<u>exercise 1</u>:

For $t > 0$, define the parametric curve C by $x(t) = t - \ln t$, $y(t) = t^{\frac{1}{2}} + \cos(\frac{\pi}{2}t)$.

(i). Find an equation of the tangent line through $P_0 = (1, 1)$.

(ii). Show that x is a function of y on C near P_0.

<u>exercise 2</u>:

Let $\mathcal{C}$ be the curve defined by $x = e^t + e^{-t}$, $y = e^t - e^{-t}$, $-1 \leq t \leq 1$.

(i). Show that $\mathcal{C}$ is a piece of hyperbola and sketch it.

(ii). Compute $\dfrac{d^2x}{dy^2}$ as a function of t and infer the concavity of $\mathcal{C}$.

<u>exercise 3</u>:

Define the parametric curve $x(t) = t^2 + 1, y(t) = t - 1$, $1 \leq t$.

(i). Show that it defines a portion of parabola. Sketch that portion.

(ii). Find an equation of the tangent line at $t = 1$.

<u>exercise 4</u>:

Find the arclength of $\mathcal{C}$ defined in exercise 2. Leave your final answer in integral form.

<u>exercise 5</u>:

The goal of this exercise is to show that the arclength is invariant under rotations.
For the vector $u = (u_1, u_2)$ we define the vector
$R_\theta u = (\cos\theta\, u_1 - \sin\theta\, u_2, \sin\theta\, u_1 + \cos\theta\, u_2)$. Let $e_1 = (1, 0)$, $e_2 = (0, 1)$.

(i). For $\theta = \dfrac{\pi}{2}$, find and sketch $R_\theta e_1$ and $R_\theta e_2$.

(ii). For any two vectors u, v in $\mathbb{R}^2$, show that $u \cdot v = R_\theta u \cdot R_\theta v$.

(iii). Let $\mathcal{C}$ be the parametric curve defined by $(x(t), y(t))$, $t \in [a, b]$ where x and y are C^1 functions of t. Let $\mathcal{C}_\theta$ be the parametric curve defined by $(R_\theta x(t), R_\theta y(t))$, $t \in [a, b]$. Show that $\mathcal{C}$ and $\mathcal{C}_\theta$ have the same arc length.

<u>exercise 6</u>:

(i). Sketch the parametric curve $x(t) = \cos t$, $y(t) = 2\sin t$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

(ii). Find the arclength of that curve. Leave your answer in integral form.

(iii). Find the area of the region between that curve and the x-axis.