

### 1. Problem Statement:

A 72-gram puck starts at rest at 2.9 meters up a ramp, creating an angle of 38 degrees above the horizontal. The coefficient of friction between the puck and the ramp is 0.17, and the bottom of the ramp is at the top of a counter that is 1.6 meters above the ground. Using this information, calculate the horizontal distance that the puck travels from the base of the counter. Additionally, by utilizing either a spreadsheet or a computer program, find the angle that maximizes the horizontal distance that the puck will travel from the base of the counter.

2. Process: Describe what you did in attempting to solve the problem, using your notes as a reminder. Include things that didn't work out or that seemed like a waste of time. Do this part of the write-up even if you didn't solve the problem. If you get assistance of any kind on the problem, you should indicate what the assistance was and how it helped you.

We started our approach to the problem by examining the forces acting upon the puck. This led to determining the directions of the forces as well as the components. We then used this to provide ourselves with a free body diagram (FBD). We used these forces to come up with several equations that would help us solve the acceleration on the ramp which could be used to find its velocity and the end of the ramp. This velocity –the velocity it has leaving the ramp- can be used to help us find the horizontal distance traveled by using the equations for projectile motion.

While developing our code to find the optimal angle, we ran into some syntax errors. We had to work to figure out where our code went wrong and adjust accordingly. After several iterations of code, we were able to get the code functioning, however, it output an integer value. We wanted a more specific value, so rather than running the code through just integer values, we ran it through every hundredth decimal place i.e. 0.01, 0.02, 0.03, and so on.

### 3. Solution:

We started by drawing a FBD of all the forces acting on the puck. We then made the following equations:

$$F_y = F_N - mg \cos \theta = 0 \text{ (acceleration perpendicular to counter equals 0)}$$

$$F_x = mg \sin \theta - F_f = ma$$

Since  $F_N - mg \cos \theta = 0$ ,  $F_N$  is equal to  $mg \cos \theta$ .

To Find  $F_f$ , you can set it equal to  $\mu * F_N$  or  $\mu * mg \cos \theta = F_f$ . We can plug this equation into our equation for  $F_x$  (F Net) rather than  $F_f$ . Since we know these values, we can solve for acceleration by rearranging the net force equation to:

$$a = (mg \sin \theta - \mu * mg \cos \theta) / m$$

The 'm' cancels out leaving you with  $a = g \sin \theta - \mu * g \cos \theta$

We then plugged in our values for  $\theta$  and  $\mu$  which got us an answer of  $a = 4.721 \text{ m/s}^2$ .

The next step was to find the velocity at the end of the ramp, which was done using the formula  $v^2 = v_0^2 + 2a\Delta x$ . This gave us  $v = 5.233 \text{ m/s}$  at the end of the ramp. We then separated this into x and y velocities, by multiplying the total velocity by cos and sin of  $\theta$  respectively. We then use the y velocity to find the time it spends in the air by plugging it into the formula  $0 = -4.9t^2 - v_y + 1.6$ ,  $v_y = 3.221$ . This gives us a value of  $t = .331 \text{ s}$ , which means that the distance away from the ramp is  $v_x = 4.123$ ,  $t * v_x = .331 * 4.123 = 1.363 \text{ m}$ . This is the distance away from the ramp that it lands on when the angle is 38 degrees.

To find the optimal angle, we wrote a Python script which checks every value to two decimal places for angles.

```

import math
ang=0
m=0
for i in range(1, 8900):
    v=0
    expression = 5.8 * (9.8 * math.sin(math.radians(i/100)) - (0.17 * 9.8 * math.cos(math.radians(i/100))))
    if expression >= 0:
        v = math.sqrt(expression)
        vx=v*math.cos(math.radians(i/100))
        vy=v*math.sin(math.radians(i/100))
        def solve_quadratic(a, b, c):
            d = b**2 - 4 * a * c

            if d > 0:
                root1 = (-b + math.sqrt(discriminant)) / (2 * a)
                root2 = (-b - math.sqrt(discriminant)) / (2 * a)
                return root1

            elif d == 0:
                root1 = -b / (2 * a)
                return root1

            else:
                return 0
        root1=solve_quadratic(4.9, vy, -1.6)
        if vx*root1>m:
            m=vx*root1
            ang=i
print(ang/100, m)

```

26.65 1.5178430764124304

The code gave us an output angle of 26.65 degrees, which gave us 1.519 meters. This means that the angle that makes the block fly the furthest is 26.65 degrees.

4. Extensions: Invent some extensions or variations to the problem. That is, write down some related problems. They can be easier, harder, or about the same level of difficulty as the original problem. (You don't have to solve these additional problems.)

Some extensions could be:

1. Having the ramp incline be up rather than down and calculating the distance the puck travels given this situation. The puck would be given an initial velocity, and from there, similar steps could be taken.
2. Getting rid of the height between the ramp and the base of the counter. This would change the values of some equations, which would change the results.
3. Having the puck starts with a force that is already moving it. This would change some of equations, as some more work would be needed to determine the acceleration.

