POW Traffic Light Design by: Chris, Rddhima, Jack

Problem Statement:

How do different factors affect the time a traffic light stays yellow?

Process:

As we began the problem, it became clear that there would be two cases described as the "go zone" and "stop zone" that should be separated during the formula derivation process. We began by approximating and making up key variables we would need. First, we set the standard for the first car at the line to be able to make it past, but that consideration neglected the fact that some cars a small distance behind the intersection can still make it past the intersection during the span of the yellow light. We tried to brainstorm ways to have our formula apply to more vehicles.

To mitigate the shortcoming, we considered two separate distances: one for the distance for a car to safely pass (x_1), and one distance for the car to safely stop (x_2). The goal was to calculate the two distances and have $x_1 > x_2$ to ensure that the cars would have some leeway. However, we were confronted by some problems during this process because many of our variables were inter-dependent, which makes the formulas impossible to solve without already knowing one of them (see Photo 1). In addition, we ran into some problems regarding the inequalities because we were unable to substitute our equations from the two different cases to arrive at a reasonable formula.

After a little bit, we were enlightened to try a different method. We re-defined our distance for the go zone and stop zone as a single distance x' instead of two separate values. This consideration was effective in reducing the problem we ran into during the second consideration because we would not have to use any inequalities. Using this method, it allowed us to eventually arrive at a solution that was dependent on factors such as length of the intersection, length of the car, and speed limit.

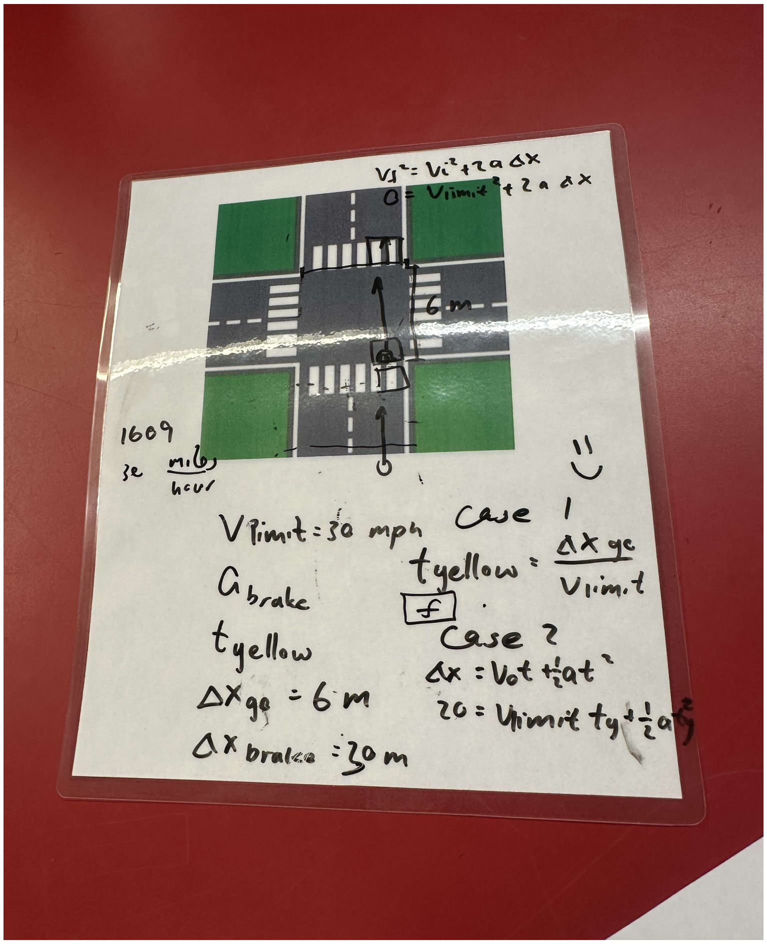

Photo 1: Our first (failed) attempt to find the solution

Solution:

To solve the problem, we assumed a couple key assumptions:

1. The Go Zone and Stop Zone do not intersect.
2. The positive direction is the vehicle's direction of motion.
3. For the Stop Zone, all of the drivers start braking immediately after seeing the light turn yellow. We will address this again at the end of the solutions section.

The variables we define for this problem are:

v_{lim}=initial velocity (max velocity or speed limit)

x'=distance between Go/Stop Zone and the intersection

x_{int}=distance of the intersection

x_L=length of the vehicle

t_y=total time of yellow light

a=braking acceleration (negative value)

Our final goal was to find the time of the yellow light, but it would be easier to calculate the x' values of each case and then set them equal to substitute and find t_y. We first considered the Stop Zone for the yellow light.

If we use the kinematic equation $\Delta x = v_0 t + \frac{1}{2}at^2$, we can find an expression for x' but there is an acceleration value that is not ideal, so we will also use the equation $v_f^2 = v_i^2 + 2a\Delta x$. In our case, we have two equations:

$$x' = v_{lim}t_y + \frac{1}{2}at_y^2 \text{ and}$$

$$0 = v_{lim}^2 + 2ax' \text{ (because the car will stop at the intersection)}$$

The second equation will simplify to $a = \frac{-v_{lim}^2}{2x'}$.

We can now substitute this into the first equation which now becomes

$$x' = v_{lim}t_y - \frac{v_{lim}^2 t_y^2}{4}$$

Now, with an equation that represents x' in the Stop Zone case, we can move on to the Go Zone case.

By definition, $V = \frac{\Delta x}{\Delta t}$ or $\frac{\Delta x}{\mathrm{v}} = \Delta t$. After substituting our variables, it is modeled that

$$\frac{x' + x_{int} + x_L}{v_{lim}} = t_y$$

This equation simplifies to $x' = v_{lim}t_y - x_{int} - x_L$.

At this point, we have two equations for x', so we can set them equal.

$$x' = v_{lim}t_y - x_{int} - x_L = v_{lim}t_y - \frac{v_{lim}^2 t_y^2}{4}$$

$$\frac{1}{4}v_{lim}^2 t_y^2 = x_{int} + x_L$$

$$t_y^2 = \frac{4(x_{int} + x_L)}{v_{limt}^2}$$

$$t_y = 2\sqrt{\frac{x_{int} + x_L}{v_{lim}^2}}$$

For our model, the time needed for the yellow light has a positive correlation with the length of the intersection and length of the vehicle since both of those terms are in the numerator of the fraction. The time will have a negative correlation with the speed of the vehicle because it is in the denominator. We can look at a couple of values to see if the time of yellow makes sense.

If, $x_{int} = 10\ m, x_L = 4\ m, \&\ v_{lim} = 20\frac{m}{s}$, then, $t_y = {\sim}1.67\ s, x' = 19\ m, \&\ a = -10.5\ m/s^2$

If, $x_{int} = 10\ m, x_L = 4\ m, \&\ v_{lim} = 40\frac{m}{s}$, then, $t_y = {\sim}1.2\ s, x' = 34\ m, \&\ a = -23\ m/s^2$

The time for the yellow light makes sense, and the amount of distance that can stop or go is reasonable. However, we did not include the time it takes to react. If we offset the time by one full second, then the equation would be close. In conclusion, our equation is:

$$t_y = 0.8 + 2\sqrt{\frac{x_{int} + x_L}{v_{lim}^2}}$$

Discussion:

a) In reality, there has to be a small overlap between the STOP and GO zones for the intersection to be safe. This is because if a car was caught in the middle ground between them, they would not be able to stop safely in time and not be able to go through the intersection, creating a dangerous situation for the driver. If there is even a slight overlap between them, the driver can decide to stop safely or go through the intersection, if possible, which is much safer. In the ideal case, it would be best for there to be no overlap within the two zones so that there is an ensured stop and start distance. Overlap might cause accidents where one driver decides to go and one doesn't, causing a crash.

b)

1) Exceeding the speed limit would affect the Velocity variable. Somebody going faster than assumed would not be able to slow down safely and require a larger STOP zone to stay safe.

2) Bald tires would affect the deceleration variable. Bald tires make it harder to slow down and therefore take more time to decelerate. They would require a larger STOP zone to slow down safely.

3) A longer vehicle would affect the variable x_L, they would require a larger STOP zone than average to be able to slow down safely in time.

4) A distracted driver would require a longer yellow light time because of their lack of attention. This would affect the GO zone because the driver might not go, even if it's safe to do so, and hold up potential drivers behind them as well. It could also affect the STOP zone because it can cause the driver to not slow down fast enough.

c) In the ideal case it would be helpful to know when to stop. However, in the real world there are cases where people go slower or higher than the speed limit, and therefore the stop/go intersection would not be helpful to them. There are also conditions such as weather, vehicle types, and other arbitrary challenges that would make marking these zones incredibly difficult to implement. So no, it would not be necessary because of the variability of the STOP and GO zones for everyone.

d) People would likely decide they could make it through the intersection when, in reality, they cannot and just end up causing traffic problems like running red lights. Some drivers might try to speed up to beat the clock, while others might brake abruptly and cause rear-end collisions, causing dangerous drivers. The intersections would be less safe.