exercise 1:

Let V be a vector space over K. Using the definition of vectors spaces show that for all v in V, $(-1)v = -v$.

exercise 2:

Let W_1 and W_2 be subspaces of the vector space V. The set $W_1 + W_2$ is by definition equal to $\{u + v : u \in W_1,\ v \in W_2\}$.
(i). Show that $W_1 \cap W_2$ and $W_1 + W_2$ are subspaces of V.
(ii). Show that $W_1 \cup W_2$ is a subspace if and only if $W_1 \subset W_2$ or $W_2 \subset W_1$.

exercise 3:

(i). 1.2.5: c, d, f, h.
(ii). Let $a_1, ..., a_n$ be n distinct real numbers and $f_i(t) = e^{a_i t}$. Show that the functions $f_1, ..., f_n$ are independent.

exercise 4:

Let $P_1, ..., P_n$ be n polynomials in $K[X]$ such that P_i is not the zero polynomial for $1 \leq i \leq n$ and if $1 \leq i \neq j \leq n$, $\deg P_i \neq \deg P_j$. Show that these n polynomials are linearly independent.

exercise 5:

Let $v_1, ..., v_p$ be p independent vectors in a vector space V and w in V such that $w \notin \text{Span}\ \{v_1, ..., v_p\}$. Show that $\{v_1, ..., v_p, w\}$ is independent.

exercise 6:

Let $a_0, ..., a_n$ be $n + 1$ distinct scalars in K. Set

$$Q = (X - a_0)...(X - a_n),$$

for $0 \leq i \leq n$, $P_i = Q/(X - a_i)$.
(i). Show that $P_0, ..., P_n$ are linearly independent.
(ii). Let $K_n[X]$ be the subspace of polynomials with degree less or equal to n. Show that $P_0, ..., P_n$ is a basis of $K_n[X]$.

exercise 7:

Let A be in $K^{m \times n}$ with $m < n$. Use a result from the first two lectures to show that $\exists x \neq 0 \in K^n$ such that $Ax = 0$.

exercise 8:

True or False:
Let V_1, V_2, V_3 be subspaces of a vector space V. If $V_i \cap V_j = \{0\}$ for $1 \leq i < j \leq 3$ then the sum $V_1 + V_2 + V_3$ is direct.

exercise 9:

True or false: there is a dimension $n \geq 2$ such that if A in $K^{n \times n}$ has only zeros on its diagonal, then A is singular.