General information

Office hours

**Instructors:** Cosme Furlong
Office: HL-151
Everyday:
9:00 to 9:50 am

Christopher Scarpino
Office: HL-153
During laboratory sessions

**Teaching Assistants:** During laboratory sessions
Wheatstone bridge

- Use for the comparison and measurement of resistances from 1 Ω to 1 M Ω
- Resistances are arranged in a “diamond” shape
- $R_2$ and $R_3$ are normally known resistors (of high-quality)
- $R_1$ is a variable resistor
- $R_x$ is the unknown resistor
Wheatstone bridge

- Voltage $E$ is applied to the bridge (by closing switch $S_1$)
- A “balanced” bridge is one with potential difference between $B$ and $D$ is equal to zero
- Balance is sensed by closing switch $S_2$ and measuring output current and voltage - to be near zero
- Bridge can be balanced by adjusting resistance $R_1$
Wheatstone bridge

- When bridge is balanced: voltage drop across $R_2$ is equal to voltage drop across $R_1$, since voltage difference between $B$ and $D$ is equal to zero. Therefore,

$$i_2 R_2 = i_1 R_1$$

Further,

$$i_2 = i_3 = \frac{E}{R_2 + R_3} \quad \text{if balanced}$$

and

$$i_1 = i_x = \frac{E}{R_1 + R_x} \quad \text{if balanced}$$

If the currents are eliminated from these relations, the result is

$$\frac{R_2}{R_3} = \frac{R_1}{R_x}$$

or

$$R_x = \frac{R_1 R_3}{R_2}$$
Wheatstone bridge: balanced bridge

Example: uncertainty analysis

- For a balanced Wheatstone bridge, determine uncertainty in the measured resistance $R_x$, as a result of an uncertainty of 1% in the known resistances

$$R_x = \frac{R_1 R_3}{R_2} \implies R_x = R_x(R_1, R_2, R_3)$$

Uncertainty:

$$\delta R_x = \left[\left(\frac{\partial R_x}{\partial R_1} \delta R_1 \right)^2 + \left(\frac{\partial R_x}{\partial R_2} \delta R_2 \right)^2 + \left(\frac{\partial R_x}{\partial R_3} \delta R_3 \right)^2\right]^{1/2}$$

$$\frac{\partial R_x}{\partial R_1} = \frac{R_3}{R_2}; \quad \frac{\partial R_x}{\partial R_2} = -\frac{R_1 R_3}{R_2^2}; \quad \frac{\partial R_x}{\partial R_3} = \frac{R_1}{R_2};$$
Wheatstone bridge: balanced bridge

Example: uncertainty analysis

Determine percentage:
\[
\frac{\delta R_x}{R_x} = \left[ \left( \frac{1}{R_1} \delta R_1 \right)^2 + \left( -\frac{1}{R_2} \delta R_2 \right)^2 + \left( \frac{1}{R_3} \delta R_3 \right)^2 \right]^{1/2}
\]

Recall:
\[
R_x = \frac{R_1 R_3}{R_2}
\]

Percentages
(Same % contributions)

Determine percentage (numerical value):
\[
\frac{\delta R_x}{R_x} = \left[ (0.01)^2 + (-0.01)^2 + (0.01)^2 \right]^{1/2} = 0.01732 \Rightarrow 1.732\%
\]
Wheatstone bridge: unbalanced bridge

- $R_1, R_2, R_3, R_4$ are different
Wheatstone bridge: unbalanced bridge

Equivalent circuit of bridge at the output:
Wheatstone bridge: unbalanced bridge

Equivalent resistance:

\[ R = \frac{R_1R_4}{R_1 + R_4} + \frac{R_2R_3}{R_2 + R_3} \]

Current at the output is:

\[ i_g = \frac{E_g}{R + R_g} \]

Recall a voltage divider:

\[ i_b = \frac{E}{R_1 + R_4} \]

\[ E_4 = \left( \frac{E}{R_1 + R_4} \right) \cdot R_4 \]
Wheatstone bridge: unbalanced bridge

Considering voltage divider on a bridge:

\[ E_g = \left( \frac{E}{R_1 + R_4} \right) R_1 - \left( \frac{E}{R_2 + R_3} \right) R_2 \]

\[ = E \left( \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right) \]

and

\[ E_g = \left( \frac{E}{R_1 + R_4} \right) R_4 - \left( \frac{E}{R_2 + R_3} \right) R_3 \]

\[ = E \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) \]
Wheatstone bridge: unbalanced bridge
What about if one resistance changes by a small amount?

Use: \[ \Delta R_4 \Rightarrow \Delta E_g \]

Therefore, \[ E_g + \Delta E_g = E \left( \frac{(R_4 + \Delta R_4) R_2 - R_3 R_1}{(R_1 + R_4 + \Delta R_4)(R_2 + R_3)} \right) \]

Divide numerator and denominator by: \( R_2 \ R_4 \)

\[
E_g + \Delta E_g = E \left( \frac{1 + \Delta R_4 / R_4 - R_3 R_1 / R_4 R_2}{(1 + R_1 / R_4 + \Delta R_4 / R_4)(1 + R_3 / R_2)} \right)
\]
Wheatstone bridge: unbalanced bridge

What about if one resistance changes by a small amount?

If all resistors are initially the same:
\[ \left( E_g = 0; \quad R_i = R \right) \]

\[ \frac{\Delta E_g}{E} = \frac{\Delta R_4 / R}{4 + 2(\Delta R_4 / R)} \]

But because changes in resistance are small, i.e., \( \Delta R_4 << 1 \)

\[ \frac{\Delta E_g}{E} \approx \frac{\Delta R_4}{4R} \]
Stress and strain (pay attention to nomenclature)

Axial strain: \[ \varepsilon = \frac{T}{AE} = \frac{\sigma_a}{E} = \frac{dL}{L} = \varepsilon_a \]

Poisson’s ratio: \[ \mu = -\frac{\varepsilon_t}{\varepsilon_a} = -\frac{dD/D}{dL/L} \]

Volume of rod is: \[ V = L \cdot A = L \cdot \frac{\pi}{4} D^2 \]

Volume is constant, therefore
\[ dV = 0 = L \, dA + A \, dL \]
\[ \Rightarrow \frac{dA}{A} = -\frac{dL}{L} \quad \Rightarrow \quad 2 \frac{dD}{D} = -\frac{dL}{L} \]

\[ dV = 0 = D \, dL + 2L \, dD \]

(i.e., \( \mu = 0.5 \), in this condition)
Strain gages

Electrical resistance: \[ R = \rho \frac{L}{A} \]

Differentiate resistance: \[ dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA \]

\[ \Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \]

\[ \Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \frac{dD}{D} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left( -\mu \frac{dL}{L} \right) \]

\[ \Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - 2 \left( -\mu \frac{dL}{L} \right) = \frac{d\rho}{\rho} + \varepsilon_a (1 + 2\mu) \]
Strain gages

Definition of gage factor: \[ F = \frac{dR}{R} \frac{1}{\varepsilon_a} \]

(From previous page) \[ \Rightarrow F = 1 + 2\mu + \frac{1}{\varepsilon_a} \frac{d\rho}{\rho} \]

If resistivity does not change \[ \Rightarrow F = 1 + 2\mu \]

And strain with change of resistance is: \[ \Rightarrow \varepsilon_a = \frac{1}{F} \frac{\Delta R}{R} \]

A typical strain gage has a gage factor \( \approx 2.095 \pm 0.5\% \).
Why? How is this possible? Open for discussions