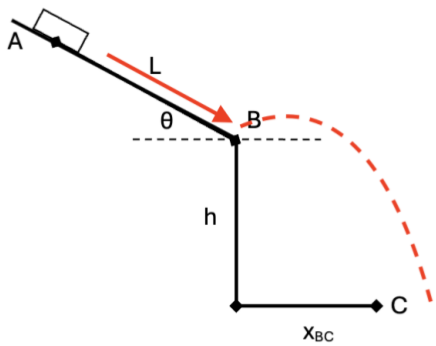
Physics POW #1

Problem Statement

For this POW, we were given a problem where a puck was sliding down an inclined plane until the ramp suddenly reaches a vertical drop, from where the puck becomes a projectile (refer to the figure to the right). We were tasked with finding out how far the puck lands away from the from the base of the counter. Then, we were tasked with finding a way to change the angle of the ramp's incline to maximize the distance that the puck travels (as a projectile).

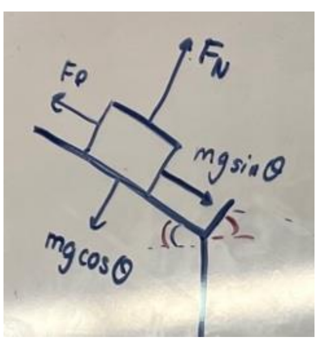


Process

General Outline of Our Process:

1. Find the forces acting on the puck. Use those forces to find the acceleration of the puck
2. Find the velocity of the puck at the exact moment where the ramp drops using the kinematic equations
3. Use the projectile equations to find the distance traveled by the puck projectile
4. Enter the equations into an Excel spreadsheet to find the angle at which the distance traveled is optimized (our solution)

1. Finding the forces on the puck and its acceleration:



We began by creating a Free-Body Diagram to identify all the forces acting upon the puck. By doing that, we were able to find out that there are four total forces acting on the puck: the normal force, the two components of gravity, and the force of friction (shown in diagram). Using all those forces, we were able to come up with a derived equation for the forces acting on the puck:

$$F = ma$$
$$mg\sin\theta - F_f = ma$$
$$mg\sin\theta - \mu\,\mathrm{mg}\cos\theta = ma$$
$$g\sin\theta - \mu g\cos\theta = a$$

We used the above equation for the next step.

2. Finding the velocity of the puck at the exact moment where the ramp drops:

Using the above equation and the ‘no t’ kinematic equation, we plugged in the equation for acceleration and found the velocity.

$$v^2 = v_0^2 + 2a\Delta x$$
$$v^2 = v_0^2 + 2(g\sin\theta - \mu g\cos\theta)\Delta x$$
$$v = \sqrt{2(g\sin\theta - \mu g\cos\theta)\Delta x}$$

We found the velocity at that point to be 5.23 m/s, which is the initial velocity of the puck as it’s launched off the ramp.

3. Find the distance traveled by the puck when it is a projectile:

We then took this initial velocity and plugged it into the vertical motion equation to find the time because the height, Δy, was given.

$$\Delta y = (sin\theta)(5.23)t - \frac{1}{2}gt^2$$

Plugging the corresponding values in and using the polynomial equation solvers on our graphing calculators we found that puck falls to the ground in 0.331 seconds.
We plugged this time into the horizontal velocity equation.

$$\Delta x = (cos\theta)(5.23)t$$

This got us **1.363 m** as our Δx.
We combined the projectile equations and dynamics equations to find the general formula for the distance traveled by the projectile puck. This was verified with the 38 degree angle we had solved for in smaller steps.

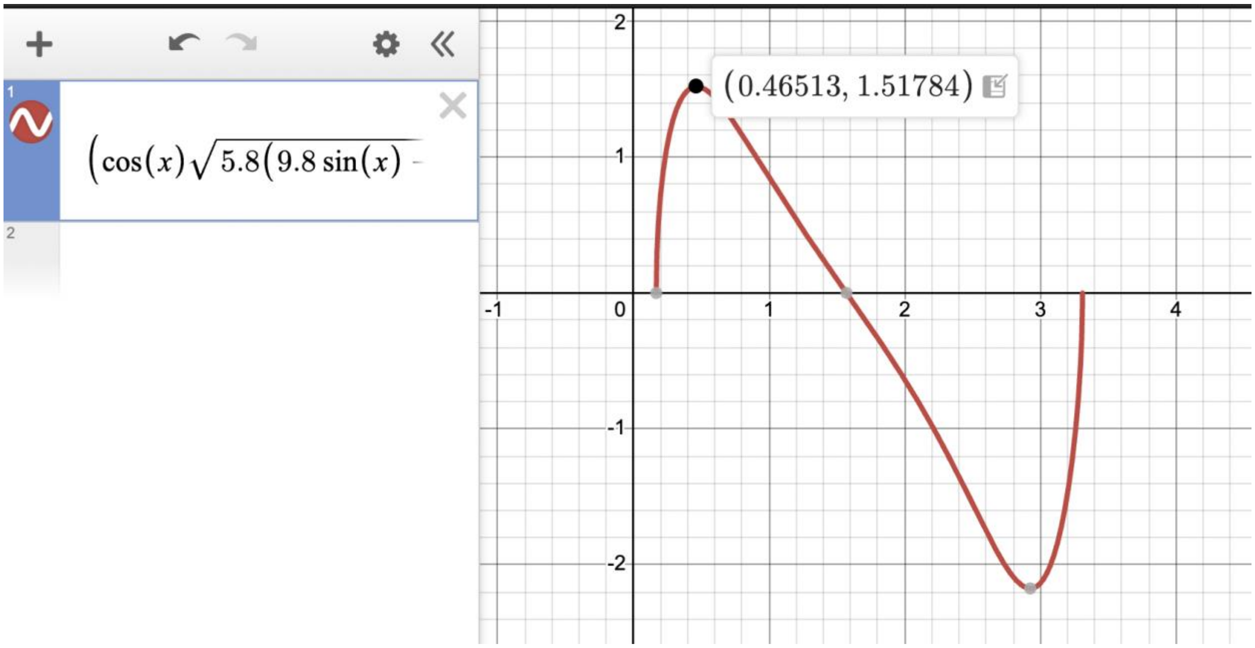
$$\Delta x = \cos\theta\sqrt{5.8(9.8\sin\theta - 1.666\cos\theta)} * \frac{-\sin\theta\sqrt{5.8(9.8\sin\theta - 1.666\cos\theta)} + \sqrt{\left(5.8(\sin\theta)^2(9.8\sin\theta - 1.666\cos\theta)\right) + 31.36}}{9.8}$$

Solution

The general solution was identified by sorting the Excel column delta x from highest to lowest Delta X, and we identified 27 as the optimal integer angle to yield the greatest distance from the base of the counter.

	A	B	C	D	E	F	G	H
1			(9.8*(SIN(B3)))-(1.666*(COS(B3)))	SQRT(5.8*C3)	D3*SIN(B3)	D3*(COS(B3)	(-E3+SQRT((E3^2)+31.36))/9.8	G3*F3
2	Angle	angle (r)	Acceleration	Velocity	Y-Velcoity	X-Velocity	T	Delta X
3	27	0.4712389	2.964690028	4.1467098	1.88256685	3.69474549	0.410754978	1.5176351
4	26	0.45378561	2.798646357	4.028914106	1.7661597	3.62116401	0.418954052	1.51710134
5	28	0.48869219	3.129830626	4.260635824	2.00024736	3.76191815	0.402679933	1.51484895
6	25	0.43633231	2.631750192	3.906936282	1.65114262	3.54088678	0.427265546	1.51289892
7	29	0.50614548	3.294017846	4.370961394	2.11908413	3.82292897	0.394739392	1.50906066
8	24	0.41887902	2.46405237	3.780410526	1.53763149	3.45357686	0.435676747	1.50464313
9	30	0.52359878	3.457201677	4.477920246	2.23896012	3.87799269	0.386942688	1.50056091

Graphing the general ΔX equation in Desmos we found that, at the maximum ΔX (on the y-axis), θ (on the x-axis) is 0.46513. Multiplying this by 180 and dividing by π, we found that the optimal angle for this problem is 26.650 degrees.



Extensions

The new puck, weighing 1078 N, travels down the 4.8m long incline that makes an angle theta relative to the horizontal. The kinetic coefficient of friction between the ramp and the new puck is 0.23. The height of the counter (beginning at the end of the incline) is 1.1m tall. The puck started from rest. Given that the puck's final horizontal position is 0.33m away from the base of the counter and that the angle of the incline is the new puck's critical angle, what is the static coefficient of friction?

Evaluation

This problem was, overall, pretty interesting. While it took my group and I a bit to get the numbers and equations right, we knew almost immediately what we needed to do for this and it improved my confidence in the subject. I think it would be interesting to see a version of this problem where x is changed instead and you'd have to find the optimal angle, although the general equation for that would not be very fun. I'd say this problem was doable, while my group and I did things a little slower and didn't do as much in the extensions as I would've liked, I still think we had productive discussions about the problem and all contributed equally.