

# WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation  
ME-3901, D'2012

Lecture 12  
25 April 2012



# General information

## Office hours

Instructors: **Cosme Furlong**

Office: HL-151

**Everyday:**

**9:00 to 9:50 am**

**Christopher Scarpino**

Office: HL-153

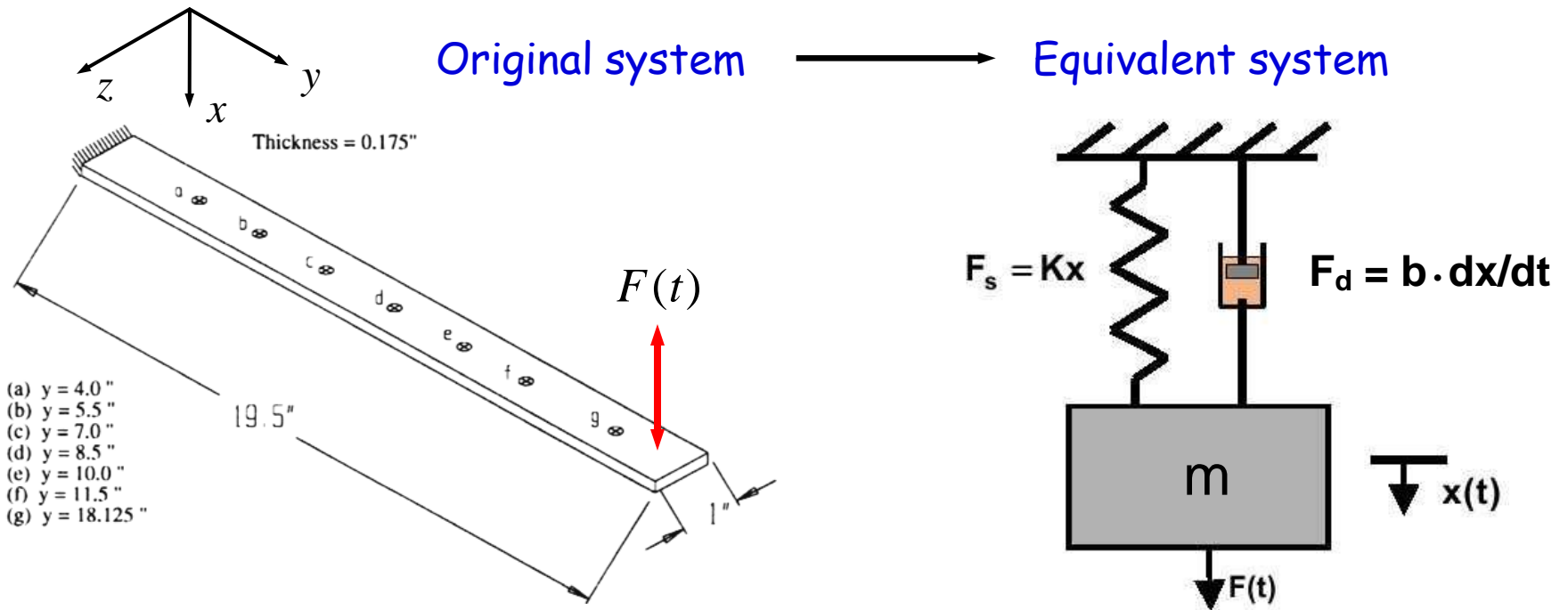
**During laboratory**

**sessions**

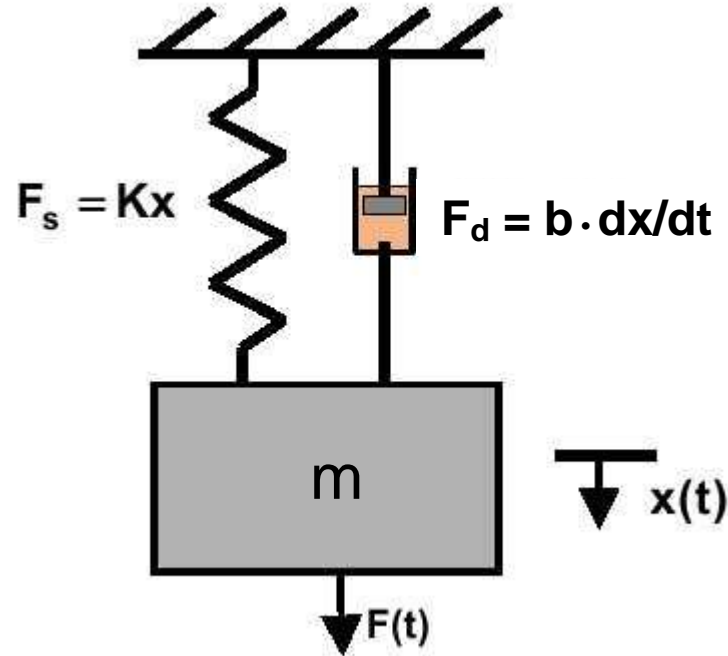
Teaching Assistants: **During laboratory sessions**



# Equivalent systems



# Analysis of a single degree of freedom system



Governing equation: 
$$m \frac{d^2 x}{dt^2} = \sum_i F_i$$

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} + F(t)$$

← External force



# Analysis of a single degree of freedom system

First case:  $F(t) \neq 0$  – Forced vibrations

Governing equation: 
$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = F(t)$$

Governing equation can be written as:

$$\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F(t)}{m} \quad \longrightarrow \quad \frac{d^2 x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = F'(t)$$

Solution has the form:

$$x(t) = x_c(t) + x_p(t)$$

$x_c(t)$  Complementary solution

$x_p(t)$  Particular solution

Solution requires the method of *undetermined coefficients*



# Analysis of a single degree of freedom system

First case:  $F(t) \neq 0$  – Forced vibrations

Governing equation:  $\rightarrow \frac{d^2 x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = F_o \sin(\gamma t)$

Solution has the form:  $x(t) = x_c(t) + x_p(t)$

$$x(t) = \underbrace{Ae^{-\lambda t} \sin(\sqrt{\omega^2 - \lambda^2} t + \phi)}_{\text{transient}} + \underbrace{\frac{F_o}{\sqrt{(\omega^2 - \lambda^2)^2 + 4\lambda^2 \gamma^2}} \sin(\gamma t + \theta)}_{\text{steady state}}$$

with:  $\sin(\phi) = \frac{C_1}{A}$  ;  $\sin(\theta) = \frac{-2\lambda\gamma}{\sqrt{(\omega^2 - \lambda^2)^2 + 4\lambda^2 \gamma^2}}$

$\cos(\phi) = \frac{C_2}{A}$  ;  $\cos(\theta) = \frac{\omega^2 - \lambda^2}{\sqrt{(\omega^2 - \lambda^2)^2 + 4\lambda^2 \gamma^2}}$



# Analysis of a single degree of freedom system

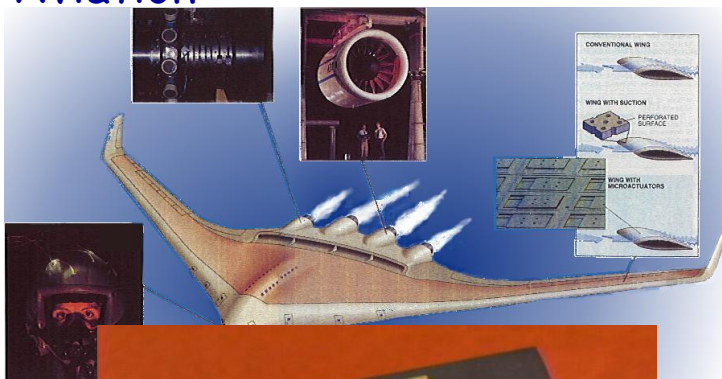
First case:  $F(t) \neq 0$  – Forced vibrations

**REVIEW - Lecture 10: section on accelerometers**

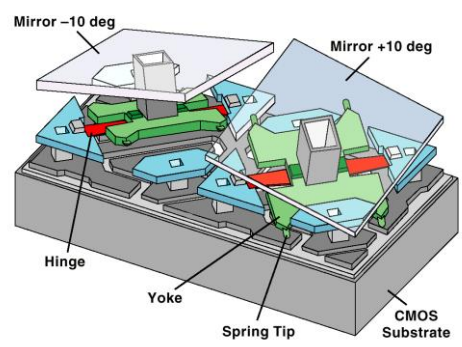
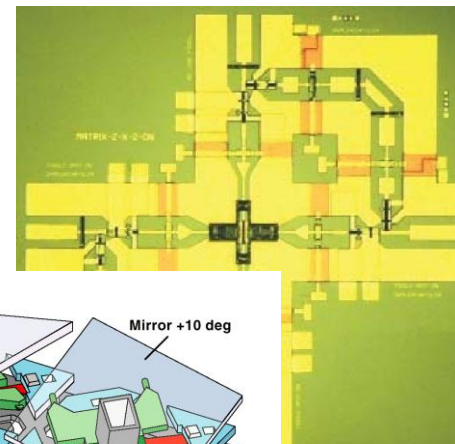


# MEMS sensors & actuators: some applications

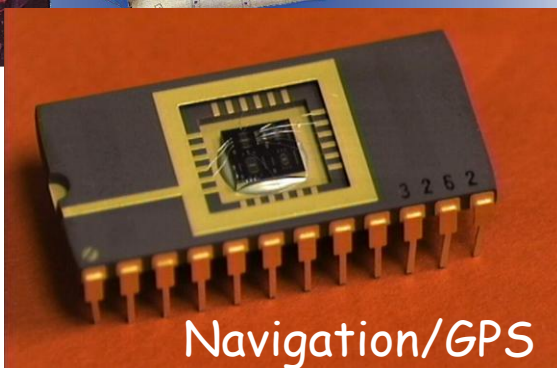
Aviation



Microelectronics: RF MEMS



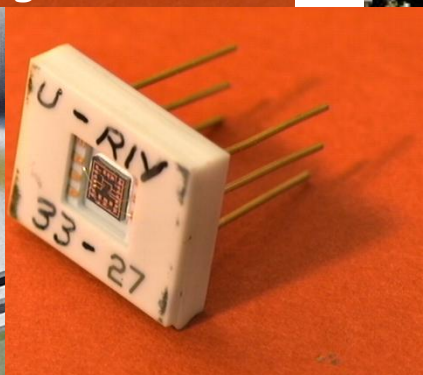
Photonics



Navigation/GPS



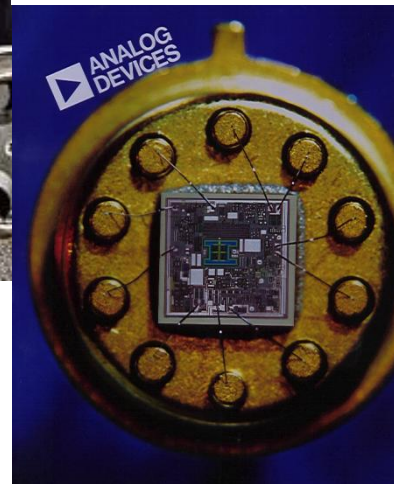
Info-tech



Controls



Segway: sensors



Automotive

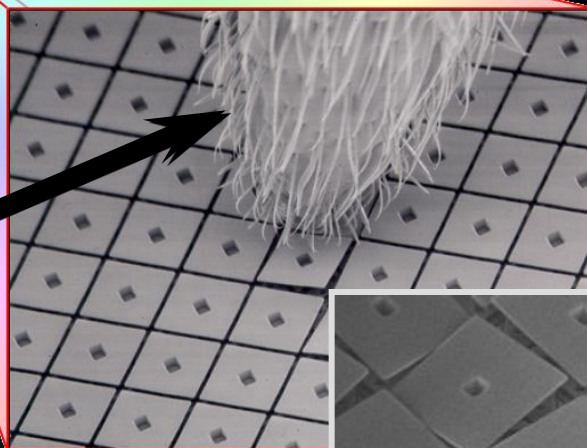
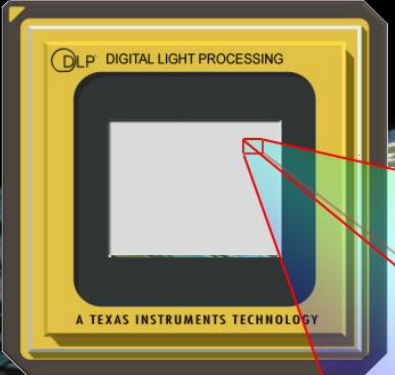


Tele-com



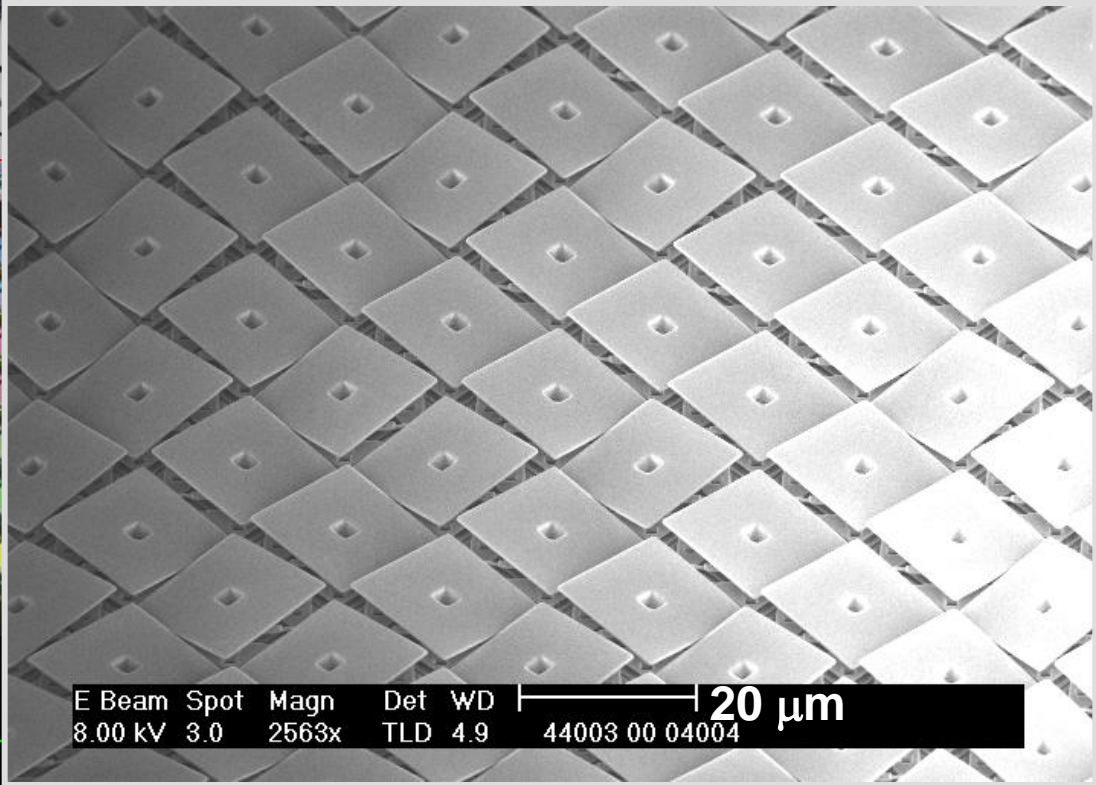


# The Digital Micromirror Device



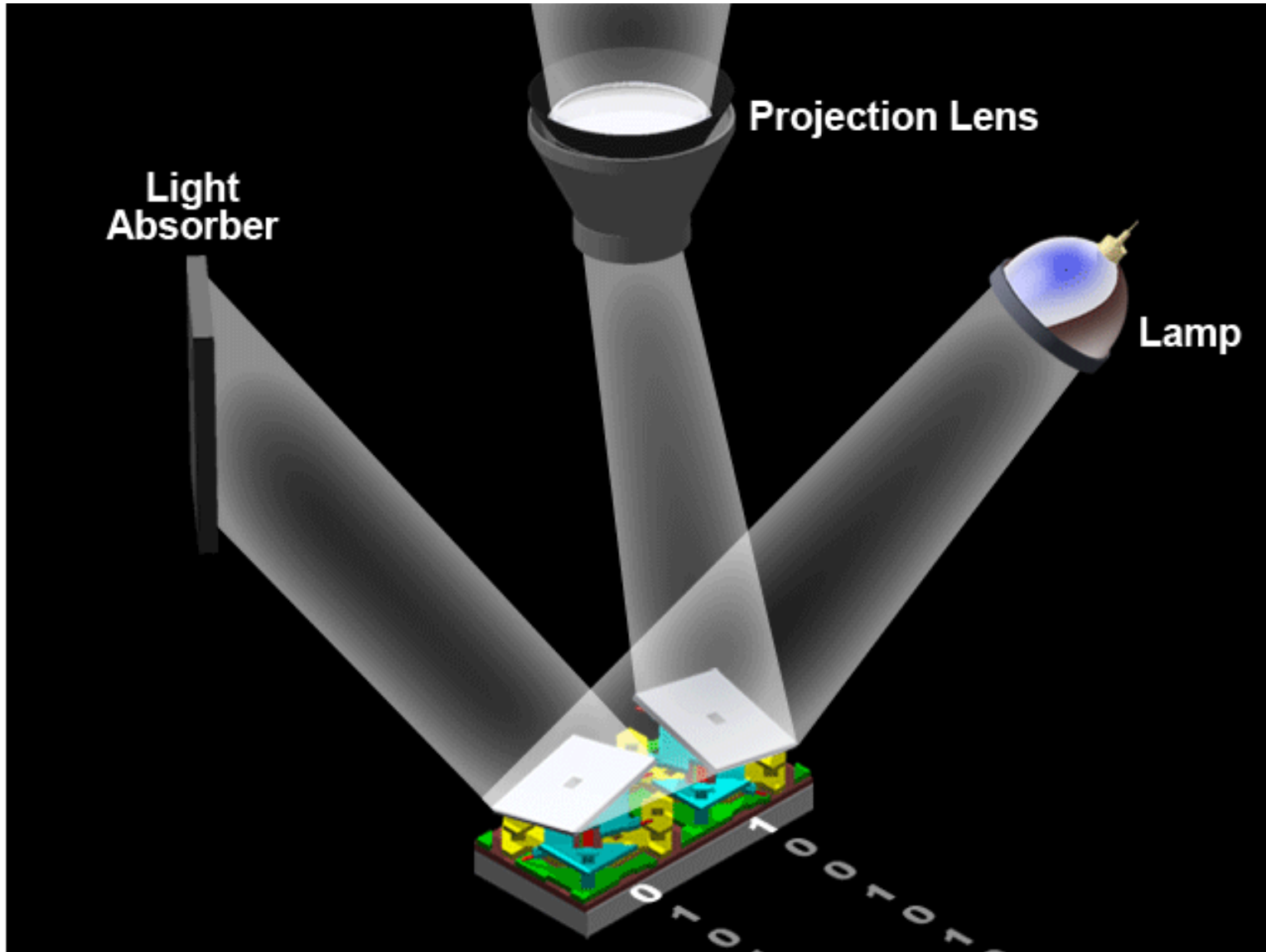
Close-up of Chip Surface

Ants Foot



<http://www.dlp.com/>

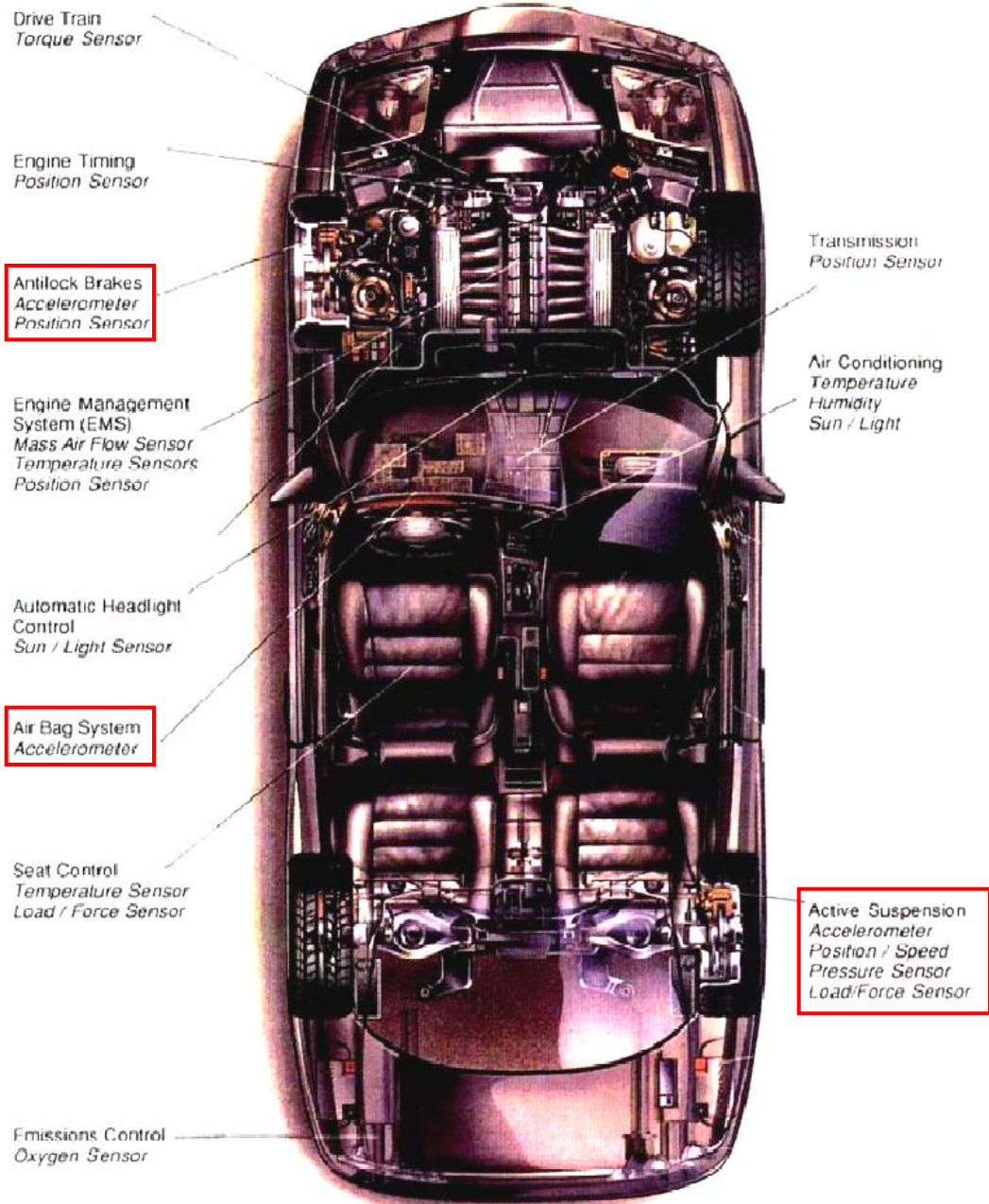
# How DMD™ Technology works



<http://www.dlp.com/>

# MEMS sensors: some applications

(MEMS - Microelectromechanical Systems)



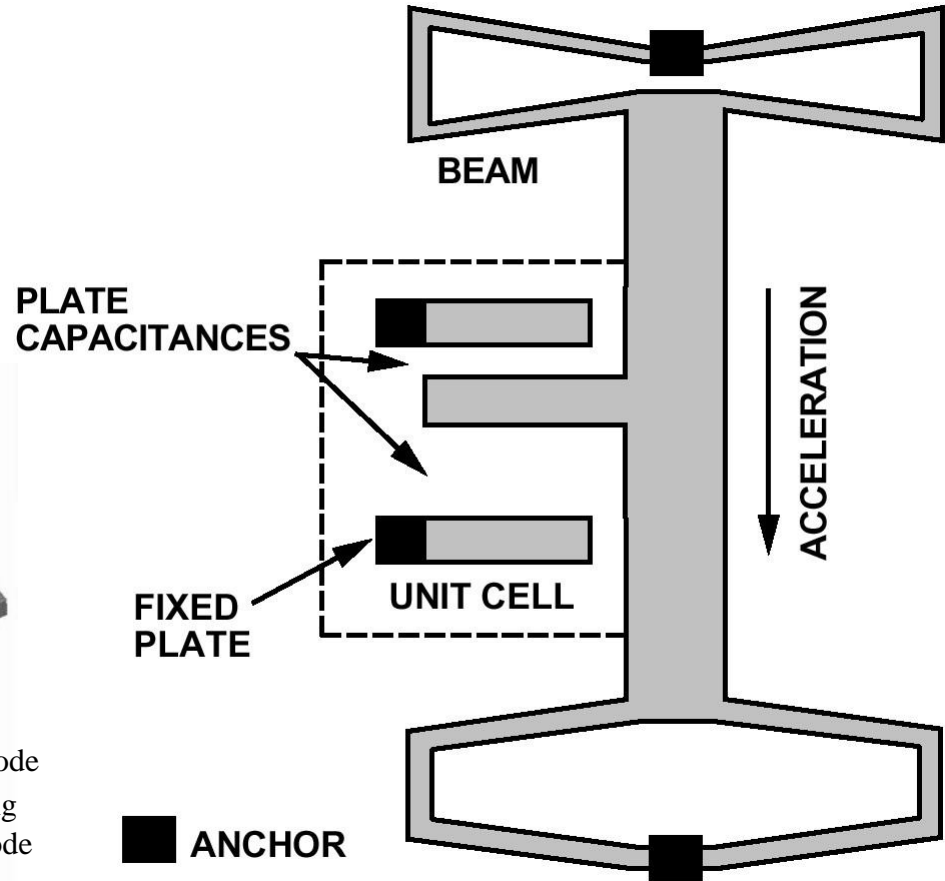
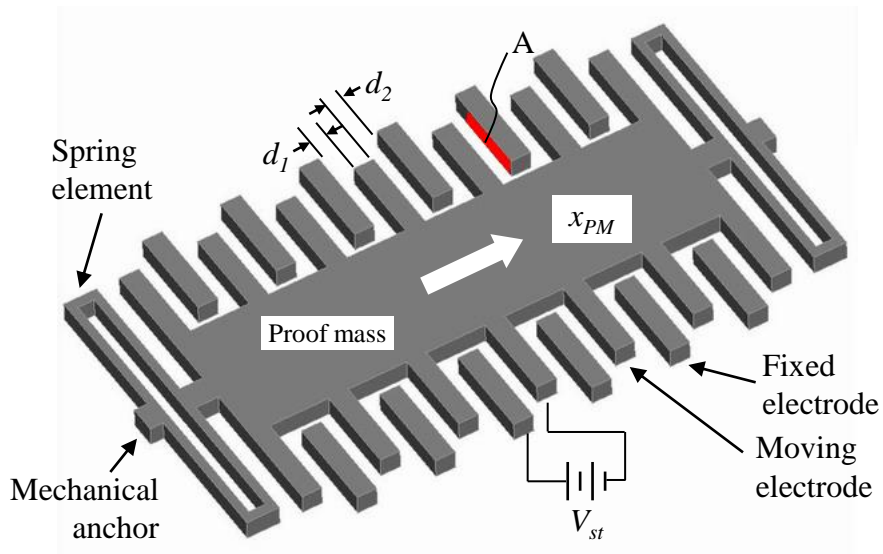


# MEMS accelerometers

Simplified view of a sensor subjected to acceleration

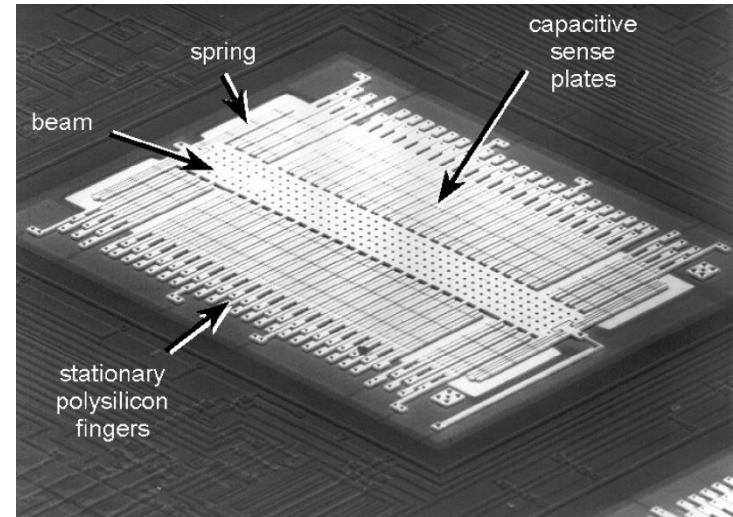
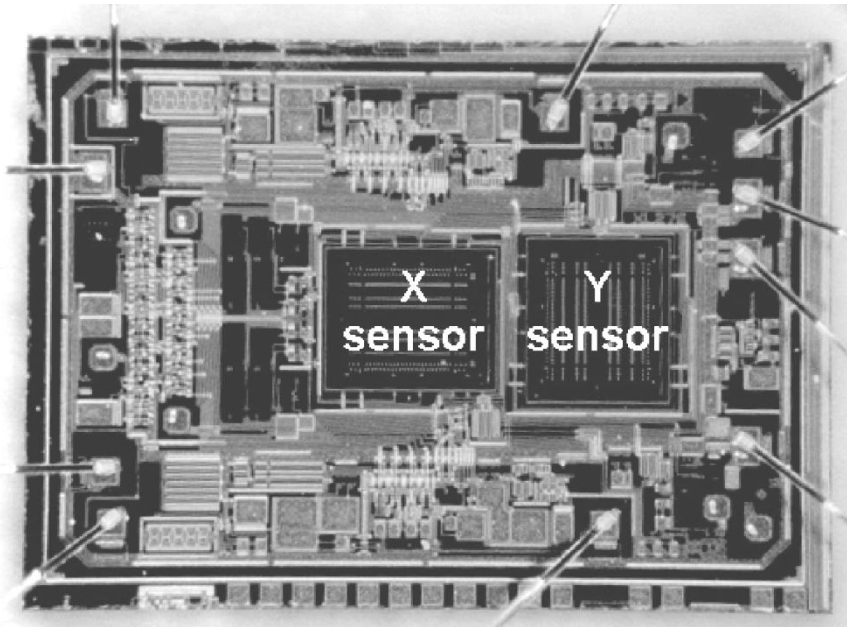
Capacitance  $\rightarrow C = \epsilon_r \epsilon_0 \frac{A}{d}$

Single axis capacitive MEMS inertial sensor

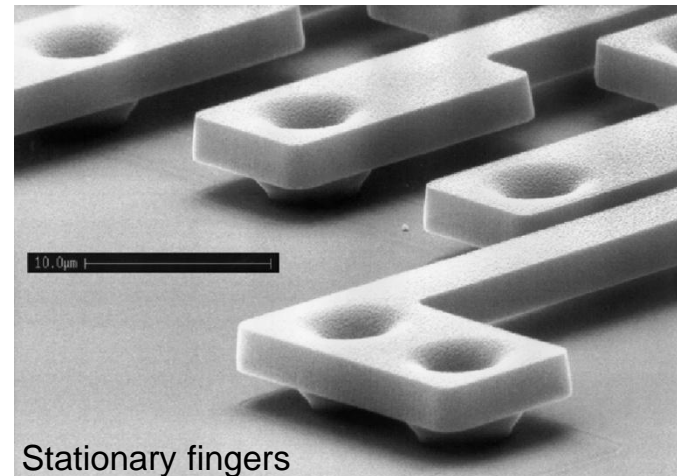
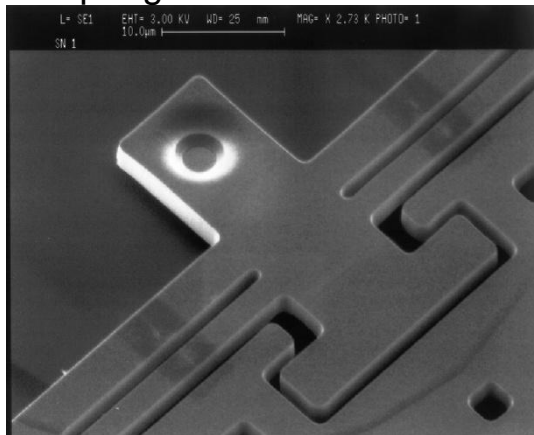


# MEMS accelerometers: ADXL 150

## Dual-axes accelerometers



Spring attachment: anchor

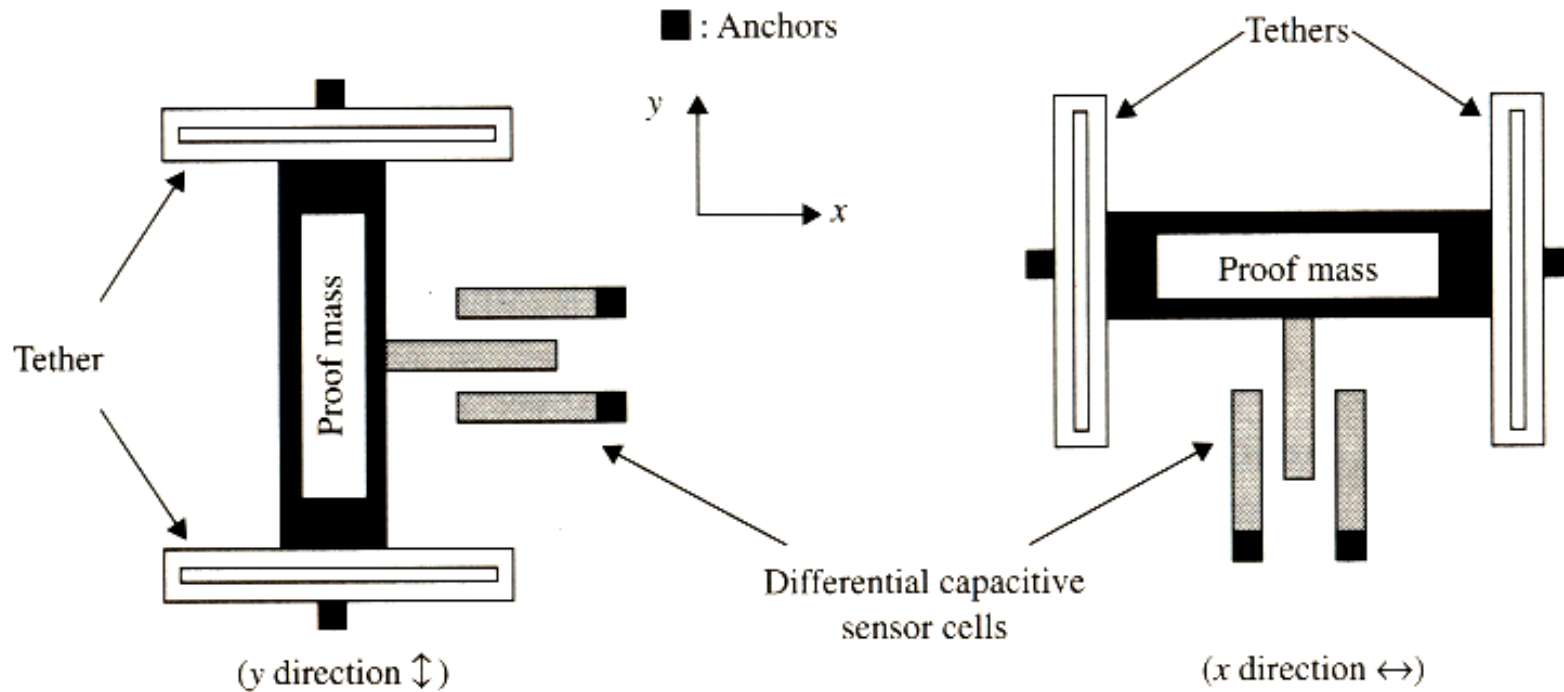


Stationary fingers



# MEMS accelerometers: ADXL 150

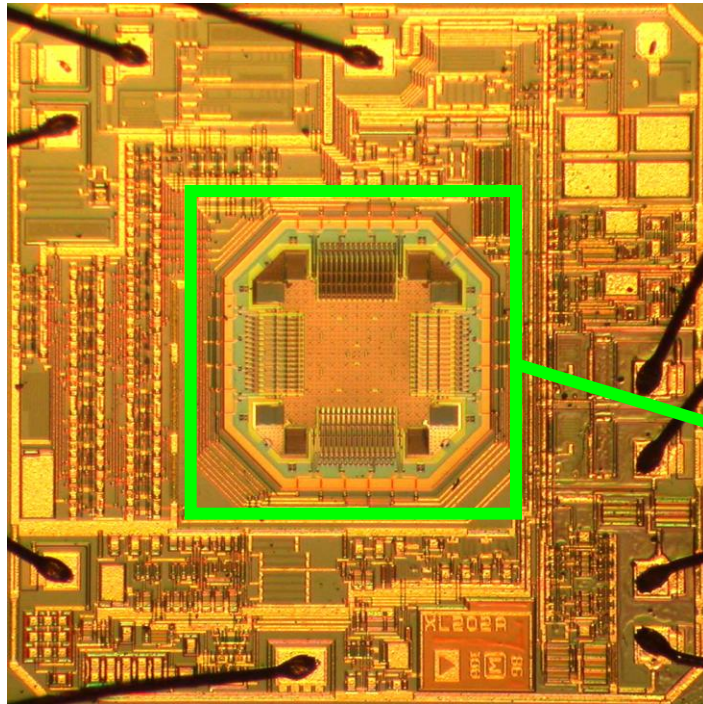
## Dual-axes accelerometers





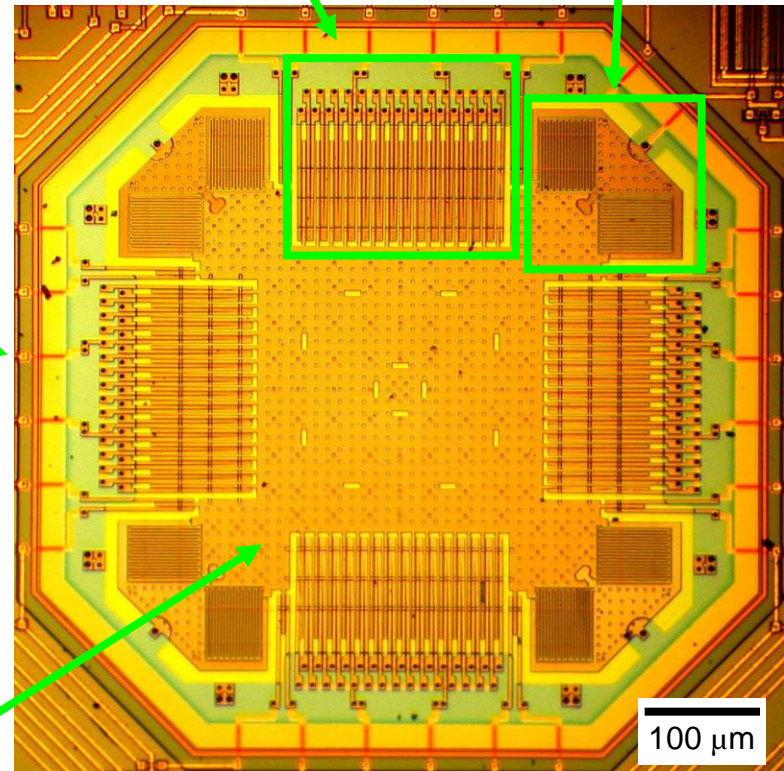
# MEMS accelerometers: ADXL 202

## Dual-axes accelerometers



Capacitive  
combs

Springs



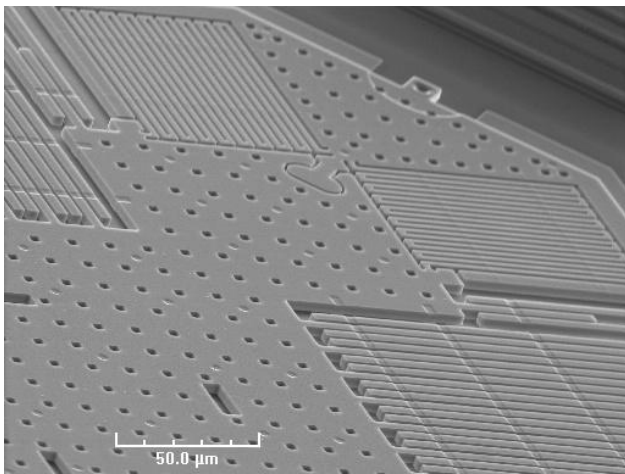
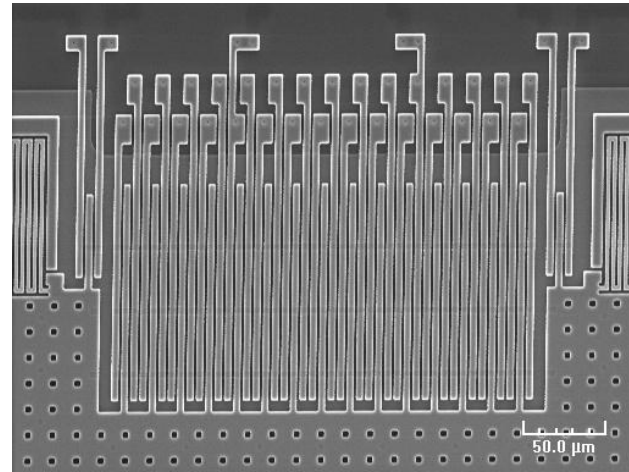
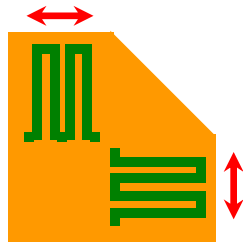
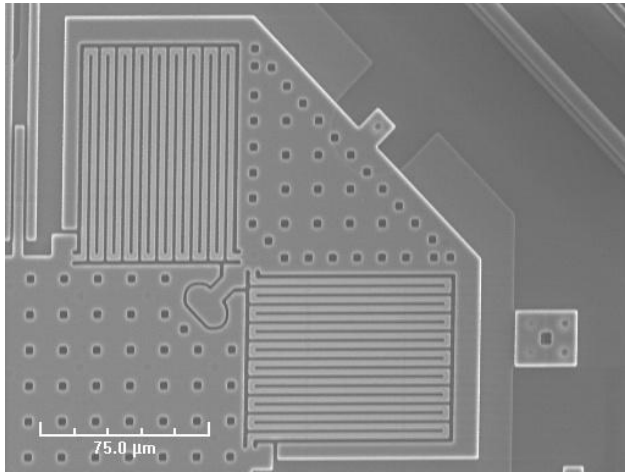
Proof mass

ADXL202 micromechanical sensor

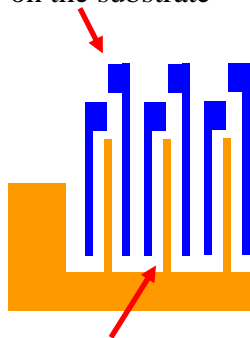


# MEMS accelerometers: ADXL 202

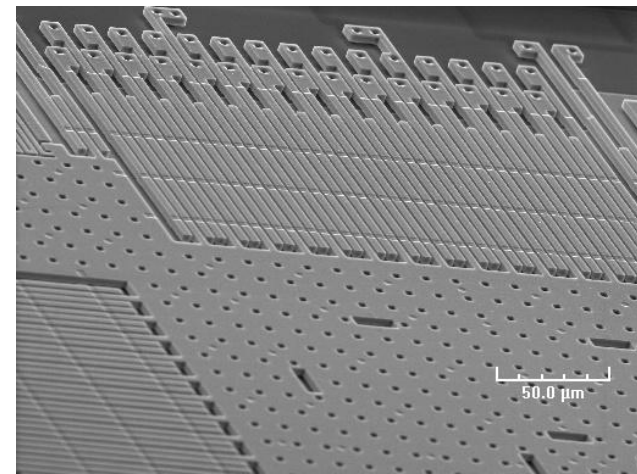
Proof mass has dimensions of  $600 \times 600 \times 3 \mu\text{m}^3$



Capacitive electrode on the substrate



Capacitive electrode on the proof mass



One set of spring elements

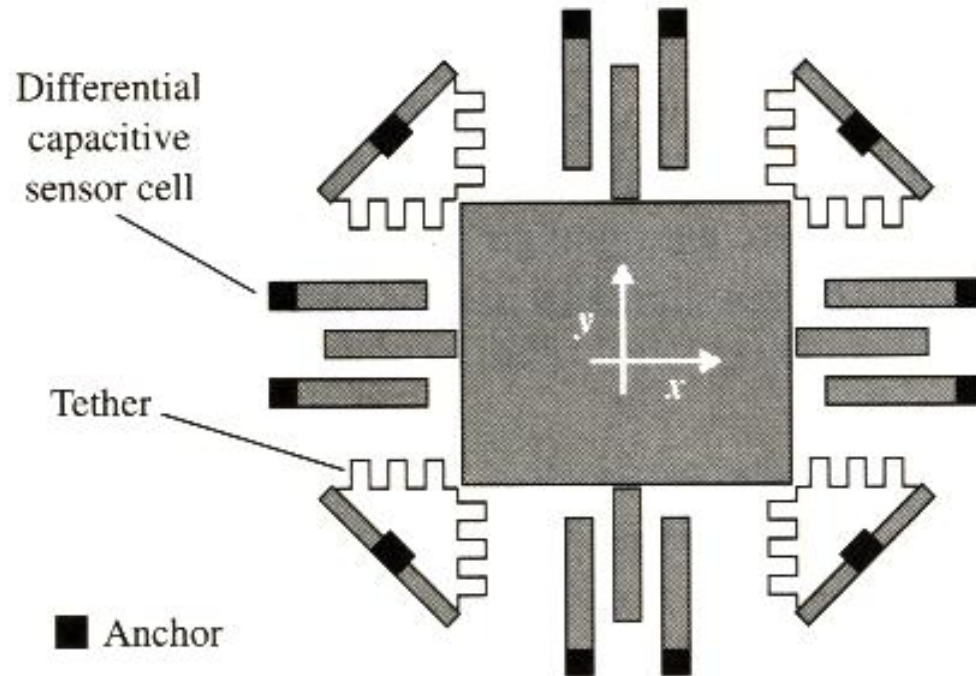
One set of capacitive combs





# MEMS accelerometers: ADXL 202

## Dual-axes accelerometers

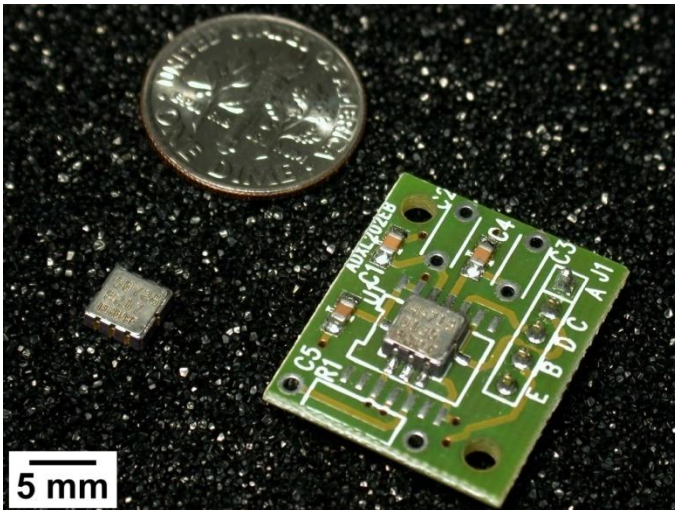


(Courtesy of Analog Devices Inc., Norwood, Massachusetts.)



# MEMS accelerometers: ADXL 202

- Provide optimum substitution for typical piezoelectric accelerometers because of their
  - ▲ high resonance frequency
  - ▲ light weight, small size, low power use
- Some properties of the Analog Devices' *ADXL202* dual-axes accelerometer include



ADXL202 packages

Specifications of Analog Devices ADXL202			
Measurement range	$\pm 2$ g	Operating voltage	3.0-5.25 V
Sensitivity	312 mV/ g	Mass	$160 \pm 10$ mgr
Resonance frequency	10 kHz	Operating temperature	$[-40,85]$ °C

