exercise 1:
Let $f : [a,b] \to \mathbb{R}$ be an increasing function. Show that f is bounded and Riemann integrable. **Hint:** Introduce the partition P, $x_j = a + \frac{j}{n}(b-a)$, $j = 0,..,n$. Evaluate $L(f,P)$ and $U(f,P)$.

exercise 2:
Let $f : S \subset \mathbb{R} \to \mathbb{R}$ be a Lipschitz continuous function. Show that f is uniformly continuous.

exercise 3:
Let $g : (0,\infty) \to \mathbb{R}$, $g(x) = \frac{1}{x}$. Show that g is not uniformly continuous.

exercise 4:
6.4.A, (a), and (b) only in the case $a \geq 0$.

exercise 5:
(i). Let $f : [a,b] \to \mathbb{R}$ be a convex and differentiable function and x_0 in (a,b). Show that the graph of f is above the the tangent line through $(x_0, f(x_0))$.
(ii). Show that

$$\frac{1}{2}(\ln(k-1) + \ln k) \leq \int_{k-1}^{k} \ln x \, dx \leq \ln(k - \frac{1}{2})$$

Hint: Use convexity of $-\ln$ for the right hand part.

exercise 6:
Define $f : [0,1] \to \mathbb{R}$, $f(x) = \begin{cases} \sin\frac{1}{x}, \text{ if } 0 < x \leq 1 \\ 0, \text{ if } x = 0. \end{cases}$
Show that f is Riemann integrable.
Hint. Fix $\epsilon > 0$. Form lower and upper sums in $[0,\epsilon]$ and in $[\epsilon,1]$ for an adequately chosen partition.

exercise 7:
Let f be in $C([a,b])$, and u and v two differentiable functions defined on an interval J and valued in $[a,b]$. Set $F(x) = \int_{u(x)}^{v(x)} f(t)dt$. Show that F is differentiable in J and find its derivative.

exercise 8:
6.4.C. Hint: set $F(x) = \int_a^x f$.

<u>exercise 9</u>:

Let $f : [0,1] \to \mathbb{R}$, $f(x) = \begin{cases} x^{\frac{3}{2}} \sin \dfrac{1}{x}, \text{ if } 0 < x \leq 1 \\ 0, \text{ if } x = 0. \end{cases}$

Show that f is differentiable and that f' is unbounded.

<u>exercise 10</u>:

Set $I_n = \displaystyle\int_0^{\pi} \sin^n x \, dx$.

(i). Compute I_0 and I_1.

(ii). Show that $0 \leq I_{n+1} \leq I_n$, for all n in $\mathbb{N}$.

(iii). Show that $\lim_{n\to\infty} I_n = 0$. **Hint**: We covered a very similar case in class.

(iv). Show that $nI_n = (n-1)I_{n-2}$, for all integer $n \geq 2$. **Hint**: $\sin^2 x = 1 - \cos^2 x$. Use integration by parts.