exercise 1:

Define a sequence a_n by setting $a_{2n} = 2^{2n}$, $a_{2n+1} = a_{2n}$. Set $b_n = a_n^{1/n}$ and $c_n = \frac{a_{n+1}}{a_n}$. Does $\lim b_n$ exist? Does $\lim c_n$ exist?

exercise 2:

Define a sequence a_n by setting $a_{2n} = \dfrac{3^n}{n!}$, $a_{2n+1} = \dfrac{(-2)^n}{n}$. Find the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} a_n x^n$.

exercise 3:

8.5.F

exercise 4:

(i). For x in $(0, \infty)$ define $f(x) = e^{-1/x}$. Show by induction that $f^{(n)}(x) = \frac{P_n(x)}{x^{2n}} e^{-1/x}$, where P_n is a polynomial function. Find a recursive formula for P_n.

(ii). Define $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 0$, if $x \leq 0$, $g(x) = e^{-1/x}$, if $x > 0$. Show that $g \in C^\infty(\mathbb{R})$. (Use an argument by induction).

(iii). Is there a positive R such that for all x in $(-R, R)$, $g(x) = \displaystyle\sum_{n=0}^{\infty} \frac{g^{(n)}(0)}{n!} x^n$?

exercise 5:

(i). Find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

(ii). Defining $\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, show that $\cos' = -\sin$ and $\sin' = \cos$.