exercise 1:
Let (V, φ) be a vector space over K where φ is bilinear, symmetric, and non-degenerate form. Let A, B be in $\mathcal{L}(V)$ and s be in $\mathbb{R}$. Show that:
(i). $(A+B)^* = A^* + B^*$.
(ii). $(sA)^* = sA^*$.
(iii). $I^* = I$.
(iv). $(AB)^* = B^*A^*$.

exercise 2:
Let V be a vector space over $\mathbb{R}$ and $<,>$ a positive definite scalar product on V. Let A, B be in $\mathcal{L}(V)$. If A and B are symmetric is AB symmetric?

exercise 3:
5.6.5.

exercise 4:
Let R be a unitary matrix in $\mathbb{R}^{2\times 2}$ such that $\det R = 1$. Show that there is a θ in $[0, 2\pi]$ such that

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

exercise 5:
Let V be a finite-dimensional vector space and P in $\mathcal{L}(V)$. Show that $P$is a projection if and only if there exists a basis of V where the matrix of P is $I_{r,n}$ for some r in $\{0, .., n\}$. Here, $I_{r,n}$ is the diagonal matrix with r ones followed by zeros on the diagonal.

exercise 6:
Let $(V, <,>)$ be a finite-dimensional scalar product space where $<,>$ is positive definite. Let W be a subspace of V, and P the orthogonal projection on W (this is equivalent to saying that $P \in \mathcal{L}(V)$, the restriction of P to W is the identity and the the restriction of P to $W^\perp$ is zero). Show that for all x in V,

$$\inf_{y\in W} \|x - y\| = \|x - Px\|.$$

exercise 7:
Let $(V, <,>)$ be a finite-dimensional Hermitian space where $<,>$ is positive definite. Let A in $\mathcal{L}(V)$ be a normal operator. Show that $\mathrm{Im}\ A = \mathrm{Im}\ A^*$.

exercise 8:
7.1.10.

exercise 9:
7.2.7. Additional question: is that still true without the assumption $AB = BA$?