exercise 1:
(i). Let f be a simple function from the measure space $(X, \mathcal{A}, \mu)$ to $[0, \infty]$. Define $\nu(A) = \int_A f$, for all A in $\mathcal{A}$. Show (by direct calculations) that ν is a measure on $(X, \mathcal{A})$.
(ii). Using the M.C.T. show that the result is still true if f is measurable and valued in $[0, \infty]$.

exercise 2:
(i). Compute $\int \liminf f_n$ and $\liminf \int f_n$ for the sequence of functions from $\mathbb{R}$ to $\mathbb{R}$ $f_n = 1_{[n,n+1]}$.
(ii). Find a sequence of measurable functions f_n from $[0, 1]$ to $[0, \infty]$ such that

$$\int \liminf f_n \neq \liminf \int f_n$$

(iii). Find a sequence of measurable functions f_n from $\mathbb{R}$ to $[0, \infty]$ such that f_n converges uniformly to zero and

$$\int \liminf f_n \neq \liminf \int f_n$$

exercise 3:
Let f be a measurable function from the measure space $(X, \mathcal{A}, \mu)$ to $[0, \infty]$. Show that

$$\sup\{\int s : s \text{ is simple and } 0 \leq s \leq f\}$$
$$= \sup\{\int s : s \text{ is simple, valued in } [0, \infty), \text{ and } 0 \leq s \leq f\}.$$

exercise 4:
From Royden, 3.1.1.

exercise 5:
From Royden, 3.1.20: just use Fatou's lemma to show that the poperty holds.

exercise 6:
From Royden, 3.1.22.

exercise 7:
Find a function $f : (0,\infty) \times [0,1] \to \mathbb{R}$ such that $x \to f(x,t)$ is in $L^1((0,\infty))$ for all t in $[0,1]$, $\frac{\partial f}{\partial t}(x,t)$ exists for all $x > 0$ and t in $[0,1]$, and $x \to \frac{\partial f}{\partial t}(x,t)$ is in $L^1((0,\infty))$ for all t in $[0,1]$, and setting $F(t) = \int_0^\infty f(x,t)dx$, F fails to be differentiable at some point in $[0,1]$.
Hint: $t^a e^{-tx}$, for some adequate value of a.

exercise 8:
Find (with proof)

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} \frac{(\sin x)^n}{x^2}\, dx$$

.

exercise 9:
Let g be in $L^1(\mathbb{R})$. Set

$$G(y) = \int_{-\infty}^{\infty} \cos(xy)g(x)dx,$$

for y in $\mathbb{R}$. Show that G is continuous.

exercise 10:
Let $(X, \mathcal{A}, \mu)$ be a measure space. We proved in class that $L^1(X)$ is complete. Show that $L^2(X)$ is complete.

exercise 11:
Let V be an open subset of $\mathbb{R}$ and f be in $C(V) \cap L^\infty(V)$. Show that $\|f\|_\infty = \sup_V |f|$, where $\| \|_\infty$ is the norm on L^∞ defined in class.