exercise 1:
*** this optional problem is the continuation of hw1 ex 10***
(iv). Find a formula for I_{2n} and a a formula for I_{2n+1} involving $n!$ and 2^n.
(v). Find $\lim_{n\to\infty} nI_nI_{n+1}$.
(vi). Find $\lim_{n\to\infty} \sqrt{n}I_n$. **Hint**: You may use that $\lim_{n\to\infty} I_n$, which was covered in class.

exercise 2:
(i). Let f be in $C([a,b])$ such that $f \geq 0$ and $f(x_0) > 0$ for some x_0 in $[a,b]$. We proved in class that $\int_a^b f > 0$ in the case where $x_0 \in (a,b)$. Prove it in the case where $x_0 = a$.
(ii). Let g be in $C([a,b])$. Show that $g = 0$ if and only if $\int_a^b |g| = 0$.

exercise 3:
8.1.A. Hint: find the maximum of f_n.

exercise 4:
8.1.B

exercise 5:
8.1.D

exercise 6:
8.2.B

exercise 7:
8.2.C

exercise 8:
Let P be a polynomial function. Show that $\sum_{n=1}^{\infty} P(n)x^n$ is a absolutely convergent if $|x| < 1$ and divergent if $|x| > 1$.

exercise 9:
Show that the series $\sum_{n=0}^{\infty} \frac{\cos(nx)}{n^2+1}$ defines a continuous function f from $\mathbb{R}$ to $\mathbb{R}$.

exercise 10:

Show that the series $\sum_{n=0}^{\infty} \frac{x^n \cos(nx)}{n+1}$ defines a continuous function f from $(-1,1)$ to $\mathbb{R}$.

exercise 11:

Let $f_n : [0,1] \to \mathbb{R}$, $f_n(x) = nx^n$, if $0 \leq x < 1$, $f_n(1) = 0$.
(i). Find, with proof, f, the pointwise limit of f_n.
(ii). Find $\int_0^1 f_n$ and $\int_0^1 f$.
(iii). Is f_n uniformly convergent?