exercise 1:
(i). Find a finite dimensional space V and A in $\mathcal{L}(V)$ such that V is not the direct sum of Ker A and Im A.
(ii). Is it possible to find such an example with the additional requirement that
Ker $A \cap$ Im A $= \{0\}$?

exercise 2:
Let $V = C^\infty(0,1)$, and T in $\mathcal{L}(V)$ defined by $Tf = f'$. Find all eigenvalues of T and corresponding eigenvectors.

exercise 3:
Let R be a unitary matrix in $\mathbb{R}^{2\times 2}$ such that $\det R = 1$. Show that there is a θ in $[0, 2\pi]$ such that

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

exercise 4:
Let V, W be two finite dimensional Hermitian spaces and A in $\mathcal{L}(V, W)$.
(i). Show that A and A^* have the same rank.
(ii). If $V = W$ do A and A^* have the same image?

exercise 5:
Let V be a vector space over $\mathbb{C}$ such that $\dim V = 2$ and T in $\mathcal{L}(V)$. Show that T is non-diagonalizable if and only if the characteristic polynomial of T has only one root and T is not a multiple of the identity.

exercise 6:
Let V be a vector space over $\mathbb{R}$ such that $\dim V = 2p$ where p is a positive integer. Show that there is a T in $\mathcal{L}(V)$ such that T has no eigenvalue.

exercise 7:
Let V be an n-dimensional vector space over K. Find a T in $\mathcal{L}(V)$ that is not diagonalizable.