

exercise 1:

Let S be the solid in \mathbb{R}^3 bounded by the cone with equation $z = \sqrt{3(x^2 + y^2)}$ and the hemisphere with equation $z = \sqrt{4 - x^2 - y^2}$. Compute the volume of S by switching to spherical coordinates.

exercise 2:

With S defined as in exercise 1, compute the volume of S by switching to cylindrical coordinates.

Hint: you may want to write S as the union of two solids S_1 and S_2 and write the volume of S as the sum of two integrals.

exercise 3:

Find the center of mass S from exercise 1, assuming that it is homogeneous. Use coordinates of your choice.