exercise 1:

Find the interval of convergence for the following power series.

(i). $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$

(ii). $\sum_{n=1}^{\infty} \frac{2^n x^{2n}}{n^2}$

(iii). $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

(iv). $\sum_{n=1}^{\infty} \frac{n! x^n}{10^n}$

(v). $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$

exercise 2:

Textbook problem: B.4.8.

Additional question: show that if $x \notin (-1, 1)$, this power series diverges.

exercise 3:

Optional. Textbook problem: B.4.15.

exercise 4:

For $x > 0$, find $\sum_{n=0}^{\infty} (-1)^n \frac{x^{\frac{n}{2}}}{n!}$.

exercise 5:

(i). Find the radius of convergence of $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and of $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

(ii). Defining $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, show that $\cos' = -\sin$ and $\sin' = \cos$.

(iii). Show that $\cos^2 x + \sin^2 x = 1$, for all x in $\mathbb{R}$. Hint: use derivatives.

exercise 6:

Starting from the formula $\sum_{n=0}^{\infty} (-1)^n x^n = \frac{1}{1+x}$, $-1 < x < 1$, integrate twice to find $(1+x)\ln(1+x) - x$ as the sum of a power series.