exercise 1:
Let n be in $\mathbb{N}$ and p, q in $\mathbb{N}$ greater or equal than 1 such that $p + q = n$.
(i). Define the matrix N in $K^{n\times n}$ by blocks

$$N = \begin{pmatrix} I_p & D \\ 0 & I_q \end{pmatrix}.$$

Find $\det N$.
Now let M be in $K^{n\times n}$

$$M = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix},$$

where $A \in K^{p\times p}$, $B \in K^{q\times q}$.
(ii). If A or B is singular, show that $\det M = 0$.
(iii). Show that $\det M = \det A \det B$. **Hint:** Use matrix block multiplication.

exercise 2:
Let A be in $\mathbb{R}^{n\times n}$ and define $\varphi(x, y) = x^T Ay$ for x, y in $\mathbb{R}^n$.
(i). Show that φ is symmetric if and only if A is symmetric.
(ii). Assume that φ is symmetric and non-negative, that is, $\varphi(x, x) \geq 0$ for all x in $\mathbb{R}^n$. Show that φ is definite if and only if A is invertible.

exercise 3:
Let V be a vector space over $\mathbb{R}$ and $<, >$ a scalar product on V.
(i). Assume that $<, >$ is positive definite. Let v, w be in V. Show that $\dim\ \mathrm{span}\ \{v, w\} \leq 1$ if and only if $| < v, w > | = \|v\|\|w\|$.
(ii). Is that still true if we don't assume that $<, >$ is definite?

exercise 4:
Find a an example of a vector space (V, φ) where φ is bilinear, symmetric, and non-degenerate, and W is a subspace such that the sum $W + W^{\perp}$ is not direct.

exercise 5:
Let (V, φ) be a finite-dimensional vector space where φ is bilinear, symmetric, and non-degenerate and W be a subspace. Show that $W^{\perp\perp} = W$.

exercise 6:
3.3. 11 and 13.

exercise 7:
5.3.2.