

exercise 1:

Let q be in $(0, 1)$. Show that the sequence $a_n = n^2 q^n$ is eventually decreasing.

exercise 2:

Let $a_1 = \frac{3}{2}$, $a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}$.

(i). Show by induction that $\sqrt{2} \leq a_n \leq \frac{3}{2}$, for all $n \geq 1$. **Hint:** First verify that the function $f : [\sqrt{2}, \frac{3}{2}] \rightarrow \mathbb{R}$, $f(x) = \frac{x}{2} + \frac{1}{x}$ is increasing.

(ii). Show by induction that a_n is decreasing.

exercise 3:

Show from the definition of divergence to infinity that the sequence $a_n = n^2$ satisfies $\lim a_n = \infty$.

exercise 4:

If a_n is a sequence such that $\lim a_n = \infty$ and b_n is bounded, show that $\lim a_n + b_n = \infty$.

exercise 5:

True or false: if c_n converges to zero and $c_n \neq 0$ for all n then $\frac{1}{c_n}$ diverges to infinity or minus infinity.

exercise 6:

Textbook problem. A.1.1: o, p. For o, first show that na_n is eventually decreasing.

exercise 7:

B.1.9.

exercise 8:

B.1.10.

exercise 9:

Find $\lim_{x \rightarrow \infty} \frac{x^2 + x^2 \cos x}{x^3 + x \sin x}$.

exercise 10:

Show that $x \cos x$ has no limit as $x \rightarrow \infty$.