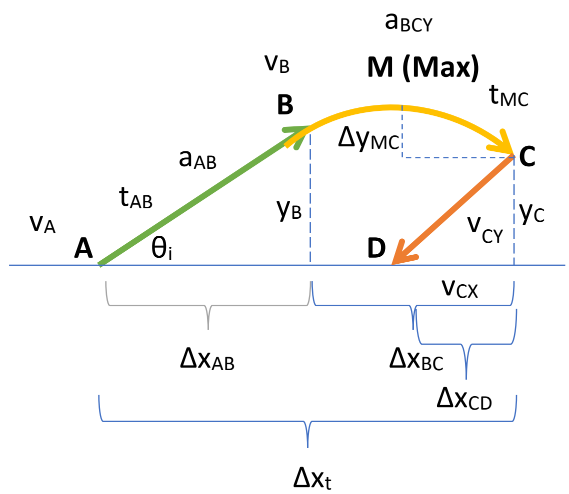
David Zhukovsky 10/3/2021 Section Q

Description:
One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Diagram:



Givens:

$\theta_i = 41°$ | $\Delta y_{MC} = 62\text{m}$ | $v_{CDX} = -17\text{m/s}$

$t_{AB} = 6.1\text{s}$

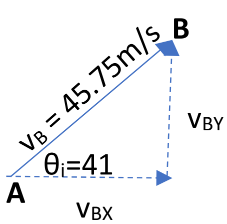
$a_{AB} = 7.5\text{m/s}^2$ | $v_{CDY} = 11\text{m/s}$ | $v_A = 0\text{m/s}$

Work:

Part A Step 1: Determine v_B using givens a_{AB}, Δt_{AB}, and v_A.

$$v_B = a_{AB}\Delta t_{AB} + v_A$$
$$v_B = 7.5 * 6.1$$
$$\underline{v_B = 45.75\text{m/s}}$$

Part A Step 2: Draw a vector triangle to determine v_{BX} and v_{BY} using θ_i and v_B. These will be substituted into equations and then put into decimal form



$$\underline{V_{BX} = 45.75\cos(41°)\ \text{m/s}}$$
$$\underline{V_{BY} = 45.75\sin(41°)\ \text{m/s}}$$

Part A Step 3: Use givens v_{AX}, v_{BX}, and Δt_{AB} to determine Δx_{AB} and x_B. This step is in x-dir. We know that $\Delta x_{AB} = x_B$ because $x_A = 0$m.

$$\Delta x_{AB} = \frac{1}{2}(v_{AX} + v_{BX})\Delta t_{AB}$$
$$\Delta x_{AB} = \frac{1}{2}(0 + 45.75\cos(41°)) * 6.1$$
$$\Delta x_{AB} = 105.31\text{m} = x_B$$
$$\Delta x_{AB} = x_B$$
$$\underline{x_B = 105.31\text{m}}$$

Part A Step 4: Use givens v_{AY}, v_{BY}, and Δt_{AB} to determine Δy_{AB} and y_B. This step is in y-dir. We know that $\Delta y_{AB} = y_B$ because $y_A = 0$m.

$$\Delta y_{AB} = \frac{1}{2}(v_{AY} + v_{BY})\Delta t_{AB}$$
$$\Delta y_{AB} = \frac{1}{2}(0 + 45.75\sin(41°)6.1$$
$$\Delta y_{AB} = 91.545\text{m} = y_B$$
$$\Delta y_{AB} = y_B$$
$$\underline{y_B = 91.545\text{m}}$$

Part B Step 1: Determine t_{BM} using y_M, a_{BC}, v_{BY}, and y_B . Here Max is signified as "M". This step is in y-dir.

$$y_M = \frac{1}{2}a_{BC}t_{BM}^2 + v_{BY}t_{BM} + y_B$$
$$y = \frac{1}{2}(-9.8)t_{BM}^2 + 45.75\sin(41°)\,t_{BM} + 91.545$$
$$Y_M = -4.9t_{BM}^2 + 30.015t_{BM} + 91.545$$
$$t_{BM} = -\frac{b}{2a} = \frac{-30.015}{-9.8}$$
$$\underline{t_{BM} = 3.0627\text{s}}$$

Part B Step 2: Plug t_{BM} back into our y_M equation to determine y_M.

$$y_M = \frac{1}{2}a_{BC}t_{BM}^2 + v_{BY}t_{BM} + y_B$$
$$y = \frac{1}{2}(-9.8)t_{BM}^2 + 45.75\sin(41°)\,t_{BM} + 91.545$$
$$y_M = -4.9 * 3.0627^2 + 30.015 * 3.0627 + 91.545$$
$$\underline{y_M = 137.508\text{m}}$$

Part B Step 3: Determine y_C using givens y_{MC}

$$\Delta y_{MC} = y_C - y_M$$
$$-62 = y_C - 137.508$$
$$y_C = 137.508 - 62$$
$$\underline{y_C = 75.508\text{m}}$$

Part B Step 4: Find t_{MC} using a_{MC}, Δy_{MC}, and v_m.

$$\Delta y_{MC} = \frac{1}{2}a_{MC}t_{MC}^2 + v_M t_{MC}$$
$$-62 = \frac{1}{2}(-9.8)t_{MC}^2 + 0t_{MC}$$
$$\frac{-62}{-4.9} = t_{MC}^2$$
$$\underline{t_{MC} = 3.5571\text{s}}$$

Part B Step 5: Find t_{BC} using t_{BM} and t_{MC}.

$$t_{BC} = t_{BM} + t_{MC}$$
$$t_{BC} = 3.5571 + 3.0627$$
$$\underline{t_{BC} = 6.6198\text{s}}$$

Part B Step 6: Find x_C using a_{BCX}, t_{BC}, v_{BX}, and x_B. This step is in x-dir.

$$x_C = \frac{1}{2}a_{BCX}t_{BC}^2 + v_{BX}t_{BC} + x_B$$
$$x_C = 0 + 45.75\cos(41°) * 6.6198 + 105.31$$
$$x_C = 0 + 34.528 * 6.6198 + 105.31$$
$$\underline{x_C = 333.879\text{m}}$$

Part C Step 1: Determine Δt_{CD} using v_{CY} and Δt_{CD}. We know that $v_{CY} = v_{DY}$ because the velocity in this leg of the trip is constant.

$$\Delta y_C = \frac{1}{2}(v_{CY} + v_{DY})\Delta t_{CD}$$
$$75.508 = \frac{1}{2}(11 + 11)\Delta t_{CD}$$
$$\Delta t_{CD} = \frac{75.508}{11}$$
$$\underline{\Delta t_{CD} = 6.8644\text{s}}$$

Part C Step 2: Determine ΔX_{CD} using v_{CX} and Δt_{CD}. We know that $v_{CX} = v_{DX}$ because the velocity in this leg of the trip is constant.

$$\Delta x_{CD} = \frac{1}{2}(v_{CX} + v_{DX})\Delta t_{CD}$$
$$\Delta x_{CD} = \frac{1}{2}(-17 - 17) * 6.8644$$
$$\underline{\Delta x_{CD} = -116.695\text{m}}$$

Part C Step 3: Find Δx_t using Δx_{CD} and x_C.

$$\Delta x_t = x_C + \Delta x_{CD}$$
$$\Delta x_t = 333.879 - 116.695$$

$$\boxed{\Delta x_t = 217.2\text{m East}}$$