WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2012

Lecture 10

09 April 2012

General information

Office hours

Instructors: Cosme Furlong — Office: HL-151 — Everyday: 9:00 to 9:50 am

Christopher Scarpino — Office: HL-153 — During laboratory sessions

Teaching Assistants: During laboratory sessions

Lab #3: RSS uncertainty

(Include uncertainty analyses in your Lab Report #3)

Derive **complete** RSS uncertainty equation for measurements of internal pressure, P, recovered from strain-measurements (Eq. 4, in description of Lab#3). Make sure to:

(a) Indicate, in order of importance, percentage contribution of all uncertainties to the overall uncertainty.

(b) Plot uncertainty in internal pressure, P, as a function of measured tangential (Hoop) strain, ε_{Hoop}.

Discuss your results.

Lab #3: RSS uncertainty

Governing equation (based on tangential - Hoop - strain):

$$P = \frac{E\,t\,\varepsilon}{r(1-\nu/2)} \quad \Rightarrow \quad P = P(E, t, \varepsilon, r, \nu)$$

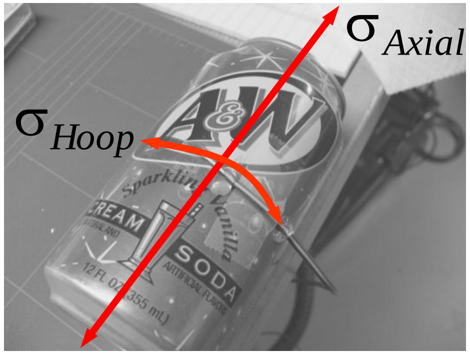
RSS uncertainty of pressure

$$\delta P = \left[\left(\frac{\partial P}{\partial E} \delta E \right)^2 + \left(\frac{\partial P}{\partial t} \delta t \right)^2 + \left(\frac{\partial P}{\partial \varepsilon} \delta \varepsilon \right)^2 + \left(\frac{\partial P}{\partial r} \delta r \right)^2 + \left(\frac{\partial P}{\partial \nu} \delta \nu \right)^2 \right]^{1/2}$$

Pressure is obtained from these equations:

$$\varepsilon_{Hoop} = \varepsilon = \frac{1}{E}(\sigma_{Hoop} - \nu\sigma_{Axial});$$

$$\sigma_{Hoop} = \frac{P\,r}{t}; \quad \sigma_{Axial} = \frac{P\,r}{2t}$$



Lab #3: RSS uncertainty

$$P = \frac{E\,t\,\varepsilon}{r\,(1-\nu/2)} \quad \Rightarrow \quad P = P(E, t, \varepsilon, r, \nu)$$

Partial derivatives

$$\frac{\partial P}{\partial E} = \frac{t\,\varepsilon}{r\,(1-\nu/2)}; \qquad \frac{\partial P}{\partial t} = \frac{E\,\varepsilon}{r\,(1-\nu/2)}; \qquad \frac{\partial P}{\partial \varepsilon} = \frac{E\,t}{r\,(1-\nu/2)};$$

$$\frac{\partial P}{\partial r} = -\frac{E\,t\,\varepsilon}{r^2\,(1-\nu/2)}; \qquad \frac{\partial P}{\partial \nu} = \frac{1}{2}\frac{E\,t\,\varepsilon}{r\,(1-\nu/2)^2}.$$

Lab #3: RSS uncertainty

Uncertainty parameters (make sure to justify values used)

$$E = 10\times10^{6}\ psi \qquad \delta E = 0.10\cdot E\ psi$$

$$t = 0.003\, inch \qquad \delta t = 5\times10^{-4}\ inch$$

$$r = 1.28\, inch \qquad \delta r = 5\times10^{-3}\, inch$$

$$\nu = 0.3 \qquad \delta\nu = 0.05$$

$$\delta\varepsilon = 25\times10^{-6} = 25\,\mu\text{Strain}$$

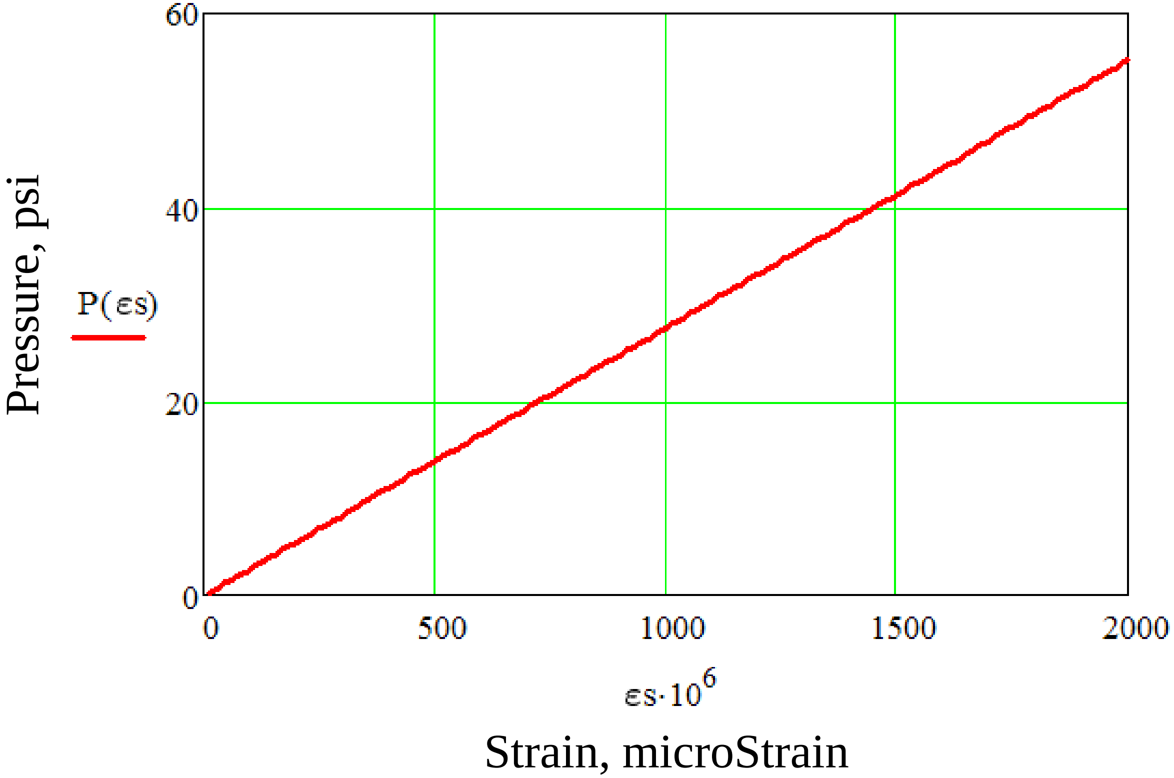
In this case, independent variable is "strain": $\delta P = \delta P(\varepsilon)$

Remember: it is possible to have more than one independent variable

Lab #3: RSS uncertainty

Pressure as a function of measured Hoop strain

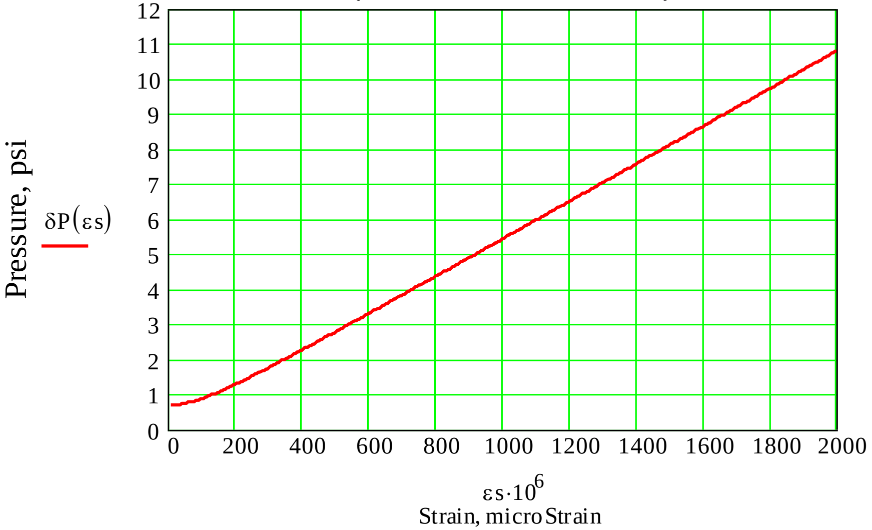
$$P = \frac{E\,t\,\varepsilon}{r\,(1-\nu/2)} \quad \Rightarrow \quad P = P(E, t, \varepsilon, r, \nu)$$



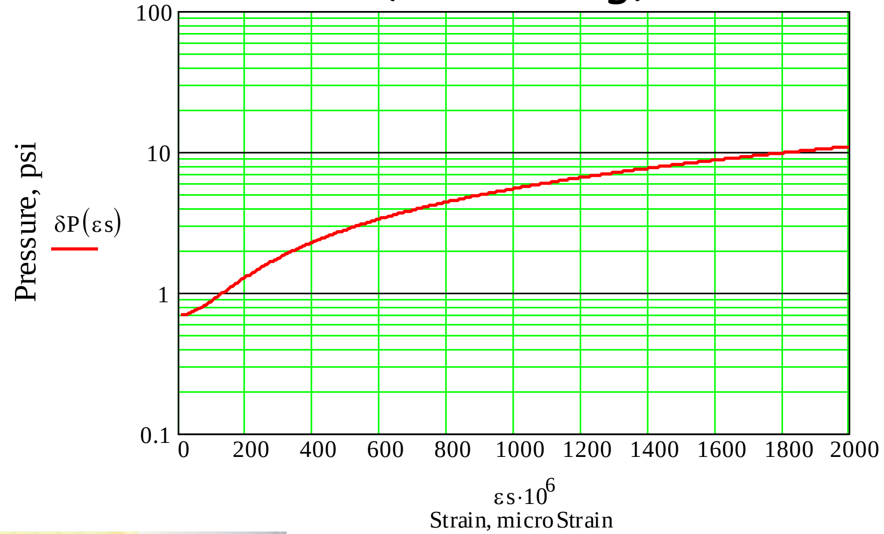
Add uncertainty analyses into your Lab Report #3

Uncertainty in pressure

(Linear-Linear)



(Linear-Log)



Add uncertainty analyses into your Lab Report #3

Percentage contribution of uncertainties

Square of uncertainty:

$$\delta P^2 = \left(\frac{\partial P}{\partial E}\delta E\right)^2 + \left(\frac{\partial P}{\partial t}\delta t\right)^2 + \left(\frac{\partial P}{\partial \varepsilon}\delta \varepsilon\right)^2 + \left(\frac{\partial P}{\partial r}\delta r\right)^2 + \left(\frac{\partial P}{\partial \nu}\delta \nu\right)^2$$

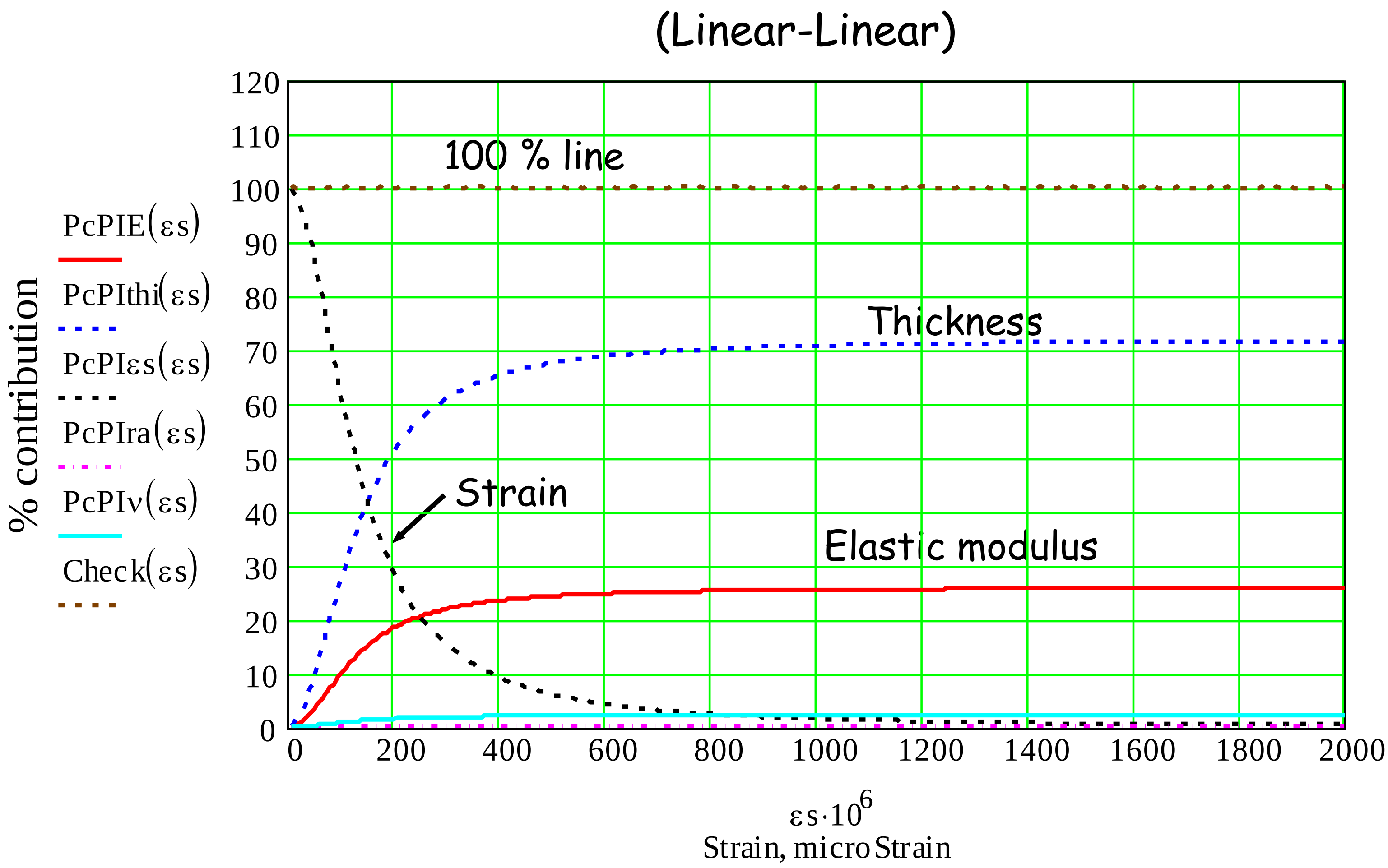
Individual contributions:

$$1 = \frac{1}{\delta P^2}\left[\left(\frac{\partial P}{\partial E}\delta E\right)^2 + \left(\frac{\partial P}{\partial t}\delta t\right)^2 + \left(\frac{\partial P}{\partial \varepsilon}\delta \varepsilon\right)^2 + \left(\frac{\partial P}{\partial r}\delta r\right)^2 + \left(\frac{\partial P}{\partial \nu}\delta \nu\right)^2\right]$$

Each contribution is a function of one independent variable (i.e., strain)

Percentage contribution of uncertainties

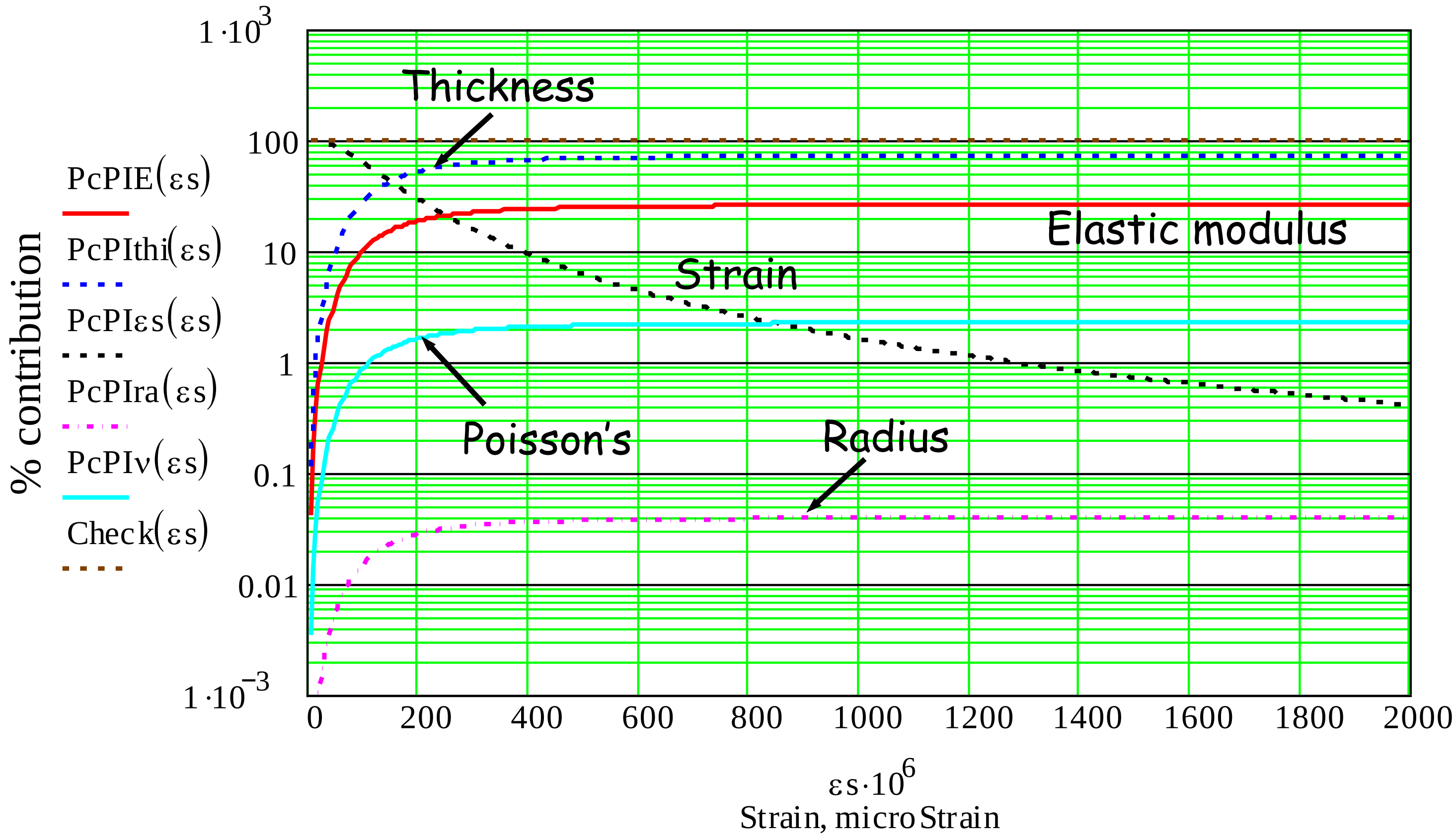
Note percentage contribution of uncertainties as a function of strain



Percentage contribution of uncertainties

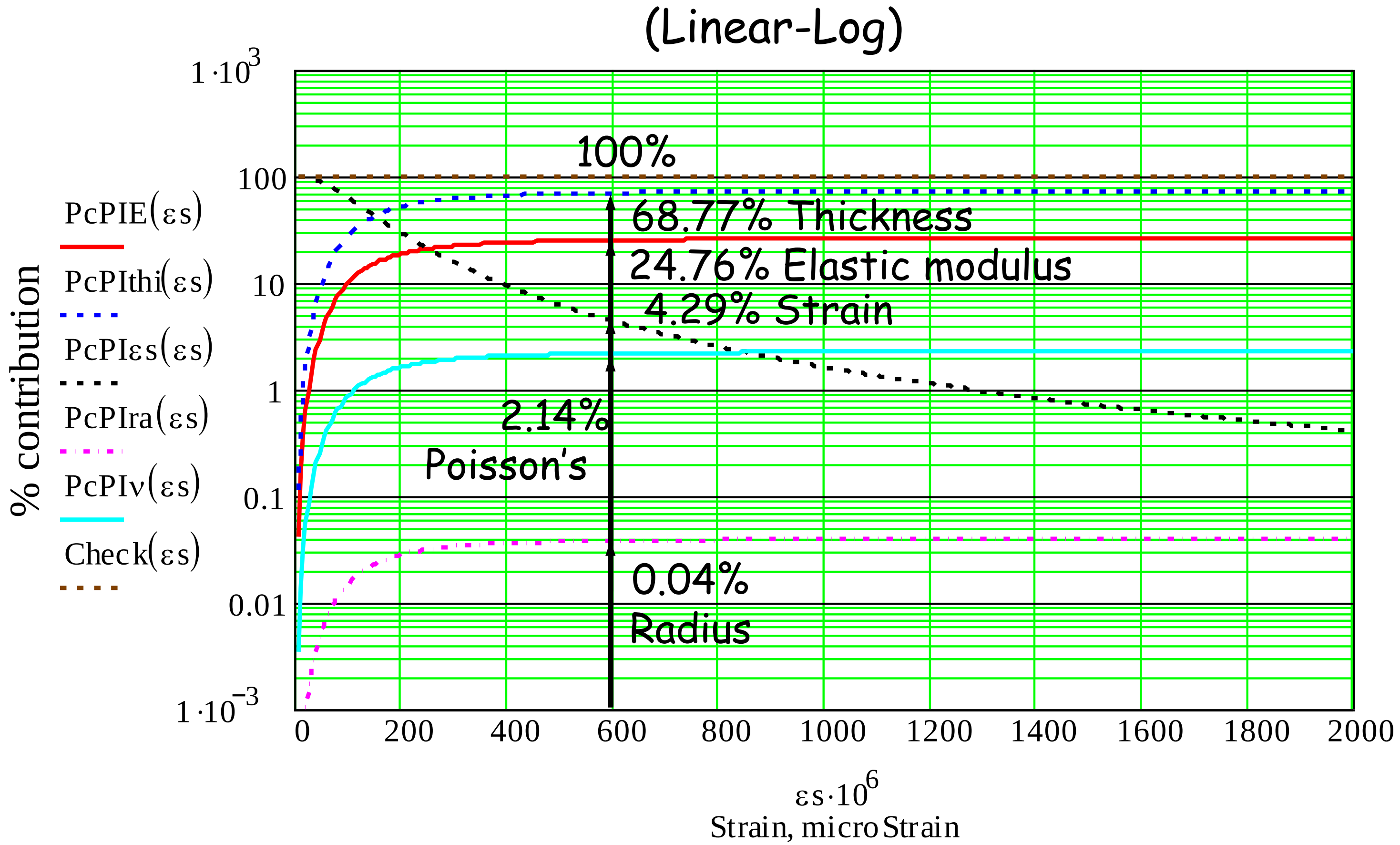
Note percentage contribution of uncertainties as a function of strain

(Linear-Log)



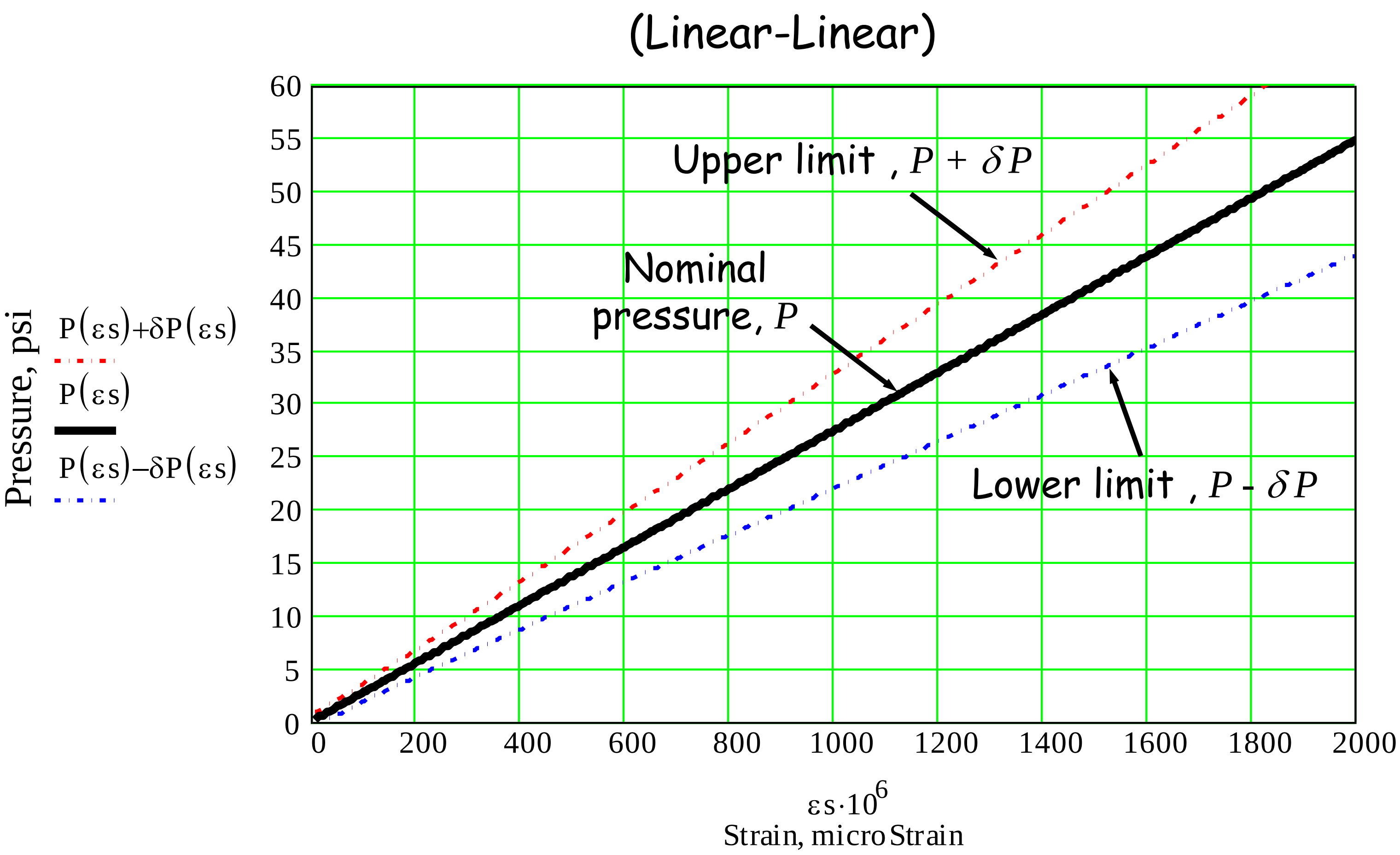
Percentage contribution of uncertainties

Note percentage contribution of uncertainties as a function of strain: see for example at the level of 600 μStrain



Uncertainty limits added to nominal pressure

For your lab report: you **must** include uncertainty limits



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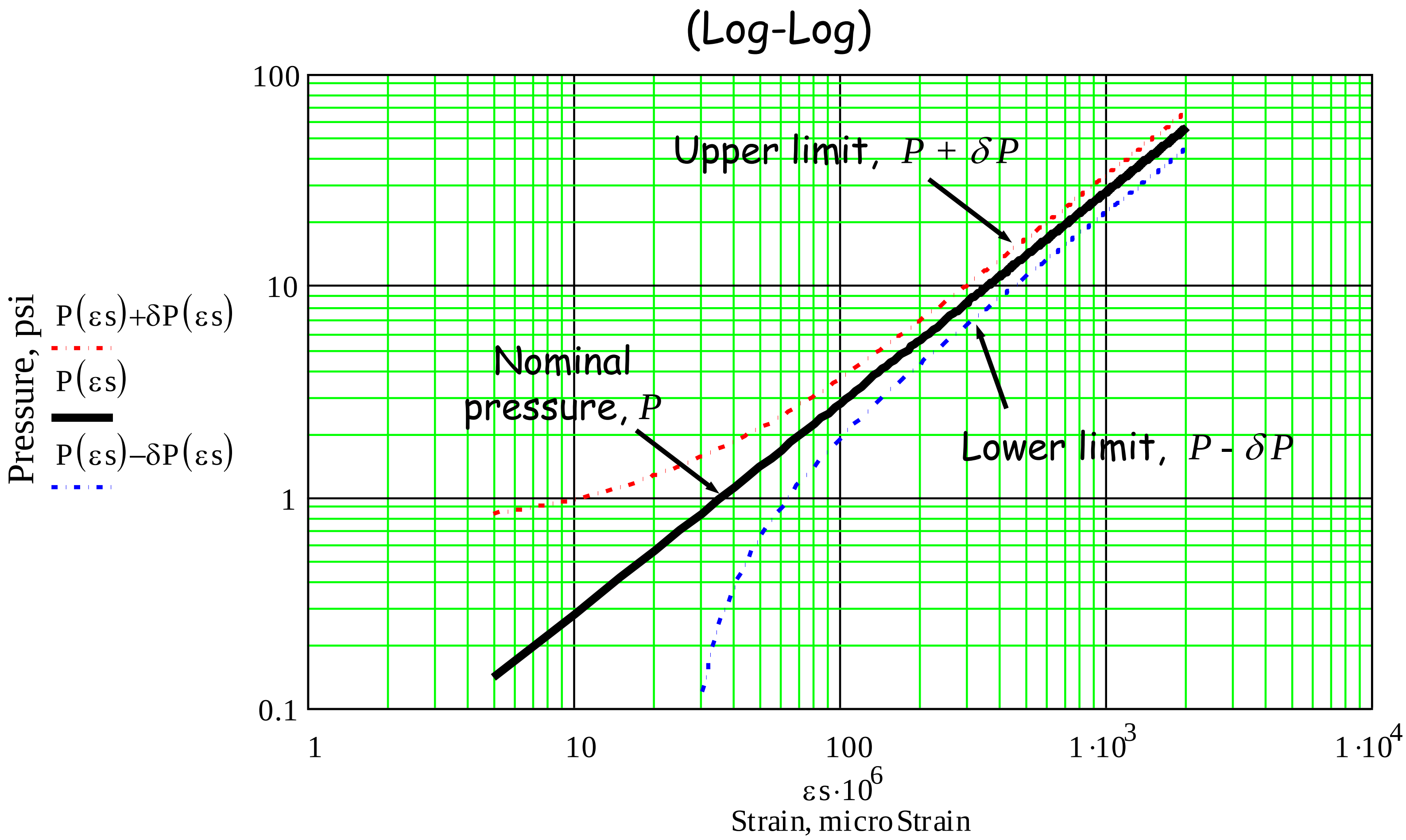
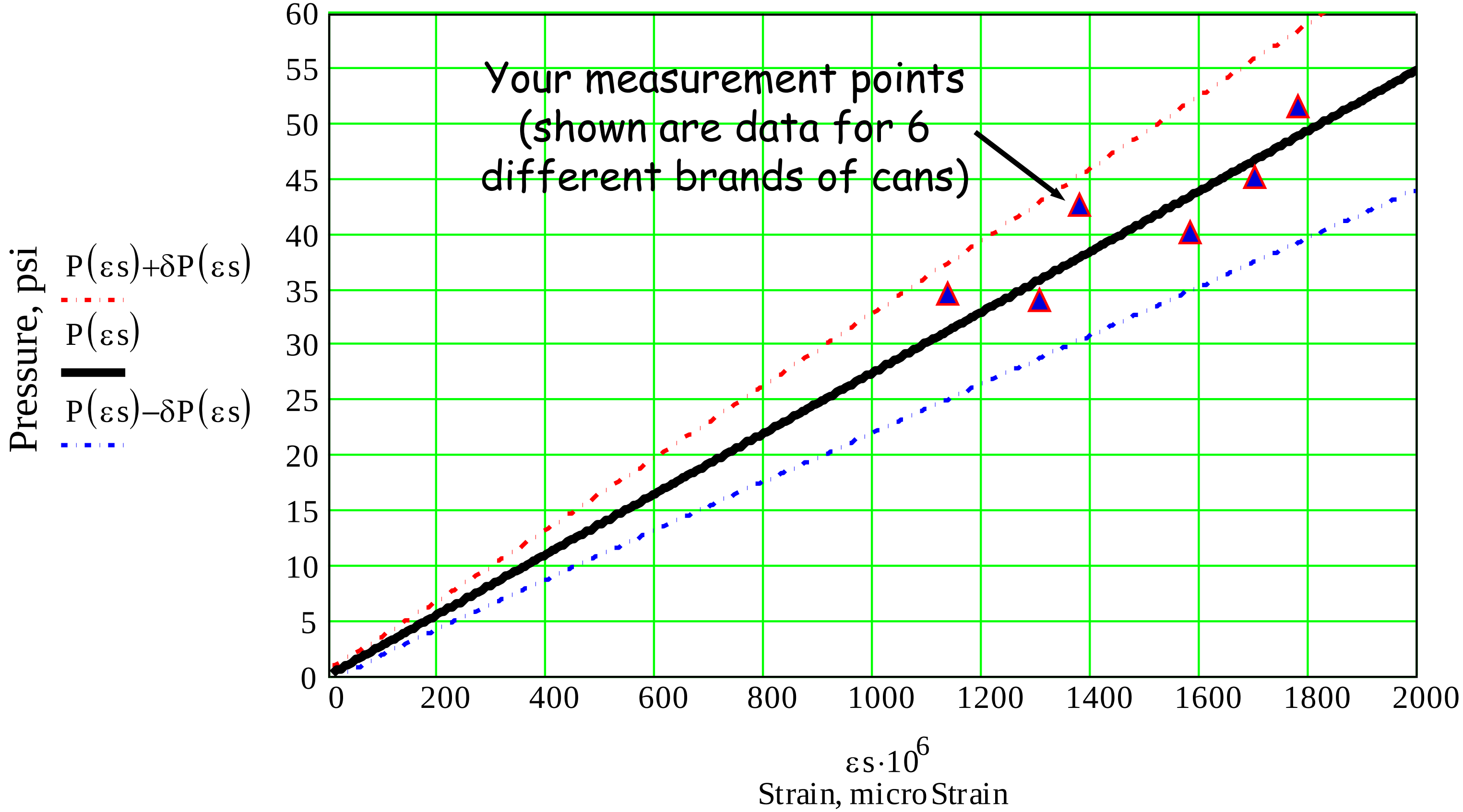


For your lab report: you must include uncertainty limits

Must also include measured data points (i.e., your measurements)



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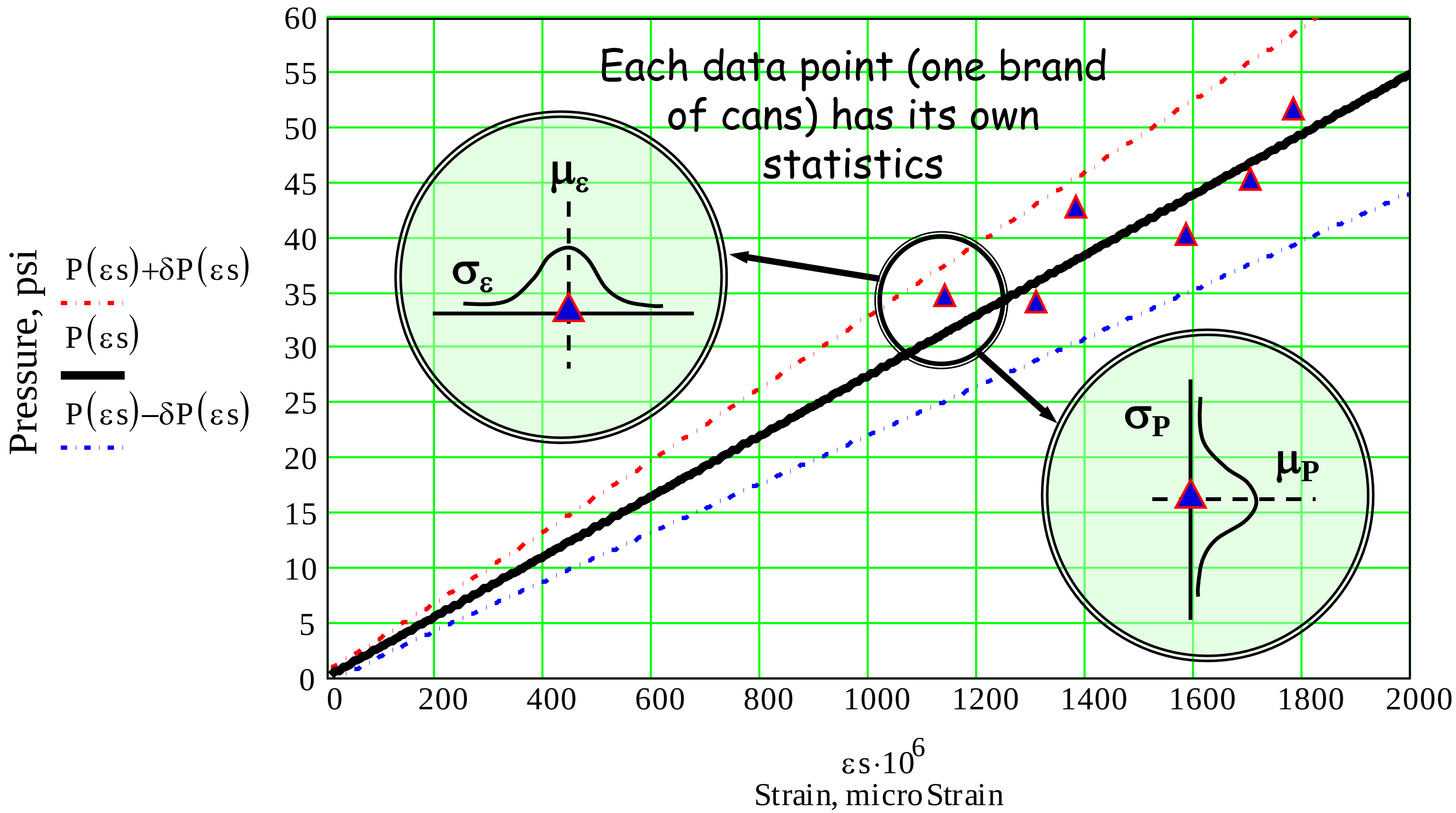
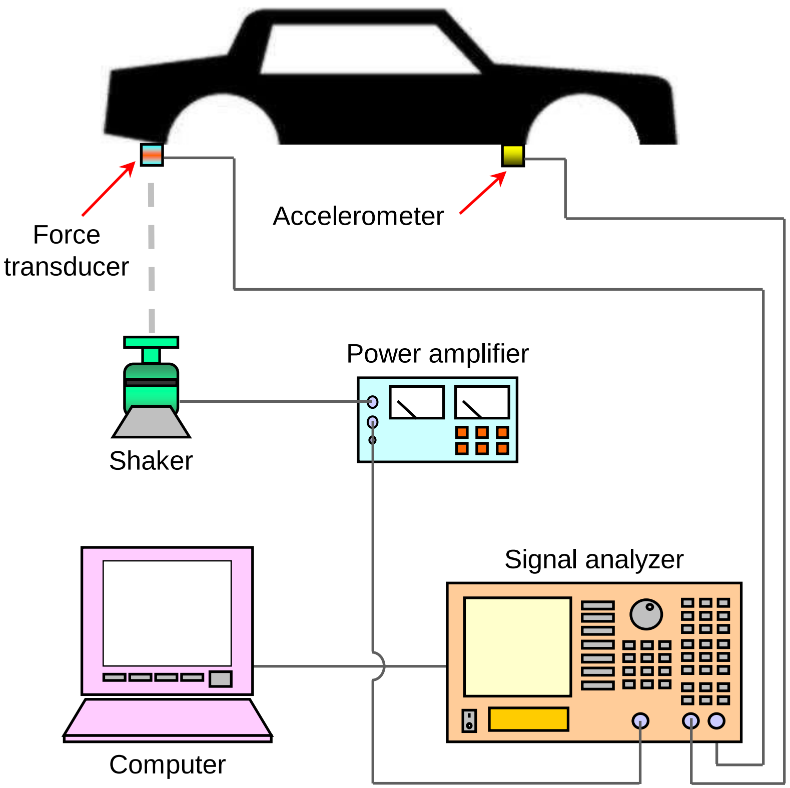
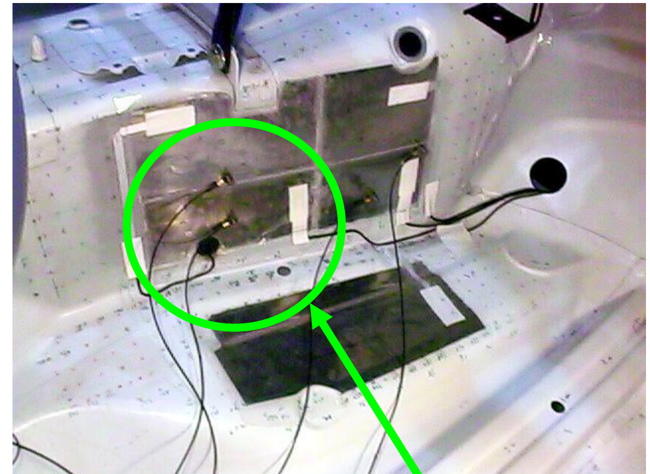


Dynamic response of structures: motion transducers

Experimental modal analysis

Typical experimental setup



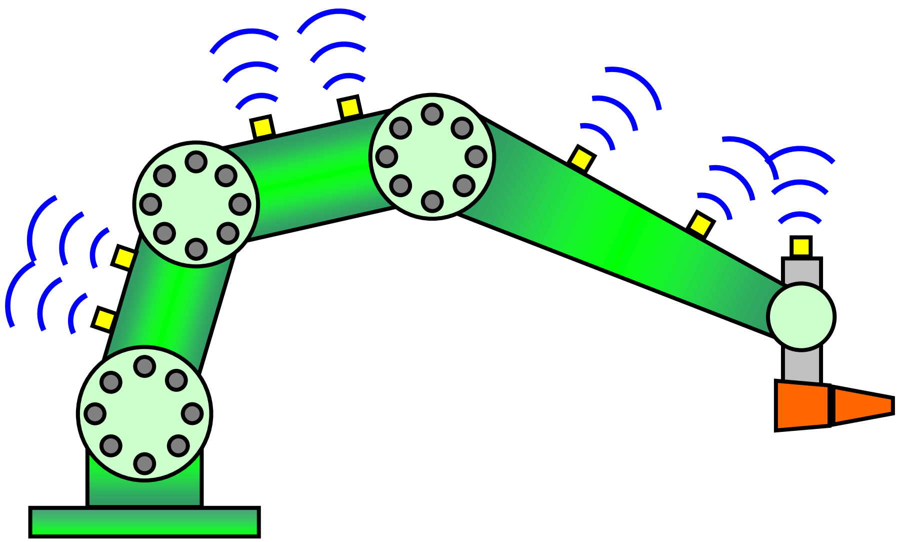


Accelerometers
(to measure
response of a
structure)

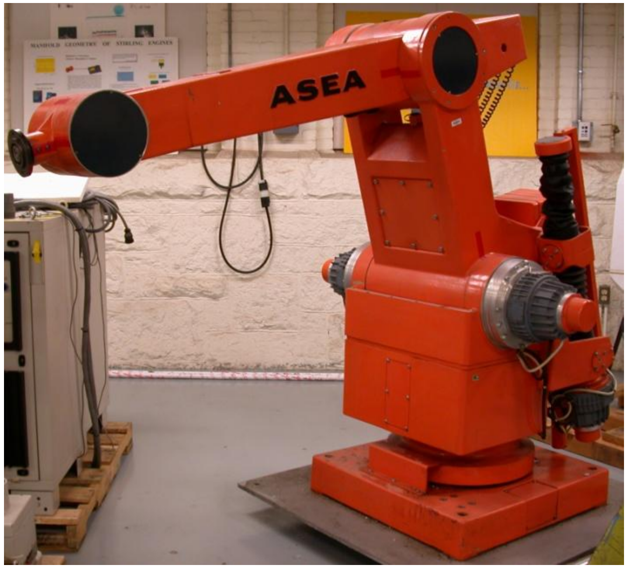
Shaker
(to dynamically
excite a structure)

Motion / position control of machinery
High-precision manufacturing and positioning

Application of motion transducers in wireless motion/position control



Robotic arm with multiple degrees of freedom



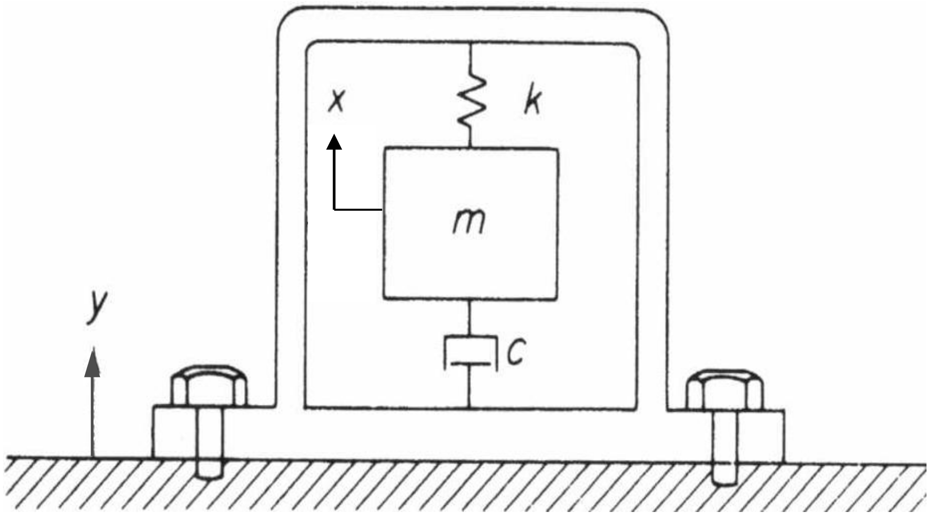
Dynamic response of structures: motion transducers

There are different types of motion transducers, which are classified based on their principle of operation:

- Strain variations
- Piezoelectric
- Piezoresistive
- Electro-mechanical
- Optical (e.g., laser vibrometer, interferometry)
- etc.

Selection of a motion transducer is based on required: *accuracy, resolution, repeatability, thermo-mechanical stability, dimensions, response-time*, etc.

Motion transducers: accelerometers



m = proof mass

c = dashpot (damper)

k = spring constant

x = displacement of proof mass

y = displacement of the vibrating body

Accelerometers: simplified SDF model

- Using Newton's 2nd law, the governing ODE of this system can be expressed as

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

- Defining the relative displacement, z, as

$$z = x - y$$

- If the vibrating body is subjected to a harmonic excitation

$$y = \beta \sin(\omega t)$$

- The governing ODE of the system can be written as

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 \beta \sin(\omega t)$$

Accelerometers: simplified SDF model

- Solution of previous equation has *steady and transient components*
- Steady state solution indicates that

$$Z = H\sin(\omega t - \phi)$$

with the amplitude H defined as

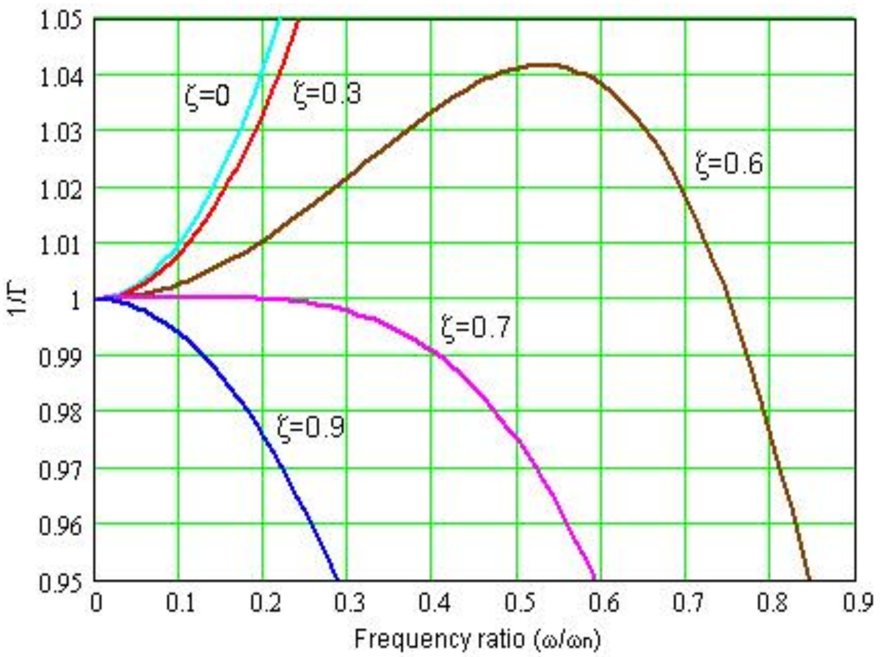
$$H = \frac{\beta\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}} \qquad \tan\phi = \frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

where

$$\zeta = \frac{c}{c_c} = \text{damping ratio}$$

$$\frac{\omega}{\omega_n} = \text{frequency ratio}$$

Accelerometers: simplified SDF model



Deviation in measurements introduced by the denominator of the previous equation

$$\frac{1}{\Gamma} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

when the frequency ratio $\frac{\omega}{\omega_n} \to 0$ it is obtained that the amplitude $H = \omega^2 \beta \frac{1}{\omega_n^{\ 2}}$

Higher resonance frequency of accelerometer = higher quality of a measurement