

exercise 1:

2.2.A: a, b,d.

exercise 2:

2.6.A

exercise 3:

For  $t > 0$ , define the parametric curve  $C$  by  $x(t) = t - \ln t$ ,  $y(t) = t^{\frac{1}{2}} + \cos(\frac{\pi}{2}t)$ .

- (i). Find an equation of the tangent line through  $P_0 = (1, 1)$ .
- (ii). Show that  $x$  is a function of  $y$  on  $C$  near  $P_0$ .

exercise 4:

Let  $\mathcal{C}$  be the curve defined by  $x = e^t + e^{-t}$ ,  $y = e^t - e^{-t}$ ,  $-1 \leq t \leq 1$ .

- (i). Show that  $\mathcal{C}$  is a piece of hyperbola and sketch it.
- (ii). Compute  $\frac{d^2x}{dy^2}$  as a function of  $t$  and infer the concavity of  $\mathcal{C}$ .

exercise 5:

Find the arclength of  $\mathcal{C}$  defined in exercise 4. Leave your final answer in integral form.

exercise 6:

Compute the curvature radius at each point of the parabola with equation  $y = x^2$ . Does this radius achieve a minimum value or a maximum value?

exercise 7:

For the vector  $u = (u_1, u_2)$  we define the vector

$R_\theta u = (\cos \theta u_1 - \sin \theta u_2, \sin \theta u_1 + \cos \theta u_2)$ . Let  $e_1 = (1, 0)$ ,  $e_2 = (0, 1)$ .

- (i). For  $\theta = \frac{\pi}{2}$ , find and sketch  $R_\theta e_1$  and  $R_\theta e_2$ .
- (ii). For any two vectors  $u, v$  in  $\mathbb{R}^2$ , show that  $u \cdot v = R_\theta u \cdot R_\theta v$ .
- (iii). Let  $\mathcal{C}$  be the parametric curve defined by  $(x(t), y(t))$ ,  $t \in [a, b]$  where  $x$  and  $y$  are  $C^1$  functions of  $t$ . Let  $\mathcal{C}_\theta$  be the parametric curve defined by  $(R_\theta x(t), R_\theta y(t))$ ,  $t \in [a, b]$ . Show that  $\mathcal{C}$  and  $\mathcal{C}_\theta$  have the same arc length.