exercise 1:

Let $f : [a, b] \to \mathbb{R}$ be a function that has $n+1$ derivatives. Let $P_n(x) = \sum_{k=0}^{n} f^{(n)}(a)\frac{(x-a)^n}{n!}$.
Show that

$$f(b) - P_n(b) = \int_a^b \frac{(b-t)^n}{n!} f^{(n+1)}(t)dt.$$

Hint: Argue by induction.

exercise 2:

This exercise is the continuation of exercise 5, homework 5.
(iii). Show that $\cos^2 x + \sin^2 x = 1$, for all x in $\mathbb{R}$.
Fix y in $\mathbb{R}$, and let

$$g(x) = \cos(x+y) - \cos x \cos y + \sin x \sin y,$$

for all x in $\mathbb{R}$.
(iv). Show that $g'' = -g$.
(v). Show that the function $g^2 + g'^2$ is constant and infer that $g(x) = 0$, for all x in $\mathbb{R}$.

exercise 3:

8.5.J