WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2012

Lecture 07

02 April 2012

General information

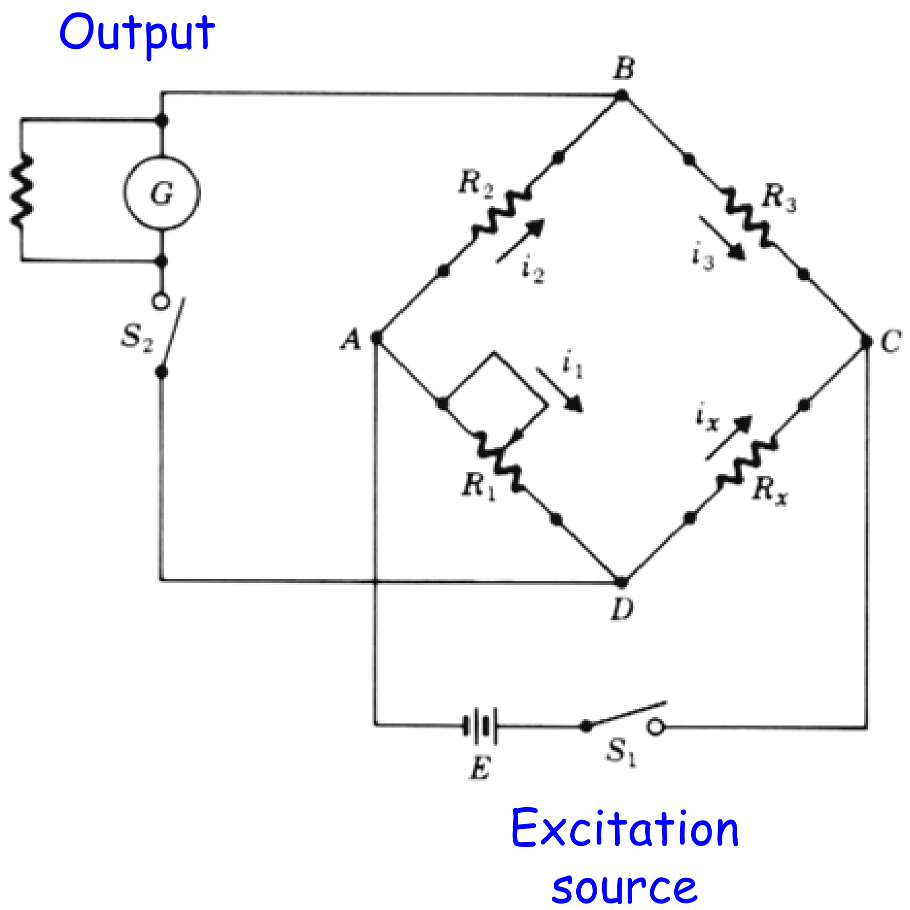
Office hours

Instructors: **Cosme Furlong** — Office: HL-151 — **Everyday: 9:00 to 9:50 am**

Christopher Scarpino — Office: HL-153 — **During laboratory sessions**

Teaching Assistants: **During laboratory sessions**

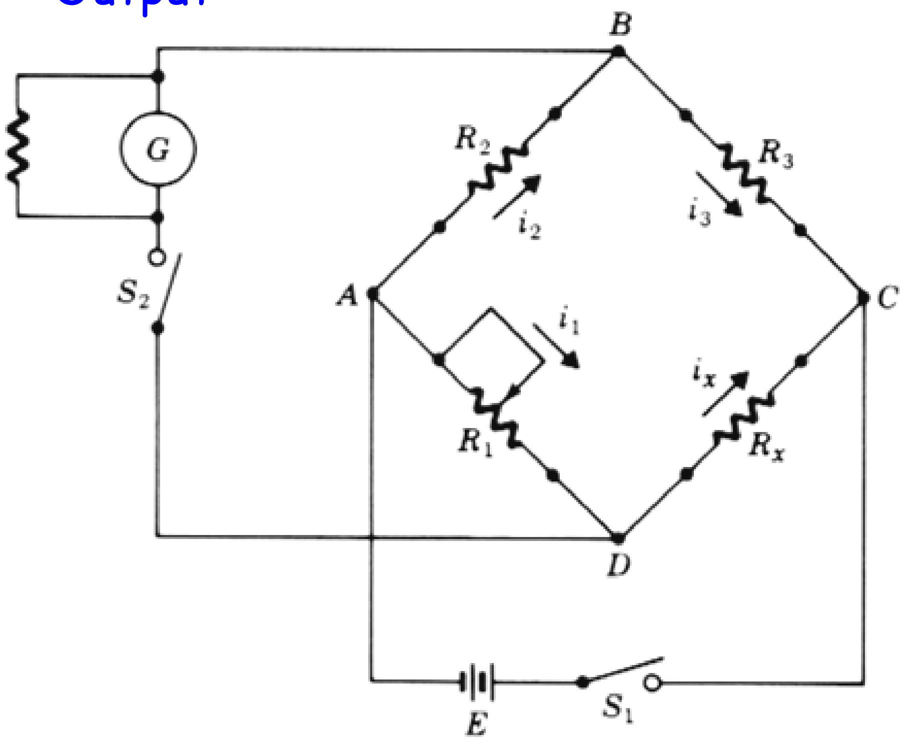
Wheatstone bridge



- Use for the comparison and measurement of resistances from 1 Ω to 1 M Ω
- Resistances are arranged in a "diamond" shape
- R_2 and R_3 are normally known resistors (of high-quality)
- R_1 is a variable resistor
- R_x is the unknown resistor

Wheatstone bridge

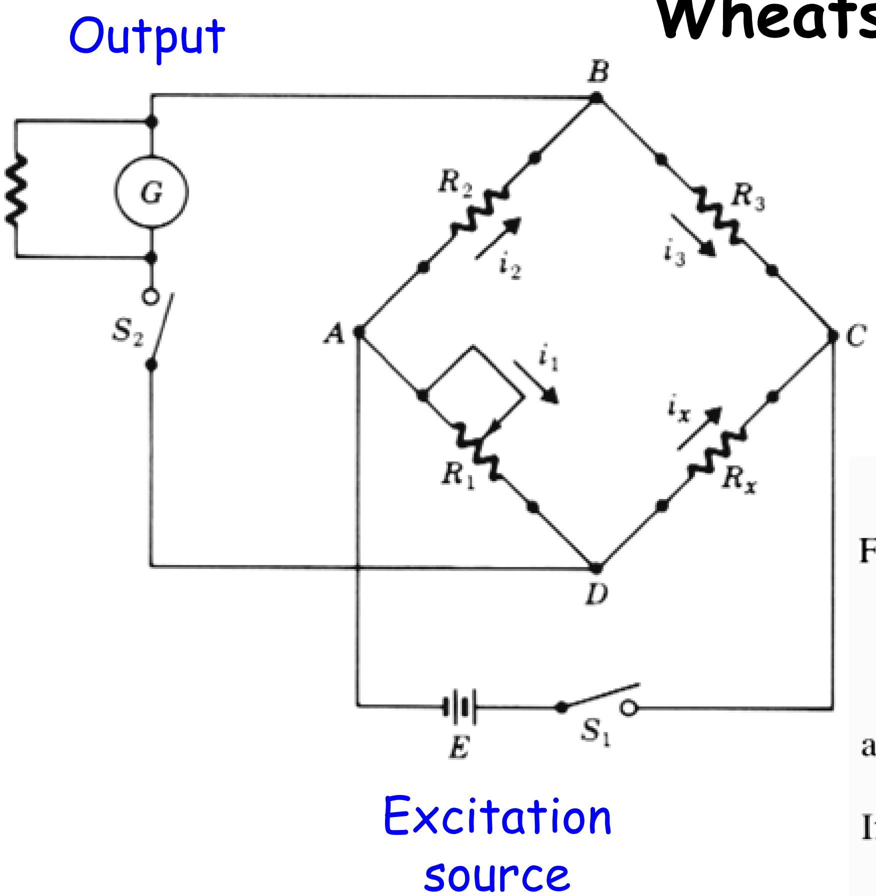
Output



Excitation source

- Voltage E is applied to the bridge (by closing switch S_1)
- A "balanced" bridge is one with potential difference between B and D is equal to zero
- Balance is sensed by closing switch S_2 and measuring output current and voltage – to be near zero
- Bridge can be balanced by adjusting resistance R_1

Wheatstone bridge



- When bridge is balanced: voltage drop across R_2 is equal to voltage drop across R_1, since voltage difference between B and D is equal to zero. Therefore,

$$i_2 R_2 = i_1 R_1$$

Further,

$$i_2 = i_3 = \frac{E}{R_2 + R_3} \qquad \text{if balanced}$$

and

$$i_1 = i_x = \frac{E}{R_1 + R_x} \qquad \text{if balanced}$$

If the currents are eliminated from these relations, the result is

$$\frac{R_2}{R_3} = \frac{R_1}{R_x}$$

or

$$R_x = \frac{R_1 R_3}{R_2}$$

Wheatstone bridge: balanced bridge

Example: uncertainty analysis

- For a balanced Wheatstone bridge, determine uncertainty in the measured resistance R_x, as a result of an uncertainty of 1% in the known resistances

$$R_x = \frac{R_1 R_3}{R_2} \quad \Rightarrow \quad R_x = R_x(R_1, R_2, R_3)$$

Uncertainty: $$\delta R_x = \left[\left(\frac{\partial R_x}{\partial R_1} \delta R_1 \right)^2 + \left(\frac{\partial R_x}{\partial R_2} \delta R_2 \right)^2 + \left(\frac{\partial R_x}{\partial R_3} \delta R_3 \right)^2 \right]^{1/2}$$

$$\frac{\partial R_x}{\partial R_1} = \frac{R_3}{R_2}; \qquad \frac{\partial R_x}{\partial R_2} = -\frac{R_1 R_3}{R_2^2}; \qquad \frac{\partial R_x}{\partial R_3} = \frac{R_1}{R_2};$$

Wheatstone bridge: balanced bridge

Example: uncertainty analysis

Determine percentage:

$$\frac{\delta R_x}{R_x} = \left[\left(\frac{1}{R_1} \delta R_1 \right)^2 + \left(-\frac{1}{R_2} \delta R_2 \right)^2 + \left(\frac{1}{R_3} \delta R_3 \right)^2 \right]^{1/2}$$

Recall: $R_x = \frac{R_1 R_3}{R_2}$

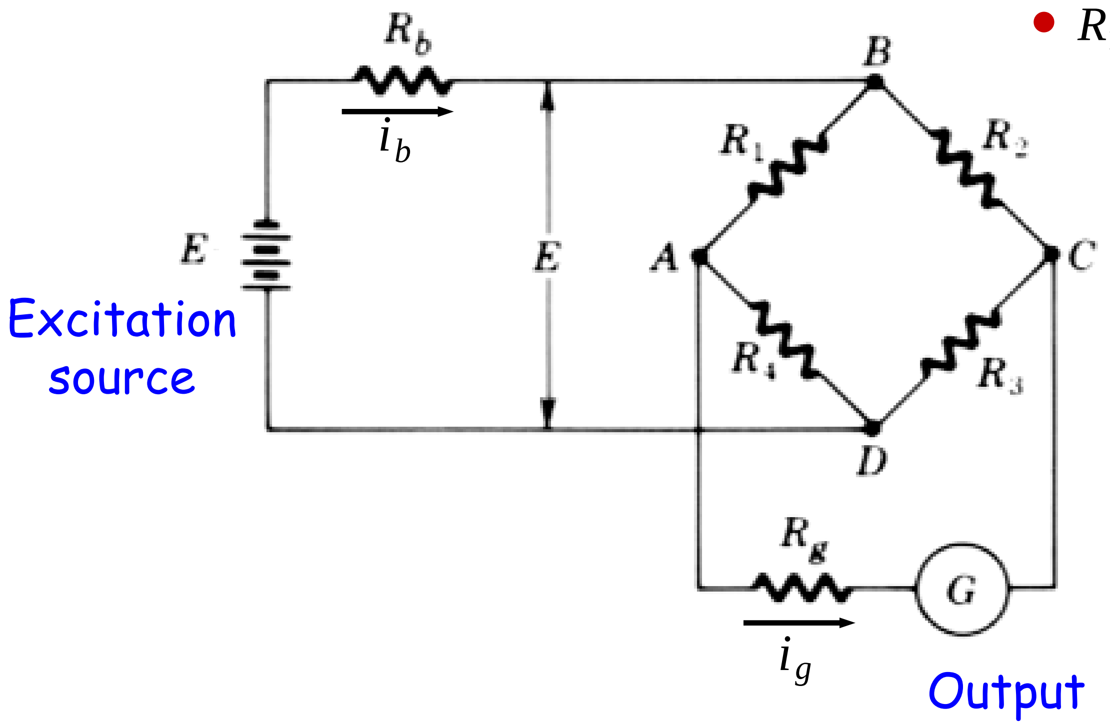
Percentages

(Same % contributions)

Determine percentage (numerical value):

$$\frac{\delta R_x}{R_x} = \left[(0.01)^2 + (-0.01)^2 + (0.01)^2 \right]^{1/2} = 0.01732 \Rightarrow 1.732\%$$

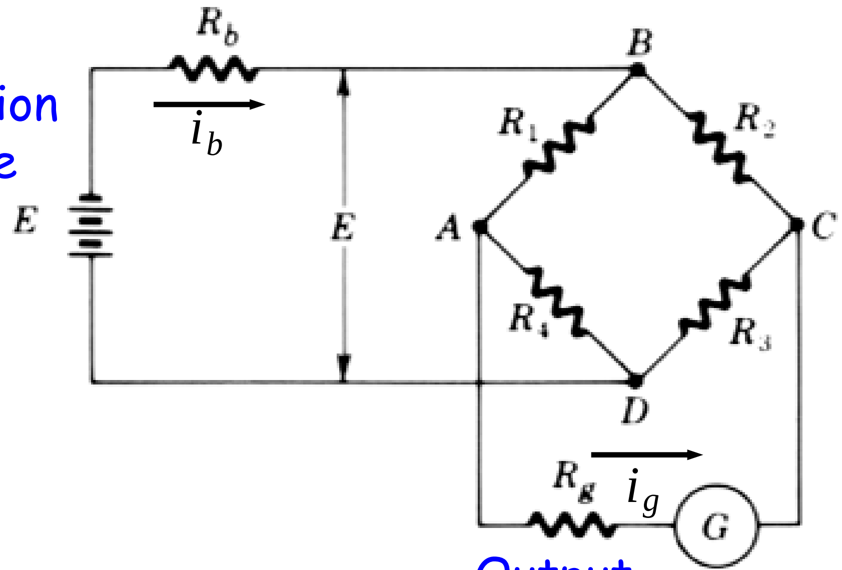
Wheatstone bridge: unbalanced bridge



- R_1, R_2, R_3, R_4 are different

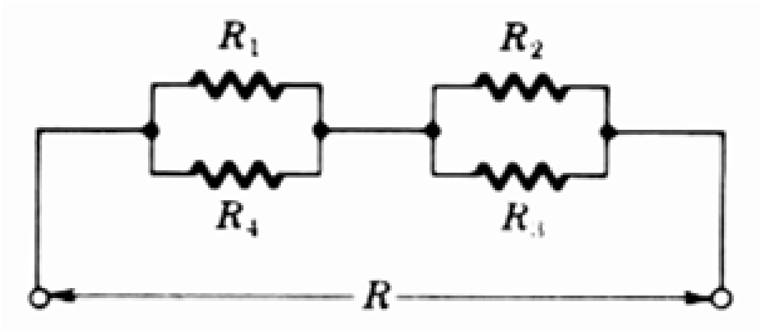
Wheatstone bridge: unbalanced bridge

Excitation source



Output

Equivalent circuit of bridge <u>at the output</u>:



Wheatstone bridge: unbalanced bridge

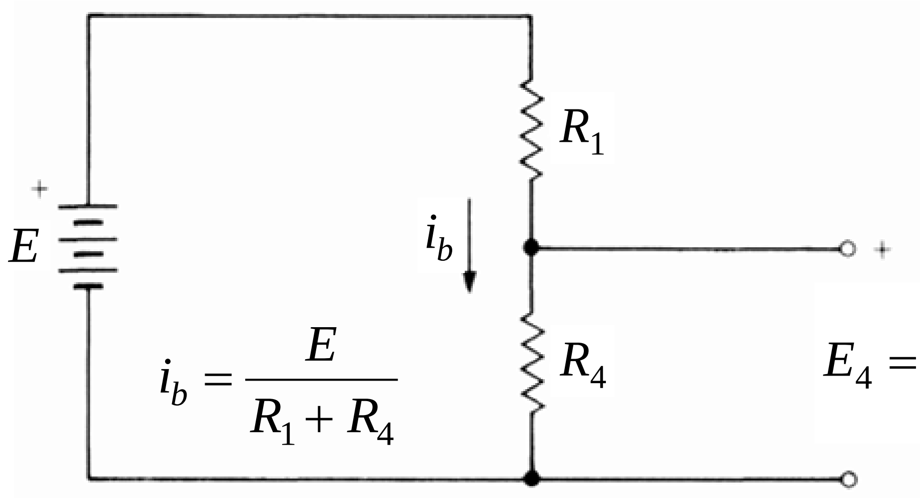
Equivalent resistance:

$$R = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3}$$

Current at the output is:

$$i_g = \frac{E_g}{R + R_g}$$

Recall a voltage divider:



$$i_b = \frac{E}{R_1 + R_4}$$

$$E_4 = \left(\frac{E}{R_1 + R_4}\right) \cdot R_4$$

Wheatstone bridge: unbalanced bridge

Considering voltage divider on a bridge:

$$E_g = \left(\frac{E}{R_1 + R_4}\right) R_1 - \left(\frac{E}{R_2 + R_3}\right) R_2$$

$$= E\left(\frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3}\right)$$

and

$$E_g = \left(\frac{E}{R_1 + R_4}\right) R_4 - \left(\frac{E}{R_2 + R_3}\right) R_3$$

$$= E\left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)$$

Wheatstone bridge: unbalanced bridge

What about if one resistance changes by a small amount?

Use: $\Delta R_4 \Rightarrow \Delta E_g$

Therefore, $$E_g + \Delta E_g = E\left(\frac{(R_4 + \Delta R_4)\, R_2 - R_3\, R_1}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)}\right)$$

Divide numerator and denominator by: $R_2\ R_4$

$$E_g + \Delta E_g = E\left(\frac{1 + \Delta R_4 / R_4 - R_3 R_1 / R_4 R_2}{(1 + R_1 / R_4 + \Delta R_4 / R_4)(1 + R_3 / R_2)}\right)$$

Wheatstone bridge: unbalanced bridge

What about if one resistance changes by a small amount?

If all resistors are initially the same:

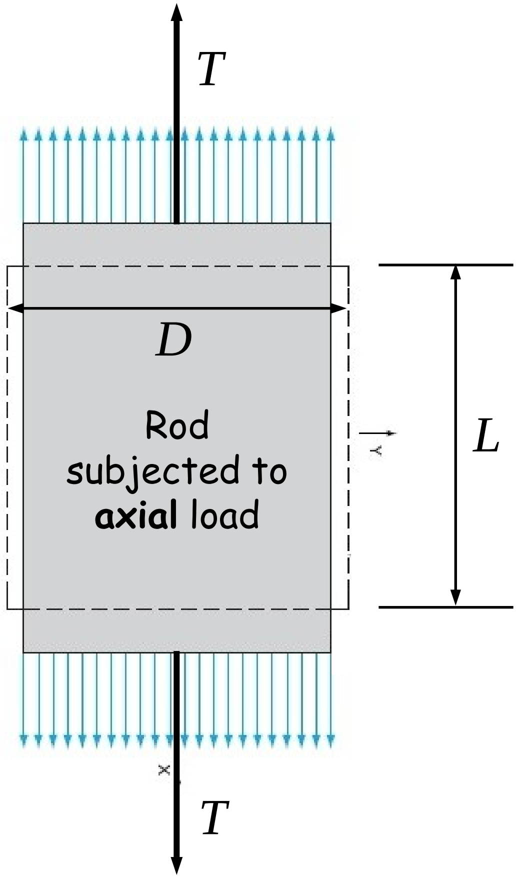
$(E_g = 0; \quad R_i = R)$

$$\frac{\Delta E_g}{E} = \frac{\Delta R_4 / R}{4 + 2(\Delta R_4 / R)}$$

But because changes in resistance are small, i.e., $\Delta R_4 << 1$

$$\frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R}$$

Stress and strain (pay attention to nomenclature)



Axial strain: $\varepsilon = \dfrac{T/A}{E} = \dfrac{\sigma_a}{E} = \dfrac{dL}{L} = \varepsilon_a$

Poisson's ratio: $\mu = -\dfrac{\varepsilon_t}{\varepsilon_a} = -\dfrac{dD/D}{dL/L}$

($\mu \approx 0.3$ for most metals)

Volume of rod is: $V = L \cdot A = L \cdot \dfrac{\pi}{4} D^2$

Volume is constant, therefore

$$dV = 0 = L\, dA + A\, dL$$

$$\Rightarrow \quad \frac{dA}{A} = -\frac{dL}{L} \quad \Rightarrow \quad 2\frac{dD}{D} = -\frac{dL}{L}$$

$$dV = 0 = D\, dL + 2L\, dD$$

(i.e., $\mu = 0.5$, in this condition)

Strain gages

Electrical resistance: $R = \rho \frac{L}{A}$

length (L), cross-sectional area (A), resistivity (ρ)

Differentiate resistance: $$dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2\frac{dD}{D} = \frac{d\rho}{\rho} + \varepsilon_a - 2\left(-\mu \frac{dL}{L}\right)$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2\left(-\mu \frac{dL}{L}\right) = \frac{d\rho}{\rho} + \varepsilon_a (1 + 2\mu)$$

Strain gages

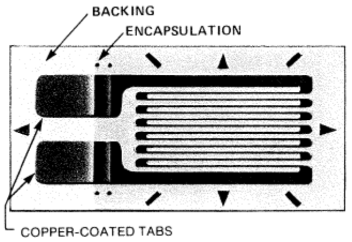
Definition of gage factor: $$F = \frac{dR/R}{\varepsilon_a}$$

(From previous page) $$\Rightarrow \quad F = 1 + 2\mu + \frac{1}{\varepsilon_a}\frac{d\rho}{\rho}$$

If resistivity does not change $$\Rightarrow \quad F = 1 + 2\mu$$

And strain with change of resistance is: $$\Rightarrow \quad \varepsilon_a = \frac{1}{F}\frac{\Delta R}{R}$$

2D view of a strain gage



A typical strain gage has a gage factor ≈ 2.095 ± 0.5%.
Why? How is this possible? Open for discussions