<u>exercise 1</u>:
Define a sequence a_n by setting $a_{2n} = 2^{2n}$, $a_{2n+1} = a_{2n}$.
(i). For $x \neq 0$, set $b_n = a_n x^n$. Does $|\frac{b_n}{b_{n+1}}|$ have a limit as $n \to \infty$?
(ii). Find the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} a_n x^n$.

<u>exercise 2</u>:
(i). For x in $(0, \infty)$ define $f(x) = e^{-1/x}$. Show by induction that $f^{(n)}(x) = \frac{P_n(x)}{x^{2n}} e^{-1/x}$, where P_n is a polynomial function. Find a recursive formula for P_n.
(ii). Define $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 0$, if $x \leq 0$, $g(x) = e^{-1/x}$, if $x > 0$. Show that $g \in C^\infty(\mathbb{R})$. (Use an argument by induction).
(iii). Is there a positive R such that for all x in $(-R, R)$, $g(x) = \displaystyle\sum_{n=0}^{\infty} \frac{g^{(n)}(0)}{n!} x^n$?

<u>exercise 3</u>:
Exercise 6.5.8 from Abbot's textbook.