1. **Problem Statement**:

Given a car approaches an intersection while driving at the speed limit, how long should the traffic light stay yellow for the car to safely pass or stop fully? What factors affect this decision?

2. **Process**:

First, we drew a diagram outlining what variables we would use. We decided on using the distance of the intersection, the length of the car, the reaction time of the driver, the deceleration of the car, and the speed the car is travelling. We started by finding some basic inequalities that must be true. For example, the time of the yellow light must be greater than or equal to the time to stop plus the reaction time. Furthermore, the distance it takes to stop must be equal to the length of the GO zone, as if the car cannot stop in time, it must go, otherwise it would be stopped in the middle of the intersection. We first looked at what the distance to stop, including the reaction time by using the formula: $v^2 = v_0^2 + 2a\Delta \mathrm{x}$

Variables:

v_s = the speed limit

t_r= the driver's reaction time

x_{stop} = the distance needed to stop

x_{go} = the length of the GO zone

t_y = how long the light stays yellow

w_{int} = the distance of the intersection

l_{car} = the length of the car

a = the acceleration of the car braking (negative value)

We got that $\Delta x = \frac{-(v_s^2)}{2a}$, and the total distance to stop is $x_{stop} = \frac{-(v_s^2)}{2a} + v_s t_r$. We then looked at the distance the car can travel during a given yellow light. This would simply be $v_s t_y$. The distance the car needs to travel in order to pass safely must be greater than the distance needed to stop, the width of the intersection, and the length of the car. So, we get $t_y \cdot v_s \geq x_{stop} + w_{int} + l_{car}$. Substituting x_{stop}, we get $t_y \cdot v_s \geq \frac{-(v_s^2)}{2a} + v_s t_r + w_{int} + l_{car}$. After simplifying by isolating t_y, we get:

$$t_y \geq \frac{-v_s}{2a} + \frac{w_{int} + l_{car}}{v_s} + t_r$$

We initially weren't sure if an inequality would be the best solution because it would result in the stop-and-go zone overlapping. If we wanted the stop-and-go zone to not overlap, instead of an inequality, we thought we should make it an equation.

3. **Solution**:

Based on the inequality made at the end of the process, the solution to this problem is:

$$t_y = \frac{-v_s}{2a} + \frac{w_{int} + l_{car}}{v_s} + t_r$$

Where:

v_s = the speed limit

t_r= the driver's reaction time

t_y = how long the light stays yellow

w_{int} = the distance of the intersection

l_{car} = the length of the car

a = the acceleration of the car braking (negative value)

Note: If t_y is longer, you would have an overlap of the stop and go zones. This creates an area where a driver can either make it through the intersection or stop safely.

If v_s increases, the first term increases linearly, and the second term approaches zero, so generally the t_y would increase.

The reaction time is constant, which makes sense, as any time would delay the traffic light by the same amount.

The decreasing acceleration (a) away from zero results in the first term decreasing, which would decrease the yellow light time. This makes sense because if a car has better braking, it will need less yellow light time to stop.

Increasing either w_{int} and l_{car} would result in t_y increasing. Longer vehicles take more time to fully exit, and wider intersections take longer to clear.

We can approach this problem in a slightly different way and check our work by calculating the lengths of the GO zone and the distance needed to stop and setting them equal to each other. In order to prevent a gap between the two zones, $x_{go} \geq x_{stop}$.

$$x_{stop} = \frac{-(v_s^2)}{2a} + v_s t_r$$

$$x_{go} = v_s t_y - w_{int} - l_{car}$$

Simplifying this gets us the same equation of

$$t_y = \frac{-v_s}{2a} + \frac{w_{int} + l_{car}}{v_s} + t_r$$

4. **Discussion Questions**: After your description of your work, directly answer the following questions. Label your answers with the question letter:

a) What must be true about the STOP and GO zones in order for the intersection to be safe? Describe what happens at the intersection if the zones do not adhere to this rule.

> The STOP zone has to be directly following or begin before the GO zone. If the zones do not adhere to this rule the cars wouldn't be able to stop in time, they would collide, or they would not pass safely. The zones do not overlap, as the GO zone is also the zone where the car is unable to safely stop in time, so they cannot be the same.

b) How would the following conditions affect the required yellow light time? Which zone would they affect? via which variable?

1) Exceeding the speed limit
    a. Increases the GO zone as the car can pass through safely in less time
    b. Causes the STOP zone to be pushed further back as it takes longer to slow down
    c. Increases the variable v_s
    d. The yellow light time must be longer
2) Bald tires
    a. Increases the GO zone
    b. Causes the STOP zone to be pushed further back as it takes more time to slow down
    c. Increases the variable a so that it gets closer to 0.
    d. The yellow light time must be longer
3) Long vehicle such as a semi-truck
    a. The GO zone would be shorter than before for longer vehicles as it would need more time to get safely through the same amount of space.
    b. Causes STOP zone to be pushed further back
    c. Increases the variable l_{car}
    d. The yellow light time must be longer in order to make sure the vehicle can get through safely.
4) Distracted driver

a. Increases the GO zone

b. Causes the STOP zone to be pushed further back

c. Increases the variable t_r

d. The yellow light time must be longer. The minimum time that the yellow light increases is the same as t_r, as it is a constant unaffected by the other variables.

c) Would it be a good idea to mark the stop and go zones on the road before the intersection? Why or why not?

No, it would not be a good idea to mark the zones on the road. This is because the length of the zones and where one zone starts and ends is dependent on multiple factors that are unique to the car and car driver. For example, the reaction speed of the driver increases the GO zone and pushes the STOP zone further back. On top of this, the deceleration and length of the vehicle are different for every car, and the car will not always be going exactly the speed limit. This means there are a multitude of variables that change on a case-by-case basis that change the length of the GO zone and either push back the STOP zone or make it longer.

d) In the 1960's traffic engineers piloted a system that would display a countdown timer to show how much time was left before the light would turn red. Why do you think the engineers decided against the idea?

I think the engineers decided against the idea because if there was a countdown timer to show how much time was left, it would've caused more problems because in order for a car to stop fully or pass safely, the time of the yellow light has to be greater than or equal to the time to stop plus reaction time. Having access to the time left before the light turned red would rush people to just go through the light. If there were 2 seconds left before the light turned red, some people would speed up and try to go past the light, increasing the likelihood of collisions.