exercise 1:
Let $f : [a,b] \to \mathbb{R}$ be an increasing function. Show that f is bounded and Riemann integrable. **Hint:** Introduce the partition P, $x_j = a + \frac{j}{n}(b-a)$, $j = 0, .., n$. Evaluate $L(f,P)$ and $U(f,P)$.

exercise 2:
6.4.A, (a), and (b) only in the case $a \geq 0$.

exercise 3:
(i). Let $f : [a,b] \to \mathbb{R}$ be a convex and differentiable function and x_0 in (a,b). Show that the graph of f is above the the tangent line through $(x_0, f(x_0))$.
(ii). Let $k \geq 2$ be an integer. Show that

$$\frac{1}{2}(\ln(k-1) + \ln k) \leq \int_{k-1}^{k} \ln x \, dx \leq \ln(k - \frac{1}{2})$$

Hint: Use convexity of $-\ln$ for the right hand part. Sketch the graph of $-\ln$ on $[k-1,k]$.

exercise 4:
Define $f : [0,1] \to \mathbb{R}$, $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } 0 < x \leq 1 \\ 0, & \text{if } x = 0. \end{cases}$
Show that f is Riemann integrable.
Hint. Fix $\epsilon > 0$. Form lower and upper sums in $[0,\epsilon]$ and in $[\epsilon,1]$ for an adequately chosen partition.

exercise 5:
Let f be in $C([a,b])$, and u and v two differentiable functions defined on an interval J and valued in $[a,b]$. Set $F(x) = \int_{u(x)}^{v(x)} f(t)dt$. Show that F is differentiable in J and find its derivative. **Hint:** $F(x) = \int_a^{v(x)} f(t)dt - \int_a^{u(x)} f(t)dt$.

exercise 6:
6.4.C. Hint: set $F(x) = \int_a^x f$.

exercise 7:
Let $f : [0,1] \to \mathbb{R}$, $f(x) = \begin{cases} x^{\frac{3}{2}}\sin\frac{1}{x}, & \text{if } 0 < x \leq 1 \\ 0, & \text{if } x = 0. \end{cases}$
Show that f is differentiable and that f' is unbounded.

exercise 8:

Let k and n be two integers greater or equal than 1.

(i). Show that $\frac{1}{k+1} \leq \int_k^{k+1} \frac{1}{x} dx \leq \frac{1}{k}$.

(ii). Infer that $\ln(n+1) \leq \sum_{k=1}^{n} \frac{1}{k}$.

(iii). **Optional:** Show that the sequence $a_n = \ln(n+1) - \sum_{k=1}^{n} \frac{1}{k}$ converges.