exercise 1:

Define $f : (-1,1) \to \mathbb{R}$, $f(x) = (1+x)^{-\frac{1}{2}}$.

(i). Set $a_n = f^{(n)}(0)$. Show that $a_0 = 1$ and if $n \geq 1$,

$a_n = \frac{(-1)^n}{2^n} 1 \cdot 3 \cdot \ldots \cdot (2n-1)$.

(ii). Find the radius of convergence of $g(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$.

(iii). Show that f and g satisfy the differential equation $2y'(1+x) + y = 0$ and infer that $f = g$.

exercise 2:

Using exercise 1, find a power series for $(1-x^2)^{-\frac{1}{2}}$ and for $\arcsin x$, for x in $(-1,1)$. Make sure to mention any theoretical result that you are using.

Exercise 3 **Optional**

With the same notation as in exercise 1 find $\sum_{n=0}^{\infty} \frac{a_n}{n!}$.

Hint: If the sequence b_n is decreasing and convergent to zero then $|\sum_{n=p}^{\infty} (-1)^n b_n| \leq b_p$. Derive uniform convergence on the closed interval $[0,1]$.

Exercise 4

(i). Define $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$. Find the radius of convergence of these two power series and show that $\frac{d}{dx} \cos x = -\sin x$ and $\frac{d}{dx} \sin x = \cos x$.

(ii). Show that $\cos^2 x + \sin^2 x = 1$ for all x in $\mathbb{R}$. **Hint:** Introduce the function $f(x) = \cos^2 x + \sin^2 x$ and find its derivative.