Happy Birthday POW

Problem Statement:

How can you figure out the day of the week someone was born on, based on their birthday?

Develop a system or model to address this using only:

- A calendar of the current month
- The "Basic Information About Calendars" handout
- Four function calculator (+, -, ×, ÷)
- Provided birthday examples

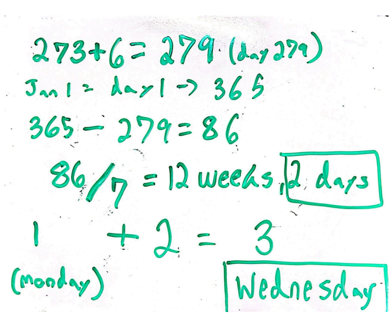
Process:

Step 1: Find January 1st in 2026
Our first idea in approaching this problem was to find a uniform start point for our final model.
But first we defined that:

- Days of the week are valued 0 - 6 (Sunday - Saturday)
- We will solve in mod 7 to get a day of the week

We decided to start from January 1st, 2026, since it is easiest to start from the beginning of a year, and also January 2026 is closer to the current date than January 2025. We solved for the day of the week on January 1st, 2026 by first finding how many days have passed in the year (279), then subtracting that value by the number of days in a year (365), to get the amount of days left in the year (86). Lastly we put that value into mod 7 and added 1 for the first day of the year (2 + 1), which we got 3.

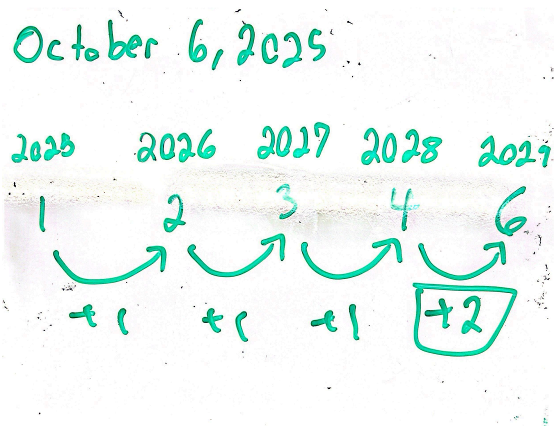
We know: Monday, October 6, 2025
Days passed before October 2025 begins: 273

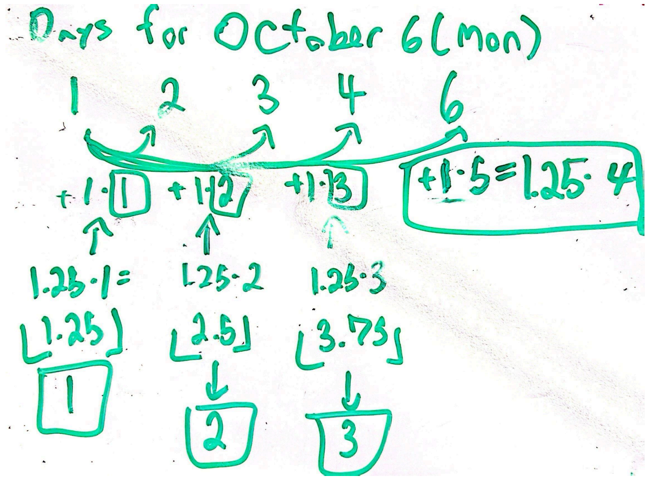


January 1st is a **Wednesday**

Step 2: Finding how many times it shifts
We noticed that each year, the day for a specific date shifts over by one day of the week. However, when there was a leap year, we noticed that the day was shifted over two days of the week. We wanted to find a way to include both leap years and normal years in the same calculation. We realized that

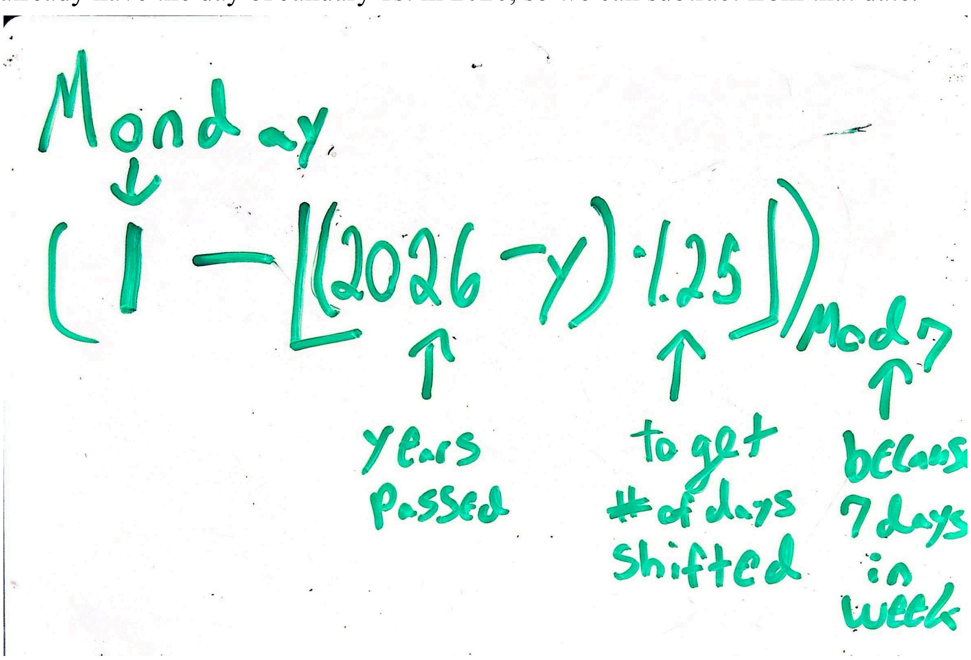
on a leap year, the date moves 5 places from the original Monday on the 4th year out. This is also equivalent to 1.25*4. To incorporate this 1.25 into the normal years as well, we took the floor, meaning only the floor of the 1.25 times the number of years between two dates to find how many days it has shifted from a given day.





Step 3: Finding Jan 1st of the birthday year

Once we knew how many days a date shifts per year, we could find out what January 1st of any given year was. We started off by trying to find January 1st in 1901, thinking that we would calculate the position adding up from that starting date in 1901. We then realized that this is unnecessary because we already have the day of January 1st in 2026, so we can subtract from that date.



Step 4: Finding the Day of the Birthday

We began by finding the number of days in a year that have already passed. To do so, we added up the days in each prior month followed by the number of days in the given month of the birthday. We soon realized that we needed to find the difference between the number of days passed in the year and the first day of the year, because we are trying to figure out how many times a day has shifted **since** January 1st.

We then tried to look to see if there were any patterns between the days passed in a year and the month number. We did so by finding the sum of the days for multiple different numbers of months and dividing that by the number of months passed. The results of those trials did not give us a specific pattern that could be implemented into our model, so, instead, we implemented a table that will calculate the days passed prior to the month's start, which can be seen in the beginning of the solutions section.

Step 5: Fixing our model

We then tried testing our model with August 10th, 2019 , and we found that our calculation for January 1st was incorrect. We then realized that our original calculation for the day was for December 31st, not January 1st. We redid our calculation to get that January 1st, 2026, was a Thursday.

We also realized that instead of taking the floor of the numbers we should have rounded it. Finally, we fixed our days since January first since it overcounted by one. This was because our model looks at what number day a given date was in a year. January 1st would be "Day 1" and February 2nd, for example, would be "Day 33", so to find the number of days passed, you would compute "Day 33 - Day 1= 32 Days".

Solution:

Variables and Definitions:

$Y_{\text{ear of birth}}$ - The year that the birthday occurs
$M_{\text{onth of birth}}$ - the month the person the person was born in
$D_{\text{ay of birth}}$ - The day of the month that the person was born on

A given person would give their birthday as $M_{\text{onth of birth}} / D_{\text{ay of birth}} / Y_{\text{ear of birth.}}$

$J_{\text{anuary first, 2026}}$ - Day of the week that fell on this date (Thursday)
$D_{\text{ays since beginning of year}}$ - The number of days from January first, year of birth to the birthday
$D_{\text{ays until beginning of month}}$ - The number of days between January first, year of birth and the first day of the birth month

- This variable is a set value based on the birth month and is dependent on whether or not the year of birth is a leap year.

$D_{\text{ays since beginning of year}} = D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1$

For instance, if someone was born on February 2nd, the days since the beginning of the year would be the days until February 1st (31) plus the day of birth (2) to get 33 days since the beginning of the year. Then, subtract 1 to get the difference between that day and January 1st.

Days of the week are assigned numbers 0 through 6, with Sunday being 0 and Saturday being 6.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
0	1	2	3	4	5	6

The table below describes the $D_{\text{ays until beginning of month}}$ based on the birth month. This is calculated by summing up the days in each month before the birth month.

m	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Norm	0	31	59	90	120	151	181	212	243	273	304	334
Leap	0	31	60	91	121	152	182	213	244	274	305	335

Equation:

$$(J_{\text{anuary first, 2026}} - (\text{round}((2026 - Y_{\text{ear of birth}})*1.25) \bmod 7) + ((D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1) \bmod 7)) \bmod 7$$

Step 1: Find the day of the week of January 1st, 2026.

We used the fact that October 6, 2025 was a Monday. Then we found out how many days were between October 6, 2025 and January 1st, 2026. Then we did that number mod 7 to find the difference of days, which ended up being 3. So, we found that January 1st, 2026, fell on a Thursday, 3 weekdays later than Monday.

Step 1 corresponds to this part of the equation: $J_{\text{anuary first, 2026}}$

Step 2: Find the difference of days between January 1st, 2026 and January 1st, Year of Birth.

In a normal year, there are 365 days. If we do 365 mod 7, we get one. That means after each normal year. January 1st is one weekday later than the previous year. For example, if January 1st, 2026 is on a Thursday, January 1st, 2027 will be on a Friday, and January 1st, 2025 was on a Wednesday. However, after a leap year, the day changes by 2, because 366 mod 7 is 2. So if January 1st, 2025 was on a Wednesday, January 1st, 2024 was on a Monday since 2024 was a leap year.

Thus, we know that for each normal year the offset is one day and each leap year the offset is two days. Over a 4-year period, the offset is 5 days. Thus, we get an average of 1.25 days per year.

By rounding 1.25 times the number of years from 2026 to Year of Birth, we will get the necessary offset that we need to subtract from the weekday of January 1st, 2026, which is our anchor date for comparison.

Note that we need to make sure that this value is in our range of 0-6, so we use this number mod 7 in our calculation.

Example:
Person A was born in 2019.

Do (2026-2019) * 1.25 = 8.75
round(8.75) = 9

9 makes sense because there were 2 leap years and 5 normal years between the beginning of 2019 and the beginning of 2026, which gets 2 + 2 + 1 + 1 +1 + 1 +1 = 9 as the offset.
9 mod 7 = 2.

So, January 1st, 2019, was two weekdays earlier than January 1st, 2026.

Step 2 correspond to this part of the equation: **round((2026 - $Y_{ear\ of\ birth}$)*1.25) mod 7**

Step 3: Subtract the difference from the day of the week of January 1st, 2026.

First, we look at our set date January 1st, 2026, which we previously calculated was on a Thursday. Using our table, Thursday is 4.

Then, we subtract the value we found in step 2. This is because we are going backwards in time instead of forwards.

In the example from step 2, we got that January 1st, 2019, was two weekdays earlier than January 1st, 2026.

Thus, we do:

4 - 2 = 2

2 mod 7 = 2

2 = Tuesday in our table

So, January 1st, 2019 was on a Tuesday.

Step 3 correspond to this part of the equation: **$J_{anuary\ first,\ 2026}$ - (round((2026 - $Y_{ear\ of\ birth}$)*1.25) mod 7)**

Step 4: Find the number of days from January 1st, Year of Birth to Birth Date.

To find the number of days between January 1st and a given birthday, you add up all the days in all the months before that month. Then, you add the day of the month the person was born. Lastly, subtract one so you don't double count January first.

For instance, if you are born on August 10th, you need to add up all the days in the months January through July and then add 10 to get the day of the year numerically. Then, you find the difference between that day of the year and the first day of the year.

We made a standardized table for the sum of all the days of months before your month. Note that if you were born on a leap year and after the end of February, you have to add one extra day to account for the leap year. This is present in our table above.

You can find if your year of birth was a leap year by seeing if it was divisible by four or seeing if Year of Birth mod 4 = 0.

This process is summarized in the equation below:

$D_{\text{ays since beginning of year}} = D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1$

Example:

Person A was born on August 10th, 2019.

Using August in the table and non-leap year, we get 212.

Do 212 + 10 = 222.

August 10th is the 222nd day of the year, so we have to subtract 1, since we want the difference between the 222nd day of the year and the 1st day of the year.

222 - 1 = 221 to get the number of days between January 1st, 2019 and August 10th, 2019.

Step 4 corresponds to this part of the equation: $\mathbf{D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1}$

Step 5: Find the offset of weekdays of the Birth Date as compared to January 1st, Year of Birth.

After you find how many days there are between your birthday and January 1st of your birth year, we need to take this number and do mod 7 to get an offset within range. Then, add this offset to January 1st, Year of Birth.

Step 5 corresponds to this part of the equation: $\mathbf{((D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1) \bmod 7)}$

Example:
Person A was born on August 10th, 2019.

There are 221 days between January 1st and your birth date, from Step 4.

221 mod 7 = 4.

In Step 3, we got January 1st, 2019, was 2, or fell on a Tuesday.

2 + 4 = 6

6 mod 7 = 6.

So, August 10th, 2019 was a Saturday.

Thus, we get the equation below:

$$\mathbf{(J_{\text{anuary first, 2026}} - (round((2026 - Y_{\text{ear of birth}})*1.25) \bmod 7) + ((D_{\text{ays until beginning of month}} + D_{\text{ay of birth}} - 1) \bmod 7)) \bmod 7}$$

Full Example:

Person A was born on August 10th, 2019.

Do (2026-2019) * 1.25 = 8.75
round(8.75) = 9

9 makes sense because there were 2 leap years and 5 normal years between the beginning of 2019 and the beginning of 2026, which gets 2 + 2 + 1 + 1 +1 + 1 +1 = 9 as the offset.
9 mod 7 = 2.

So, January 1st, 2019, was two weekdays earlier than January 1st, 2026.

Thus, we do:

4 - 2 = 2

2 mod 7 = 2

2 = Tuesday in our table

So, January 1st, 2019 was on a Tuesday.

Using August in the table and non-leap year, we get 212.

Do 212 + 10 = 222.

August 10th is the 222nd day of the year, so we have to subtract 1, since we want the difference between the 222nd day of the year and the 1st day of the year.

222 - 1 = 221 to get the number of days between January 1st, 2019 and August 10th, 2019.

221 mod 7 = 4.

We got January 1st, 2019, was 2, or fell on a Tuesday.

2 + 4 = 6

6 mod 7 = 6.

So, August 10th, 2019 was a Saturday.

Extensions:

Extension 1: How can we find what day someone's birthday will be in the future?

In order to predict the day of the week of an individual's birthday in the future, we need to create an equation. We can start with the variables and equations from our solution above.

This might be possible using our current equation. We would end up getting a negative number for 2026 - y, and since we subtract it it might end up working out. However, this idea would have to be explored further and edge cases need to be tested.

Extension 2: How will the solution change if lengths are changed?

- 10 day weeks?

- Leap year occurs every 8 instead of 4 years?
- 40 day months?

To solve this, we would have to account for using different mods and establishing a different schedule. We would also have to change the factor we multiply by to account for the fact that leap years occur every 8 years. Also, if the number of days in a year changes, the yearly shift will change too.

Extension 3: How will the solution change if we are going back as far as 1800, instead of 1900?

For this, we would have to deal with the fact that 1900 is not a leap year. Our current model is based on the assumption that there is a leap year every 4 years. If 1900 is not a leap year, then, that would mean that we skip a leap year when counting 1900, meaning our current model would be inaccurate before 1900.