph of the incline angle vs. the acceleration data for this experiment shows that this relationship is governed by the combined equation on the previous page.

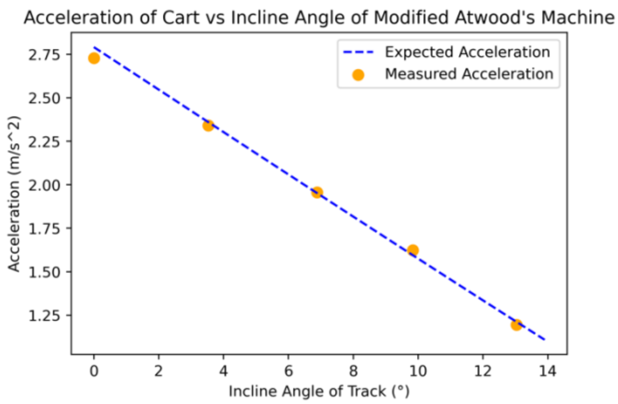

Figure 3: Acceleration vs Incline Angle Graph

When the measured accelerations are compared to the expected accelerations for each angle measurement, a percent error of 1.314% (or -0.508% when taking the sign of the acceleration into account). This indicates that the experimental data effectively supports this experiment's hypothesis. The slightly negative error of the signed percent error signifies that the accelerations yielded in the experiment are on average smaller than expected. The most likely source for this discrepancy is friction from the wheels of the cart, as any friction would reduce acceleration.