exercise 1:

Set $I_n = \displaystyle\int_0^{\pi} \sin^n x\, dx$.
(i). Compute I_0 and I_1.
(ii). Show that $0 \leq I_{n+1} \leq I_n$, for all n in $\mathbb{N}$.
(iii). Show that $\lim\limits_{n\to\infty} I_n = 0$. **Hint**: We covered a similar case in class.
(iv). Show that $nI_n = (n-1)I_{n-2}$, for all integer $n \geq 2$. **Hint**: $\sin^2 x = 1 - \cos^2 x$. Use integration by parts.
*** the remaining questions are optional***
(iv). Find a formula for I_{2n} and a formula for I_{2n+1} involving $n!$ and 2^n.
(v). Find $\lim\limits_{n\to\infty} nI_nI_{n+1}$.
(vi). Find $\lim\limits_{n\to\infty} \sqrt{n}I_n$. **Hint**: You may use that $\lim\limits_{n\to\infty} I_n$, which was covered in class.

exercise 2:

(i). Let $f : [a,b] \to \mathbb{R}$ be a continuous function such that $f \geq 0$ and $f(x_0) > 0$ for some x_0 in $[a,b]$. Show that $\int_a^b f > 0$. **Hint**: Use the definition of continuity and $\epsilon = \frac{f(x_0)}{2}$.
(ii). Let g be in $C([a,b])$. Show that $g = 0$ if and only if $\int_a^b |g| = 0$.

exercise 3:

8.1.A. Hint: find the maximum of f_n.

exercise 4:

8.1.B

exercise 5:

8.1.D

exercise 6:

8.2.B

exercise 7:

8.2.C