WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2012

Lecture 13 – Part 2

25 April 2012

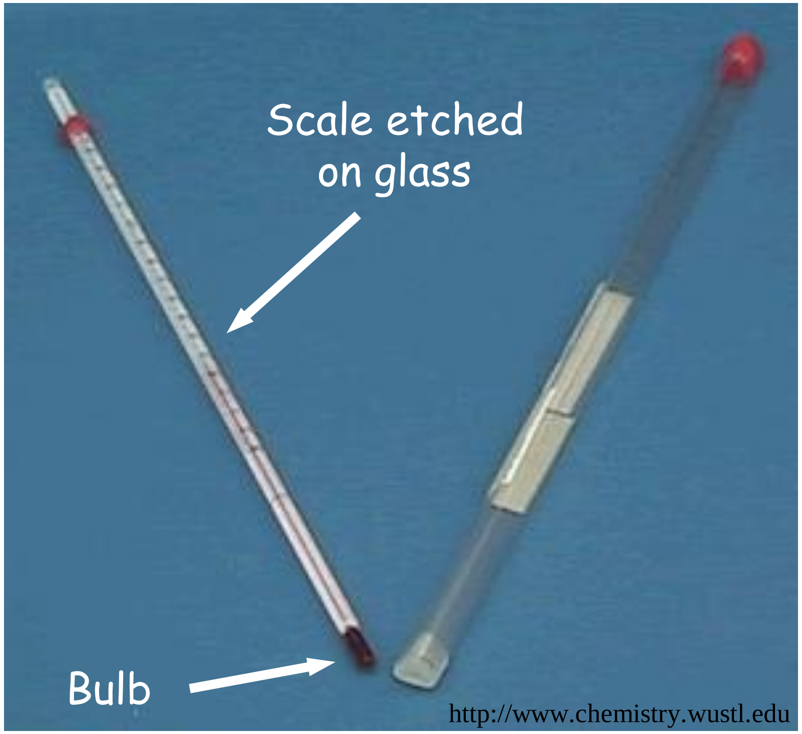
Temperature measurements

Typical devices used, which are characterized by (among other features) specific resolution, accuracy, measuring range, response time, etc:

- Thermometers
- Thermocouples
- Resistance-temperature detectors
- Thermistors and IC sensors
- Pyrometers and infrared thermometers

Thermometers

Liquid-in-glass thermometers



http://www.chemistry.wustl.edu

Estimate of a correct reading:

$$T = T_{obs} + kn\,(T_1 - T_\infty)$$

T = Corrected temperature

T_{obs} = Observed temperature

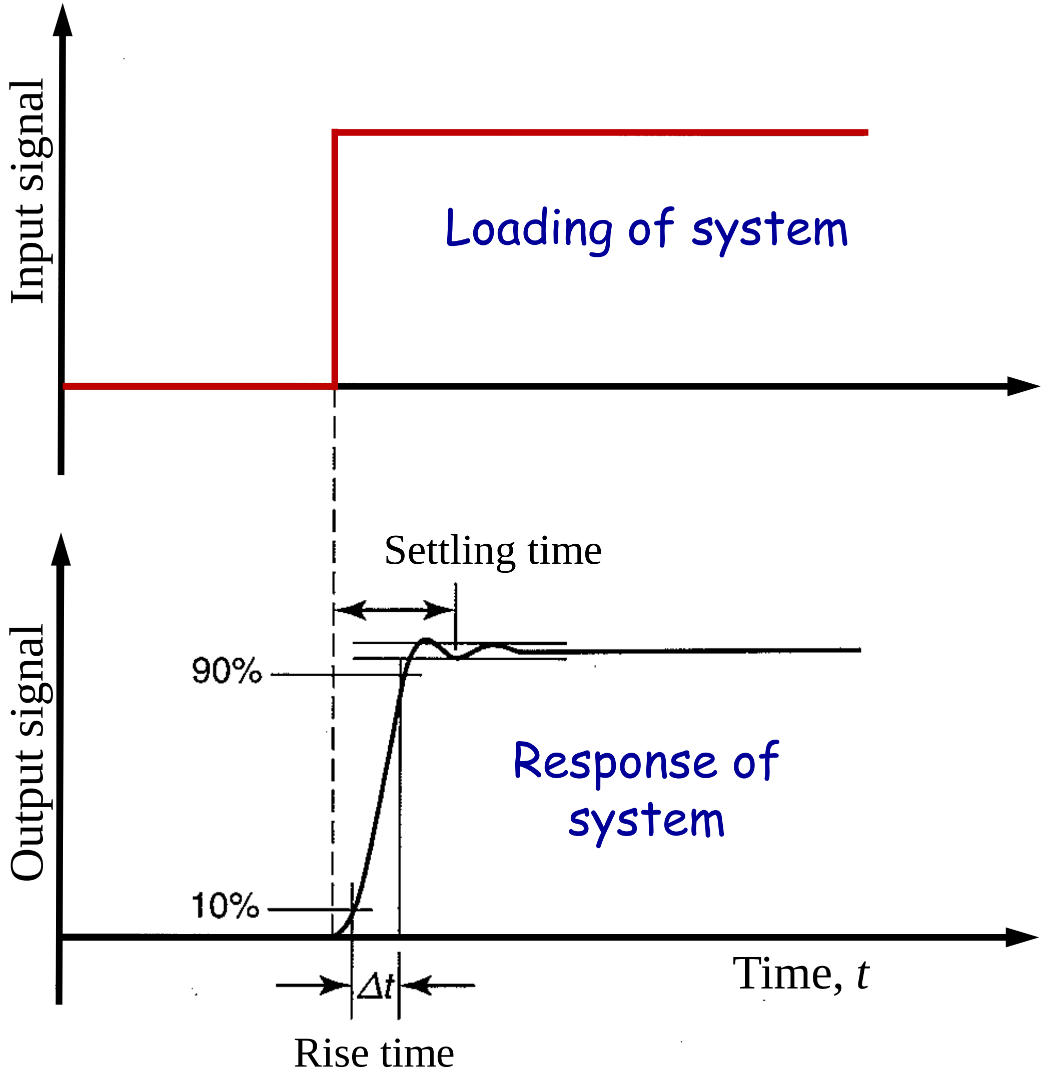
T_1 = bath temperature at total immersion

T_∞ = ambient temperature

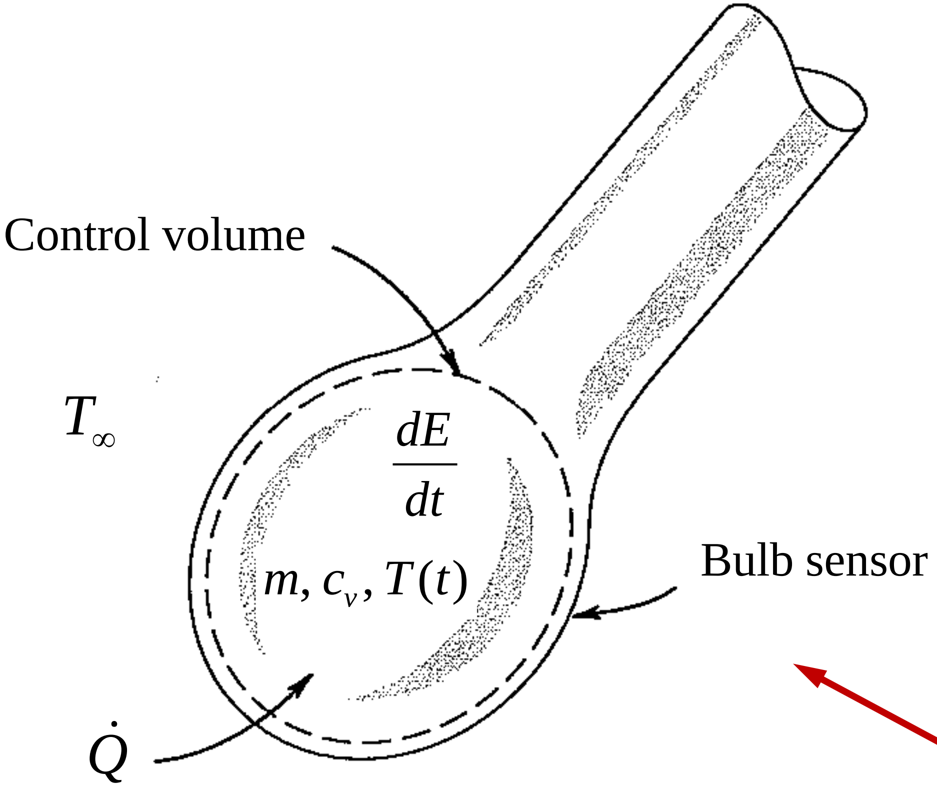
k = differential expansion coefficient between liquid and gas

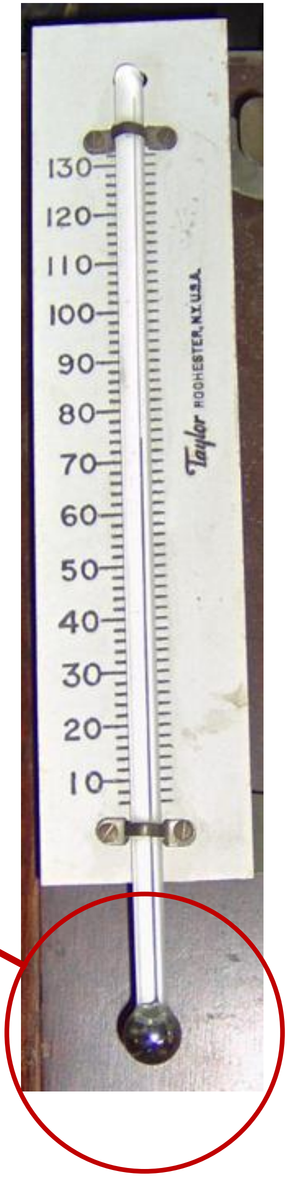
n = number of scale degrees

Response time of a system (e.g., thermometer)

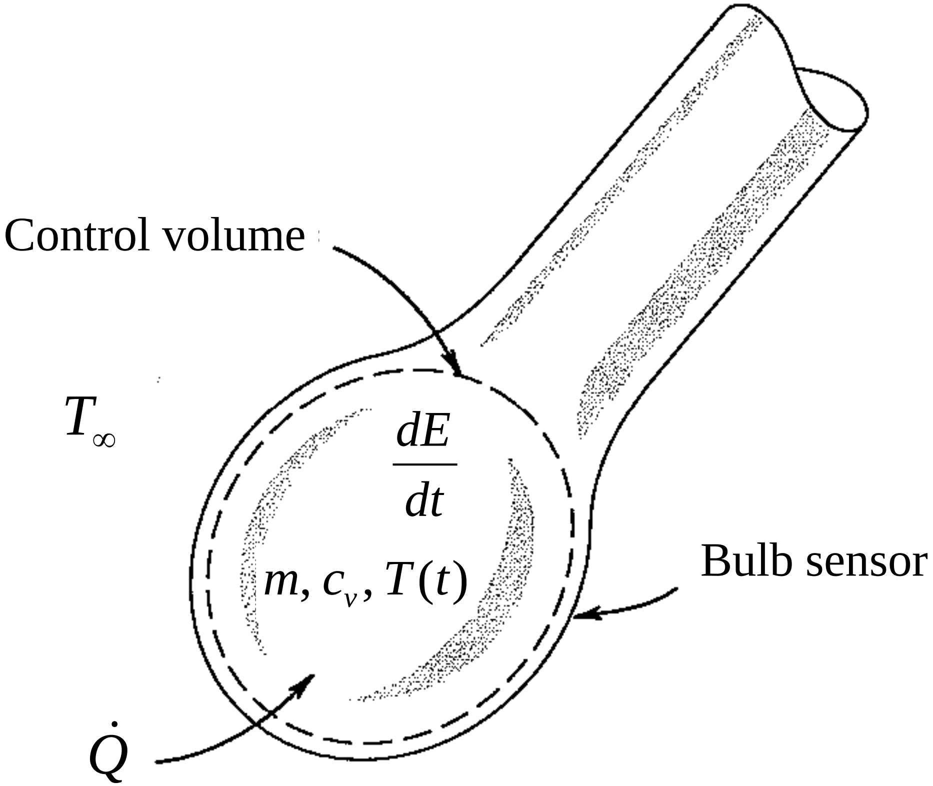


Response time of a system (e.g., thermometer)





Response time of a system (e.g., thermometer)



Conservation of energy:

$$\frac{dE}{dt} = \dot{Q}$$

$\frac{dE}{dt}$ = Energy storage within bulb

$\dot{Q}$ = Rate of energy transferred by convection

Response time of a system (e.g., thermometer)

Concept to remember: time-constant

Conservation of energy: $$\frac{dE}{dt} = \dot{Q}$$

(first order system) → $$m\,c_v \frac{dT}{dt} = h\,A_s\left[T_\infty - T(t)\right]$$

(solved in class) $$\frac{m\,c_v}{h\,A_s}\frac{dT}{dt} = T_\infty - T(t)$$

Temperature distribution: $$T(t) = T_\infty + \left[T(0) - T_\infty\right]e^{-\frac{t}{\tau}}$$

with $$\tau = \frac{m\,c_v}{h\,A_s}$$ time-constant of this system

Response time of a system (e.g., thermometer)

Example. For a bulb thermometer subjected to a step change input, calculate the 90% rise time.

Define, from previous model:

$$\Gamma(t) = \frac{T(t) - T_\infty}{T(0) - T_\infty} = e^{-\frac{t}{\tau}}$$

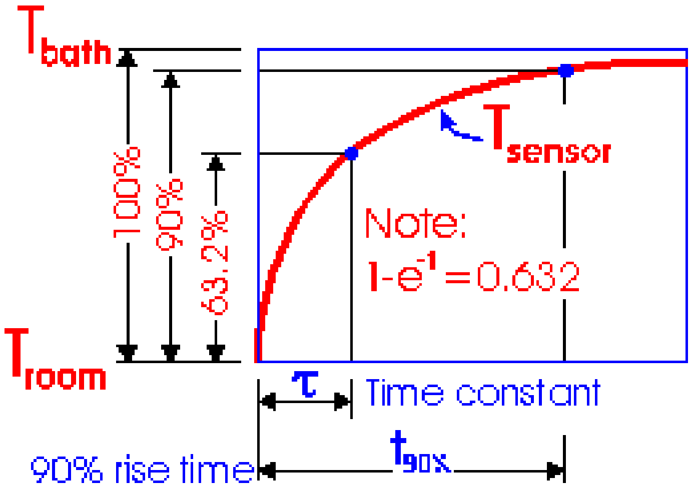
Percentage response of a system is: $[1 - \Gamma(t)] \times 100$

Therefore: $\Gamma = 0.1 = e^{-\frac{t}{\tau}}$ or $\frac{t}{\tau} = 2.3$ to achieve 90% of the applied step

Note that for: $t = \tau$ (i.e., one time constant)

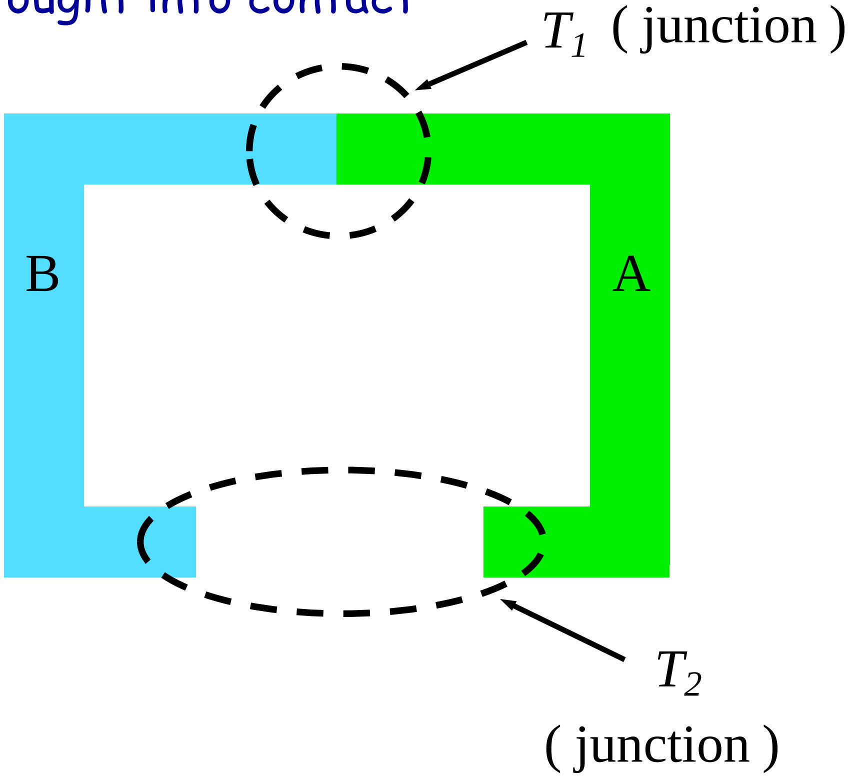
$$\Gamma = 0.368 = e^{-1}$$

or $[1 - \Gamma] \times 100 = 63.2\%$



Thermoelectricity (thermocouples)

Two different metals (A and B)
are brought into contact



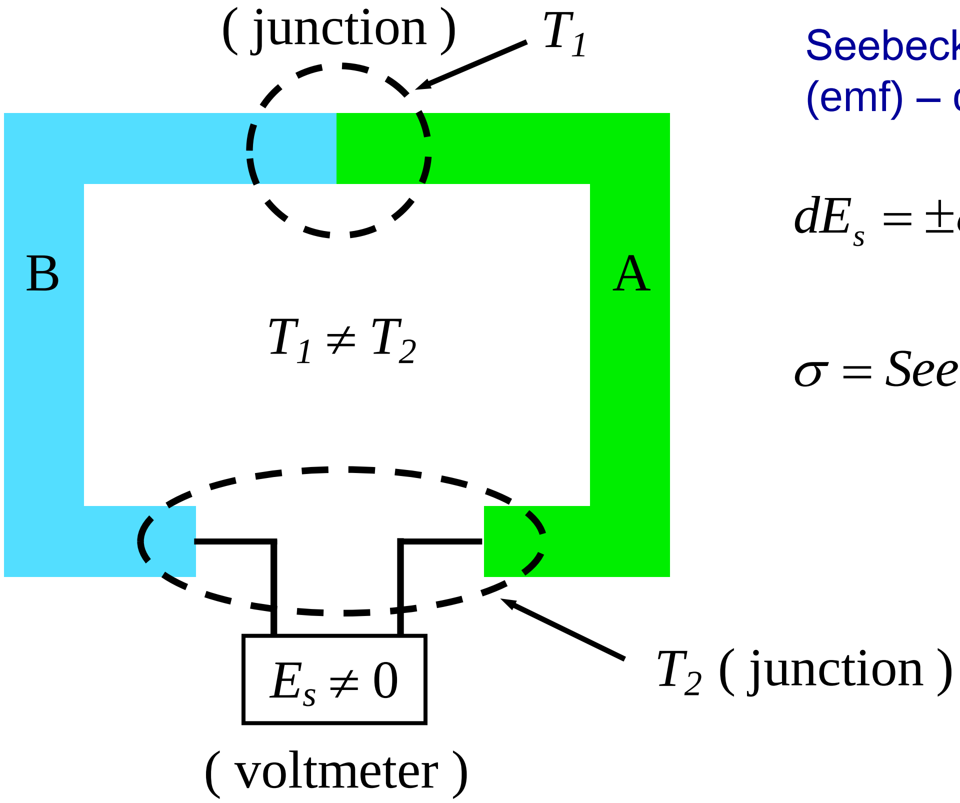
If $T_1 \neq T_2$ the following thermoelectric effects might be observed:

- Seebeck
- Peltier, and
- Thomson effects

Application: thermoelectricity (thermocouples)

Seebeck effect (reversible):

is the generation of a voltage in a circuit made with two different materials, or semiconductors, by keeping the junctions between them at different temperatures



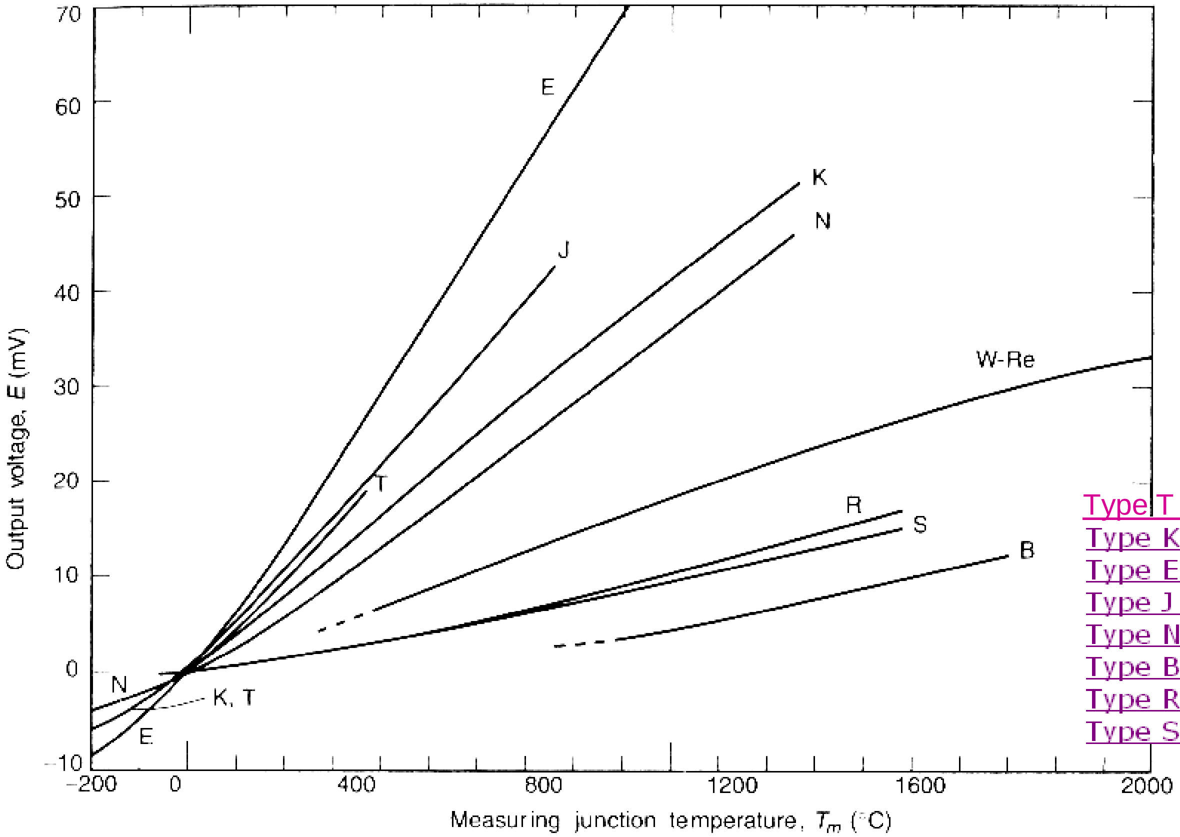
Seebeck's electromotive force (emf) – or voltage is:

$$dE_s = \pm \sigma dT \quad \Rightarrow \quad E_s = \pm \int_{T_1}^{T_2} \sigma dT\,,$$

$$\sigma = Seebeck's\ coefficient, \quad [=]\left[\frac{V}{°C}\right]$$

Application: thermoelectricity (thermocouples)

Thermocouple voltage versus temperature for reference junctions at 0 °C



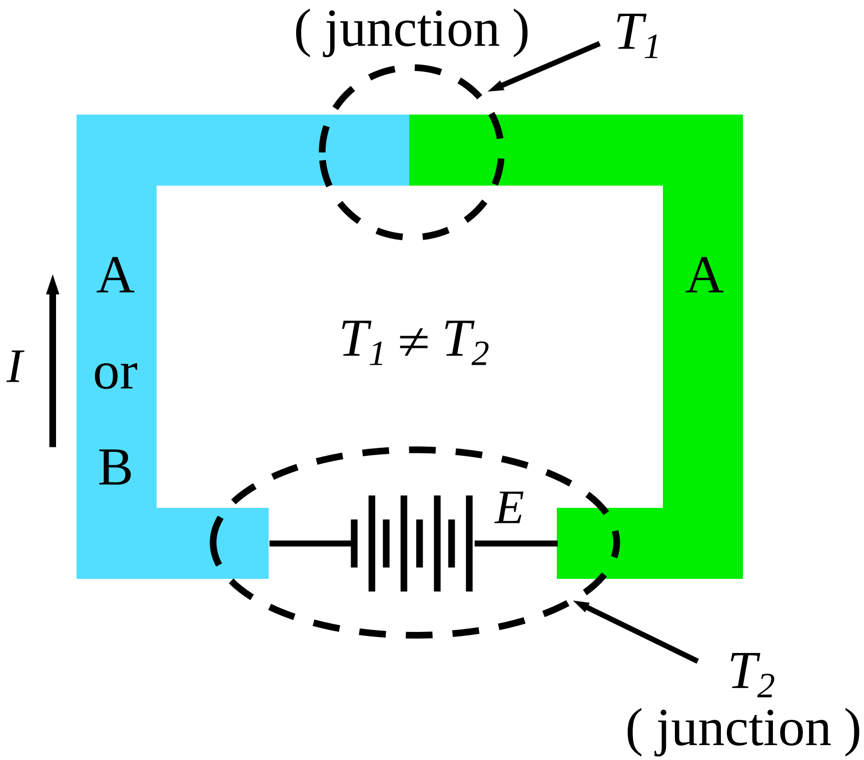
Type T (Copper / Constantan)
Type K (Chromel / Alumel)
Type E (Chromel / Constantan)
Type J (Iron / Constantan)
Type N (Nicrosil / Nisil)
Type B (Platinum / Rhodium)
Type R (Platinum / Rhodium)
Type S (Platinum / Rhodium)

Constantan is an alloy of copper and nickel with a typical composition Cu57Ni43 plus the addition of small percentages of Mn and Fe.

Thermoelectricity (TECs)

Peltier effect (reversible):

is the generation of temperature difference between the junctions of different metals as a results of an electrical current flow



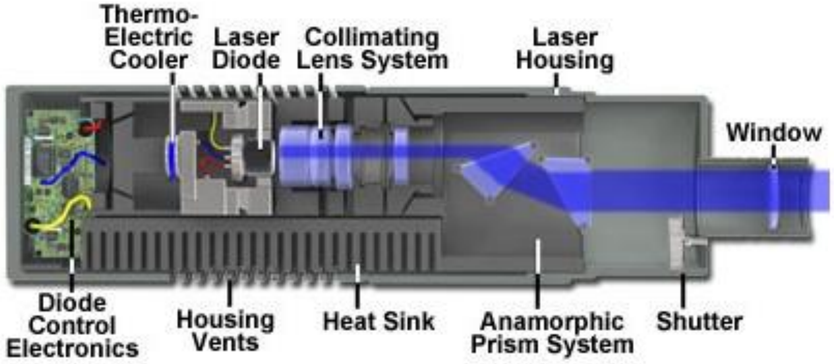
Peltier's heat rate is:

$$\frac{dQ_P}{dt} = \pm \pi I \quad [=][Watts],$$

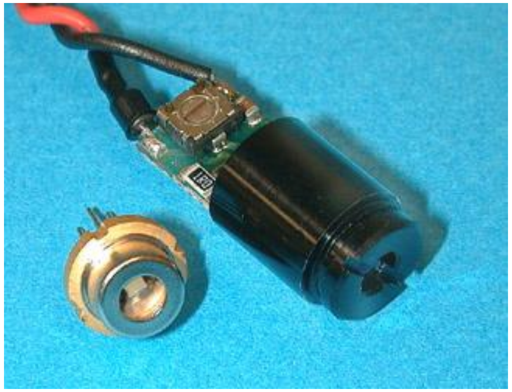
$$\pi = Peltier's\ coefficient, \quad [=][V]$$

Application: thermoelectric coolers (TECs)

Cutaway of a solid-state laser diode



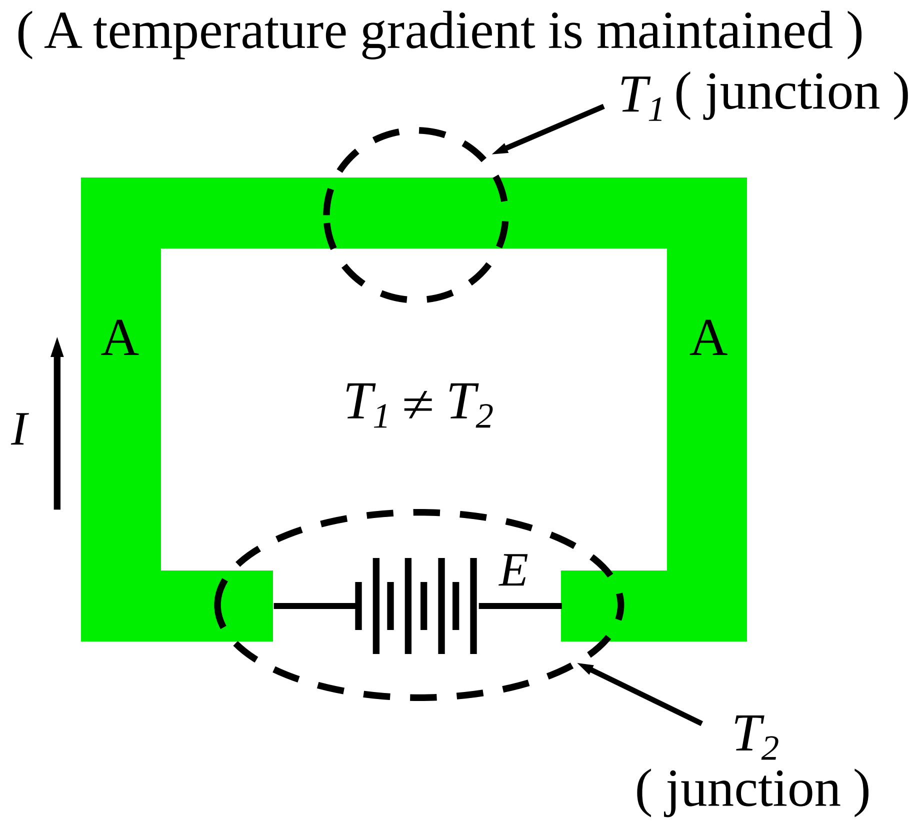
Laser diode packages



Thermoelectricity

Thomson effect (reversible):

is the the heating or cooling of a current-carrying conductor with a temperature gradient



Thomson's heat rate is:

$$\frac{dQ_T}{dt} = \pm\tau I\Delta T \qquad [=][Watts],$$

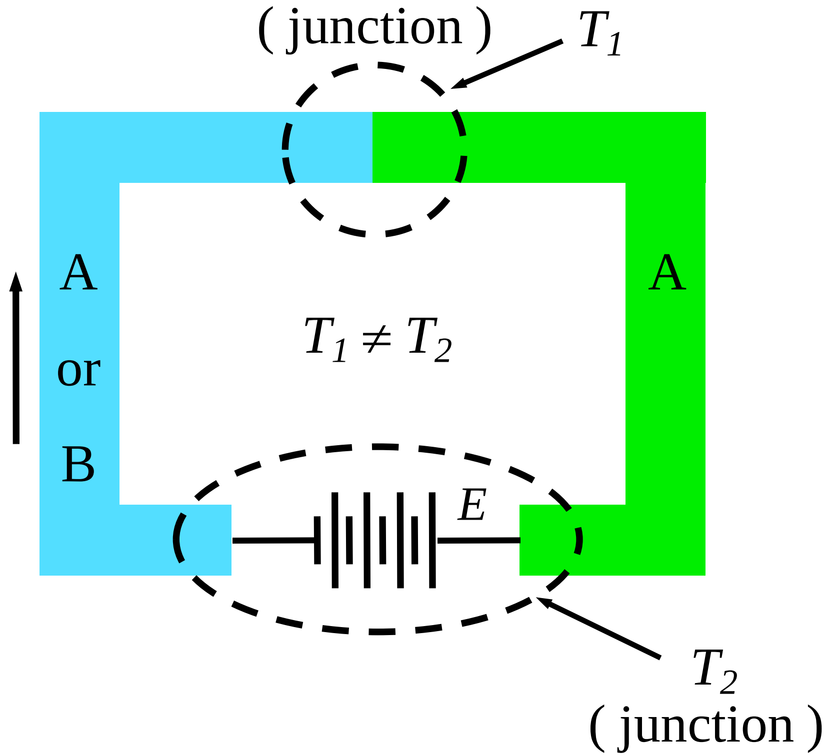
$$\tau = Thomson's\ coefficient.$$

Thomson's emf:

$$E_T = \pm\int_{T_1}^{T_2}\tau\, dT\,,$$

Thermoelectricity

The Peltier and Seebeck coefficients are related by the Thomson relation, given as: $\pi = \sigma T$ $(T \text{ expressed in absolute degrees})$

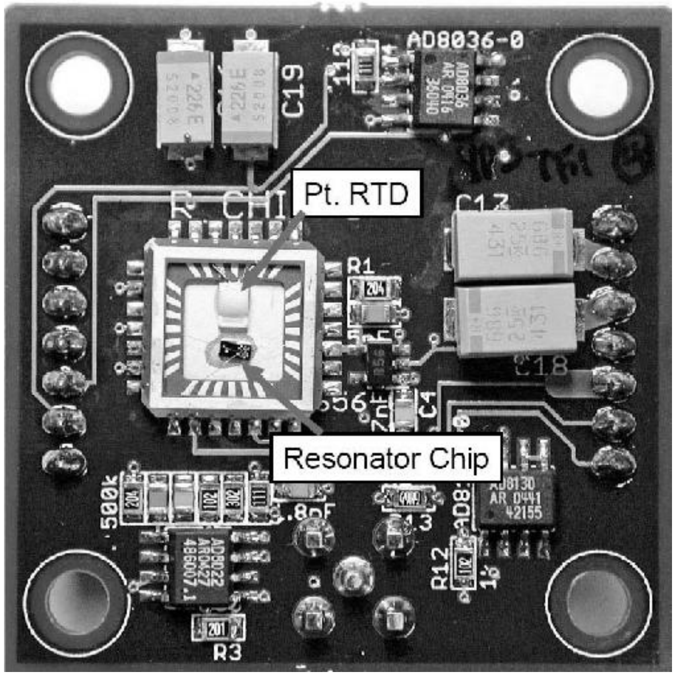


- ***Seebeck emf***: caused by the junction of different materials
- ***Peltier emf***: caused by a current flow in the circuit, and
- ***Thomson emf***: results from a temperature gradient

Resistor temperature detectors (RTDs)

Principle of operation: changes of “electrical resistance” as a function of temperature

MEMS resonator with a Pt RTD



Ref. Hopcroft et al., MEMS 2006

MEMS resonant frequency as a function of temperature

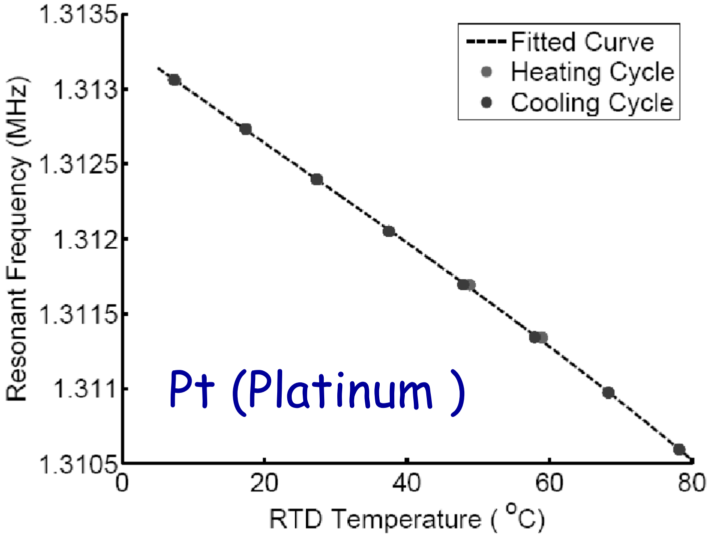
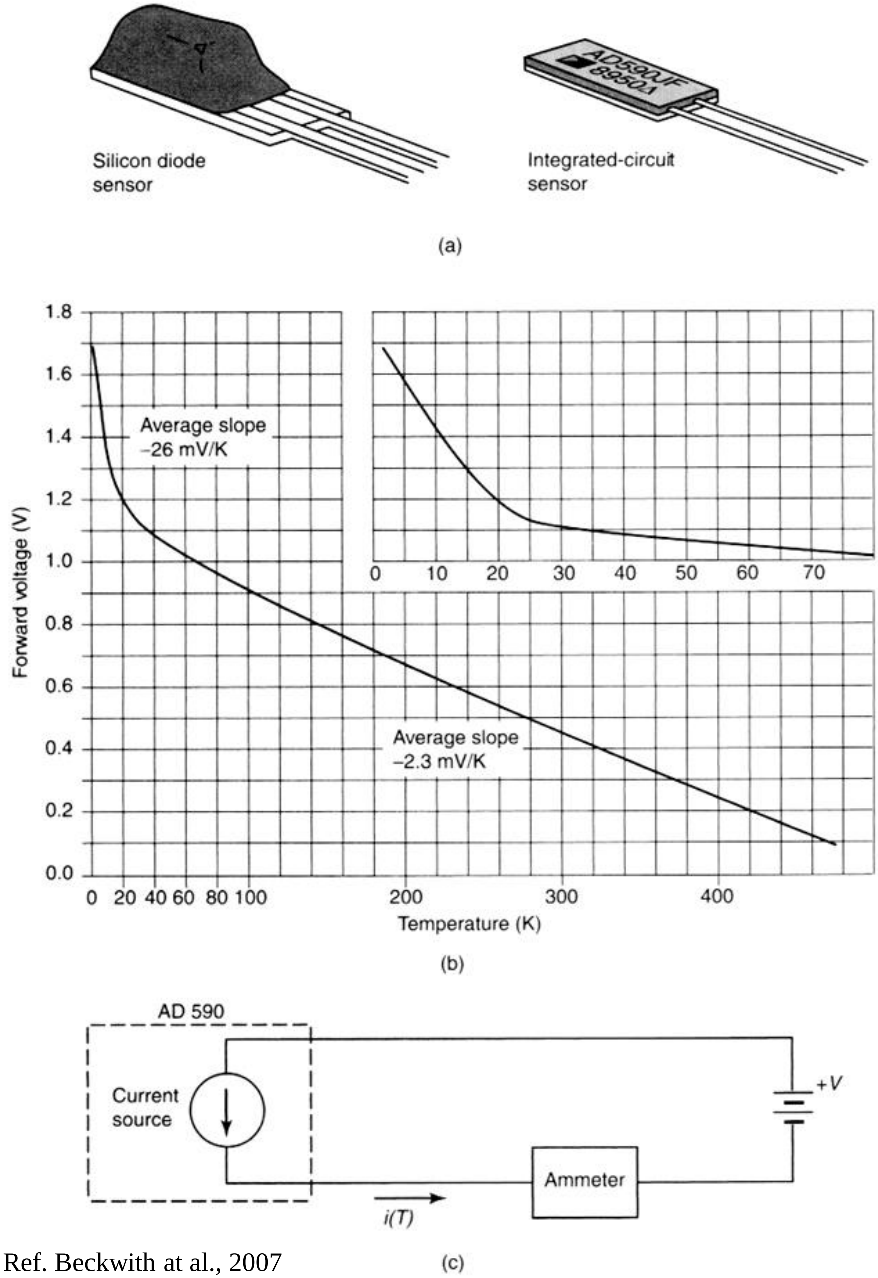


Figure 5: Resonant Frequency vs. Ambient Temperature. A single heating and cooling cycle is plotted. Each point on the graph is actually a cluster of 50 measurements taken at that temperature.

Quartz thermometers

Principle of operation: changes of “natural frequency” of quartz crystals as a function of temperature

Accuracy as high as ± 0.040 °C

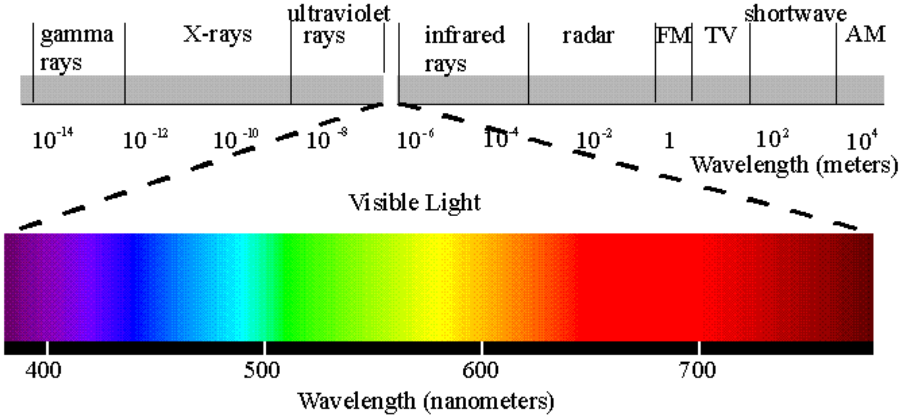


Ref. Beckwith at al., 2007

FIGURE 16.21: (a) Semiconductor junction sensors. (Courtesy: Lake Shore Cryotronics, Inc., and Analog Devices, Inc.); (b) diode forward voltage versus temperature (Lake Shore Standard Curve 10); (c) typical AD590 measuring circuit.

Pyrometers (Photodetectors)

Electromagnetic spectrum



Spectral-band pyrometer

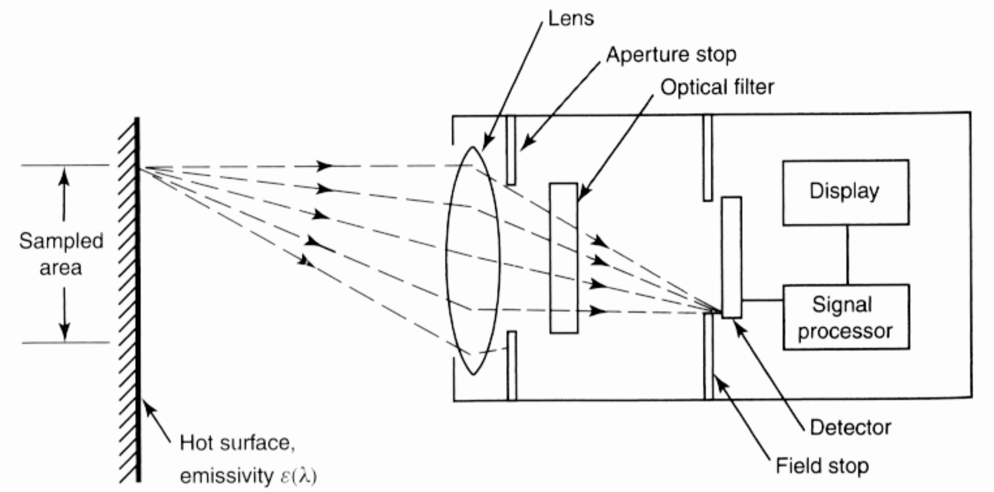
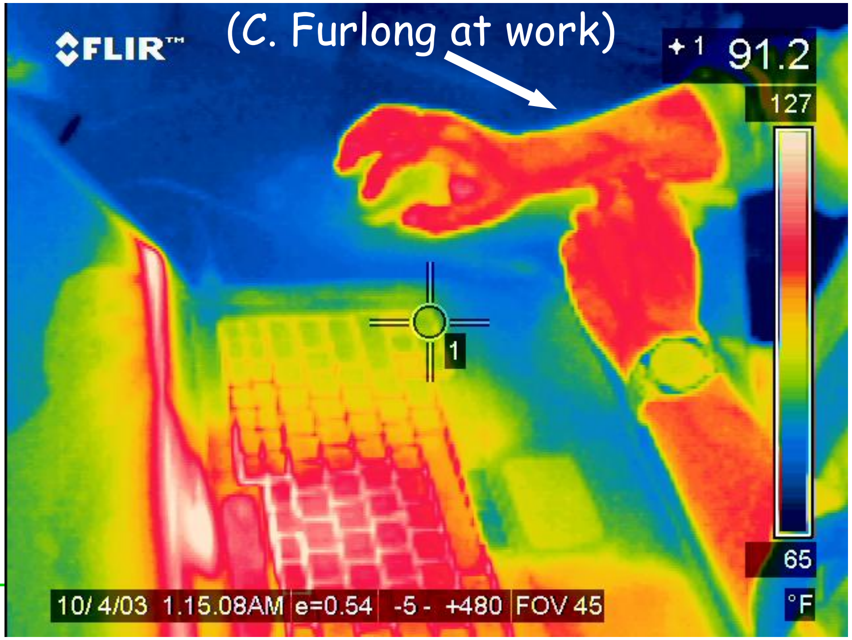


FIGURE 16.26: Schematic diagram of a spectral-band pyrometer. Light rays are shown leaving on edge of the sampled area to illustrate that the field stop limits the image size whereas the aperture stop limits the amount of light collected. Similar rays may be drawn from any point in the sampled area.

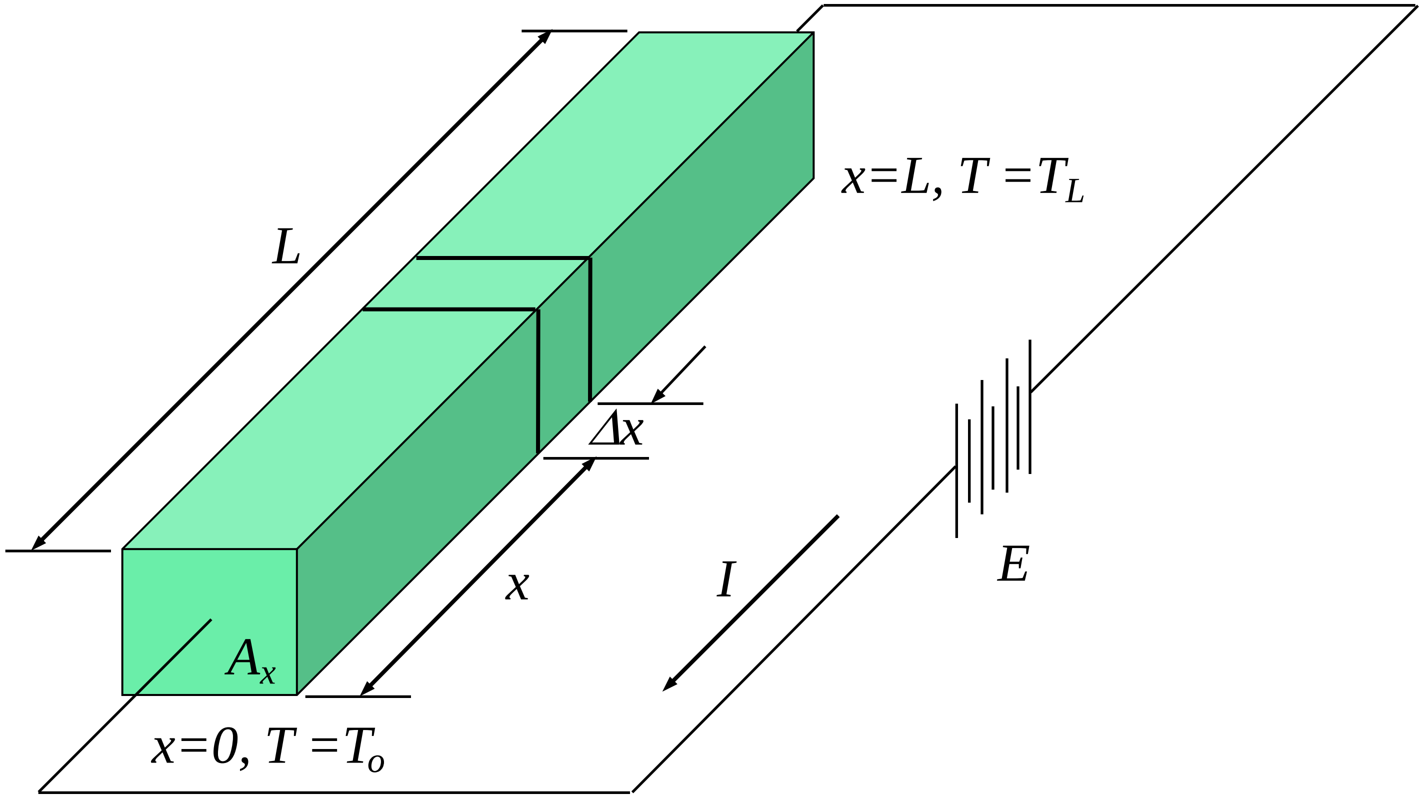
Infrared camera



(C. Furlong at work)

Heat absorbed at the cold junction of TECs

Single thermoelectric element



Heat absorbed at the cold junction of TECs

Single thermoelectric element: *energy balance*

$$\left(Heat\ entering\ x\right)+\left(Heat\ generated\ within\ \Delta x\right)=\left(Heat\ exiting\ at\ x+\Delta x\right),$$

therefore, $$-kA_x\left.\frac{dT}{dx}\right|_x+I^2\frac{\rho_e\Delta x}{A_x}=-kA_x\left.\frac{dT}{dx}\right|_{x+\Delta x},$$

re-ordering and diving by Δx, $$\frac{kA_x\left.\frac{dT}{dx}\right|_{x+\Delta x}-kA_x\left.\frac{dT}{dx}\right|_x}{\Delta x}=-I^2\frac{\rho_e}{A_x},$$

taking the limit when $\Delta x\rightarrow 0$, $$\lim_{\Delta x\rightarrow 0}\left\{\frac{kA_x\left.\frac{dT}{dx}\right|_{x+\Delta x}-kA_x\left.\frac{dT}{dx}\right|_x}{\Delta x}\right\}=-I^2\frac{\rho_e}{A_x},$$

governing ODE is: $$\frac{d}{dx}\left(kA_x\frac{dT}{dx}\right)=-I^2\frac{\rho_e}{A_x}.$$

Heat absorbed at the cold junction of TECs

Single thermoelectric element: *governing ODE*

Governing ODE is:

$$k A_x \frac{d^2T}{dx^2} + I^2 \frac{\rho_e}{A_x} = 0 \quad \Rightarrow \quad \frac{d^2T}{dx^2} + \frac{I^2 \rho_e}{k A_x^2} = 0.$$

Solution requires
double integration
and use of BCs:

$$\begin{cases} T(x=0) = T_0 \\ T(x=L) = T_L \end{cases}$$

Corresponding
temperature
distribution is:

$$\boxed{T(x) = -\frac{I^2 \rho_e}{2k A_x^2} x^2 - \left(\frac{T_0 - T_L}{L} - \frac{I^2 \rho_e L}{2k A_x^2} \right) x + T_0 \ ,}$$

with maximum
value at:

$$\frac{dT(x)}{dx} = 0 \quad \Rightarrow \quad x_{\max} = \frac{L}{2} - \frac{k(T_0 - T_L) A_x^2}{I^2 \rho_e L} = \frac{L}{2} - \frac{k \Delta T A_x^2}{I^2 \rho_e L}.$$

Heat absorbed at the cold junction of TECs

Single thermoelectric element: *introducing cold junction*

With the cold junction (CJ) located at $x = L$, fraction transferred to the CJ is:

$$f_c = \frac{L - x_{\max}}{L} = \frac{1}{2} + \frac{k \Delta T A_x^2}{I^2 \rho_e L^2} = \frac{1}{2} + \frac{k \Delta T A_x}{I^2 R_e L},$$

Net heat absorbed = $\textit{Peltier heat} - \textit{Joule loss}\,(\textit{attributable to the CJ}),$

$$\boxed{Q = Q_{net} = \pi I - f_c I^2 R_e = \sigma T_c I - f_c I^2 R_e\,,}$$

$$T_c = \textit{temperature in absolute scale}.$$

Heat absorbed at the cold junction of TECs

Single thermoelectric element: *introducing cold junction*

Net heat absorbed = $Peltier\ heat - Joule\ loss\ (attributable\ to\ the\ CJ),$

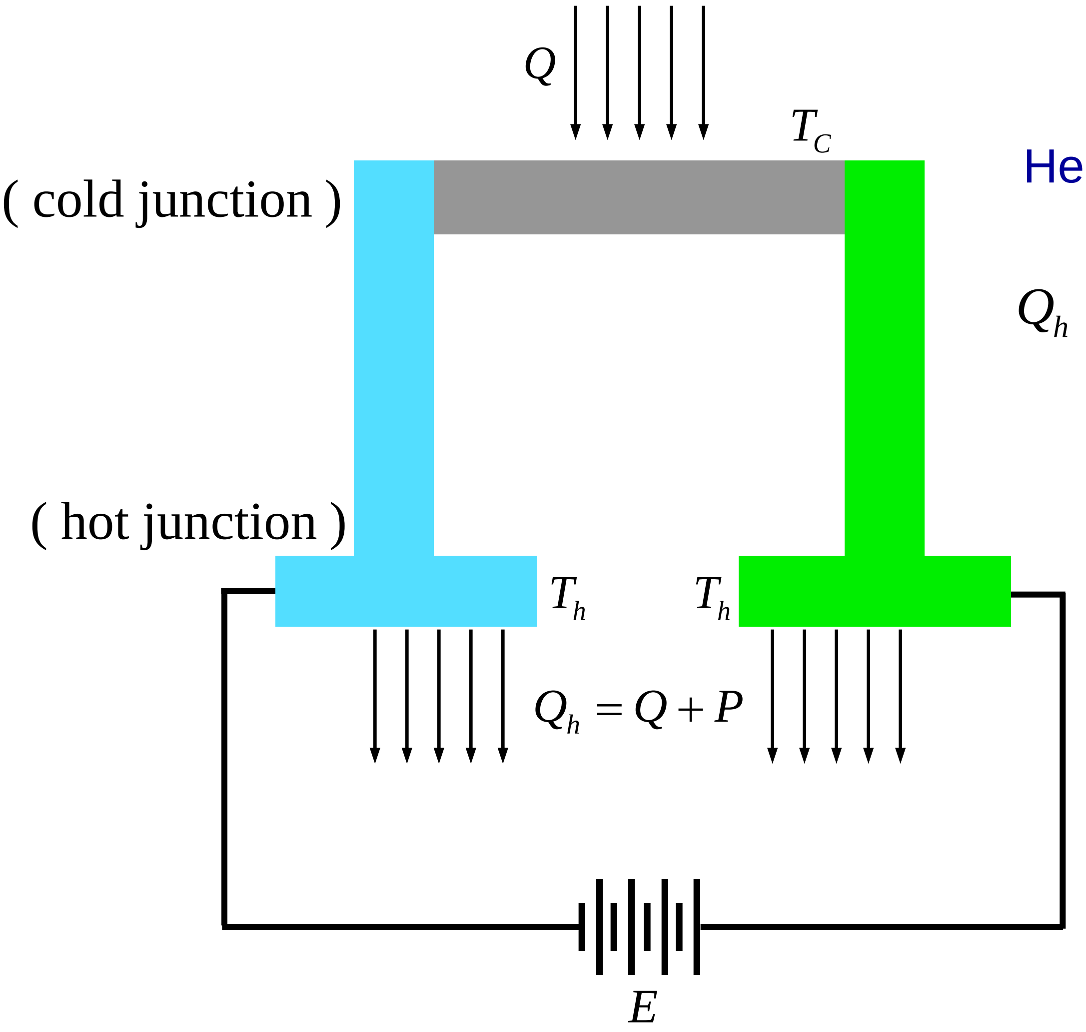
$$\boxed{Q = \sigma T_c I - \frac{I^2 R_e}{2} - \frac{k A_x \Delta T}{L},}$$

Heat at the CJ

Joule heat

Conduction
(leak)

Maximum heat pumping



Heat removed at the *hot junction* is:

$$Q_h = Q + P = Q + IV,$$

Maximum heat pumping

Heat removed at the *hot junction* is:

$$Q_h = Q + P = Q + IV, \quad \left(V = \sigma T_h - \sigma T_c + IR_e\right)$$

$$= Q + I(\sigma \Delta T + IR_e) = Q + \sigma I \Delta T + I^2 R_e \, .$$

Coefficient of performance (COP) = $\dfrac{desired\ effect}{effort\ to\ achieve\ desired\ effect} = \dfrac{Q}{P}$,

$$\boxed{COP = \frac{\sigma T_c I - \dfrac{I^2 R_e}{2} - \dfrac{k A_x \Delta T}{L}}{\sigma I \Delta T + I^2 R_e} ,}$$