<u>exercise 1</u>:
Let $A$ be in $K^{n\times n}$. Show that there exist two invertible matrices $P, Q$ in $K^{n\times n}$, $r$ in $\{0, .., n\}$ such that $A = PI_{r,n}Q$. Here $I_{r,n}$ is the diagonal matrix with $r$ ones followed by zeros on the diagonal. **Hint:** Consult the proof of the dimension formula.

<u>exercise 2</u>:
Let $A$ be in $K^{n\times n}$ such that for all $B$ be in $K^{n\times n}$, $AB = BA$. Show that $A$ is a multiple of the identity matrix. **Hint:** In a first step use the basis matrices $E_{ij}$ to show that $A$ is diagonal. Alternatively, you may use elementary matrices.

<u>exercise 3</u>:
True or false: there is a dimension $n \geq 2$ such that if $A$ in $K^{n\times n}$ has only zeros on its diagonal, then $A$ is singular.

<u>exercise 4</u>:
2.3.27. Suggestion: Introduce $C = AB$ and $D = BA$. Write the $i, j$ entries of $C$ and of $D$ using the entries of $A$ and $B$.

<u>exercise 5</u>:
2.3.35. Suggestion: prove by induction that on $k$ that $A^k e_j = 0$ for all $j = 1, .., k$.

<u>exercise 6</u>:
Write the permutation $(4 \quad 1 \quad 2 \quad 3)$ as a product of transpositions. What is its sign?

<u>exercise 7</u>:
Let $A$ be the matrix in $K^{n\times n}$ with $e_n$ in the first column, and $e_{j-1}$ in column $j$, $2 \leq j \leq n$. Find $\det A$.

<u>exercise 8</u>:
Let $[a, b]$ be an interval in $\mathbb{R}$ and $x_0 < x_1 < ... < x_n$ be in $[a, b]$. Let $y_0, ..., y_n$ be in $\mathbb{R}$. Use the Vandermonde determinant $V$ to show that there is a unique polynomial $P$ in $\mathbb{R}[X]$ with $\deg P \leq n$ such that $P(x_j) = y_j$, for $j = 0, ..., n$.
If $x_j = j\frac{b-a}{n}$, for $j = 0, ..., n$, and $0 < b - a \leq 1$, show that $|V| \leq (\frac{b-a}{n})^n$.