Question: How does a difference in angles of planes affect the acceleration of 2 equal masses?
Hypothesis: The relationship between differences of angles and acceleration will be linear.
Strategy: Two tracks were leveled and set next to each other at 2 degrees. Two identical carts, which were attached by a string, were placed on either side of the track with a pulley in between the tracks. It was observed that the carts were not moving at this stage when there was no initial velocity, confirming that the tracks were level and had the same weight. Next, textbooks were added below one of the tracks to increase the angle and to examine how the changes in angles affected acceleration. Each combination of θ_1 and θ_2 was tested 3 times to produce the average acceleration (see Figure 1). Once the data was obtained, it was graphed in Excel (see Figure 3), and the expected results were calculated with an equation solving for a, which was derived from the formula $F_{net} = ma$. Percent error was then calculated using the formula $\frac{value_{observed} - value_{expected}}{value_{expected}} \times 100$.

Data:

Theta 1	Theta 2	Avg Acceleration	Theoretical Acceleration	% Error
3	2	0.15807	0.08544	85.01
5	2	0.31783	0.25606	24.13
7	2	0.47233	0.42615	10.84
11	2	0.63590	0.76396	-16.76
9	7	0.15607	0.16937	-7.85
15	13	0.16533	0.16595	-0.37
2	3	0.15330	0.08544	79.43
2	5	0.31090	0.25606	21.42
2	7	0.48413	0.42615	13.61
2	11	0.64950	0.76396	-14.98

Figure 1: Data from experiment

Analysis:
The free body diagrams in the following figure depict the forces on the masses in the modified Atwood's machine.

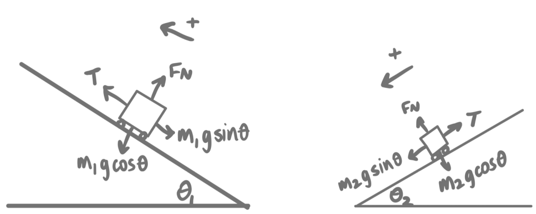


Figure 2: Free body diagrams

Assuming friction between the cart and the track is negligible and the masses are equal ($m_1 = m_2$), the following equations are based on the free body diagrams. Positive motion is described as up the incline for m_1 and down the incline for m_2.

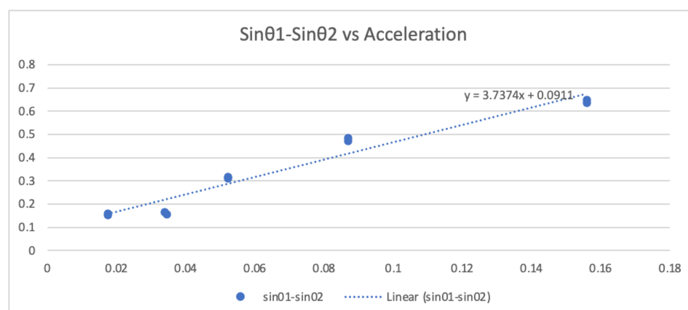
$$F_{net} = ma$$
$$T - mg\sin\theta_1 = ma$$
$$mg\sin\theta_2 - T = ma$$
$$mg\sin\theta_2 - mg\sin\theta_1 = 2ma$$

These equations can be combined and simplified to the following equation:

$$\frac{g}{2} \times (\sin\theta_1 - \sin\theta_2) = a$$

This equation indicates that a constant is multiplied by a variable, or x. This means that if the acceleration is graphed on the y-axis against $(\sin\theta_1 - \sin\theta_2)$ on the x-axis then the slope would be $\frac{g}{2}$, or 4.9.

Figure 3: $\sin\theta_1 - \sin\theta_2$ vs a graph



The actual slope of the graph is 3.7374, which means that the acceleration found from the angles is 23.73% smaller than expected. The fact that it is too small indicates friction in the wheels of the cart because any friction would reduce the acceleration. In addition, the accuracy of acceleration to expected acceleration was more accurate when larger angles were used, meaning that there was also error in the measurement of small angles. The angles were only able to be measured to the nearest degree which provides another level of uncertainty to the data collection and analysis.