WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2012

Lecture 06

26 March 2012

General information

Office hours

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Office: HL-151	Office: HL-153
Everyday: 9:00 to 9:50 am	**During laboratory sessions**

Teaching Assistants: **During laboratory sessions**

Error versus uncertainty

- Errors are based on knowledge of actual values (or standard values). Statistics are used
- Uncertainties are evaluated analytically
  - there are a number of procedures for determination of the overall uncertainty
  - the most popular and most widely used relationship for determination of uncertainty is known of the RSS-type
  - the relationship defining the RSS-type overall uncertainty is based on a partial differential representation

Precision uncertainty

Governing equation to use is:

$$R = \frac{V}{I}$$

Phenomenological equation is, therefore, $R = R(V, I)$

ASME method: Square-root of the sum-of-the squares (RSS) approach indicates that the uncertainty, δR, in R, can be determined as

$$\delta R = \left[\left(\frac{\partial R}{\partial V} \delta V \right)^2 + \left(\frac{\partial R}{\partial I} \delta I \right)^2 \right]^{1/2}$$

Uncertainty in measured voltage is: δV
Uncertainty in provided voltage is: δI

Precision uncertainty

Individual partial derivates are: $\frac{\delta R}{\partial V} = \frac{1}{I}$; $\frac{\delta R}{\partial I} = -\frac{V}{I^2}$

Substitute: $$\delta R = \left[\left(\frac{1}{I} \delta V \right)^2 + \left(-\frac{V}{I^2} \delta I \right)^2 \right]^{1/2}$$

Normalized uncertainty is: $$\frac{\delta R}{R} = \left[\left(\frac{1}{V} \delta V \right)^2 + \left(-\frac{1}{I} \delta I \right)^2 \right]^{1/2}$$

Problem: estimate uncertainty and normalized uncertainty in measurements of electrical resistance (as done in Lab #1). Assume: (a) measuring range $[-10,10]\ \mathrm{V}$; (b) 12-bit digitization; (c) measured voltage of $0.11\ \mathrm{V}$; (d) $I = 1\ \mathrm{mA}$; half the least significant digit for δI; (e) determine $R \pm \delta R$

Discuss your results.

Precision uncertainty

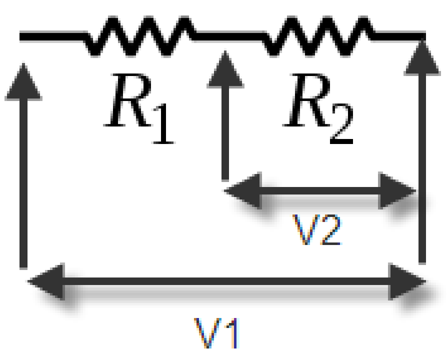
Consider the problem of measuring the electrical resistance of a component (as done in Lab #1); governing equation used to determine R_2 is:

$$R_2 = \frac{V_2 \cdot R_1}{V_1 - V_2}$$

Determine: $R_2 \pm \delta R_2$

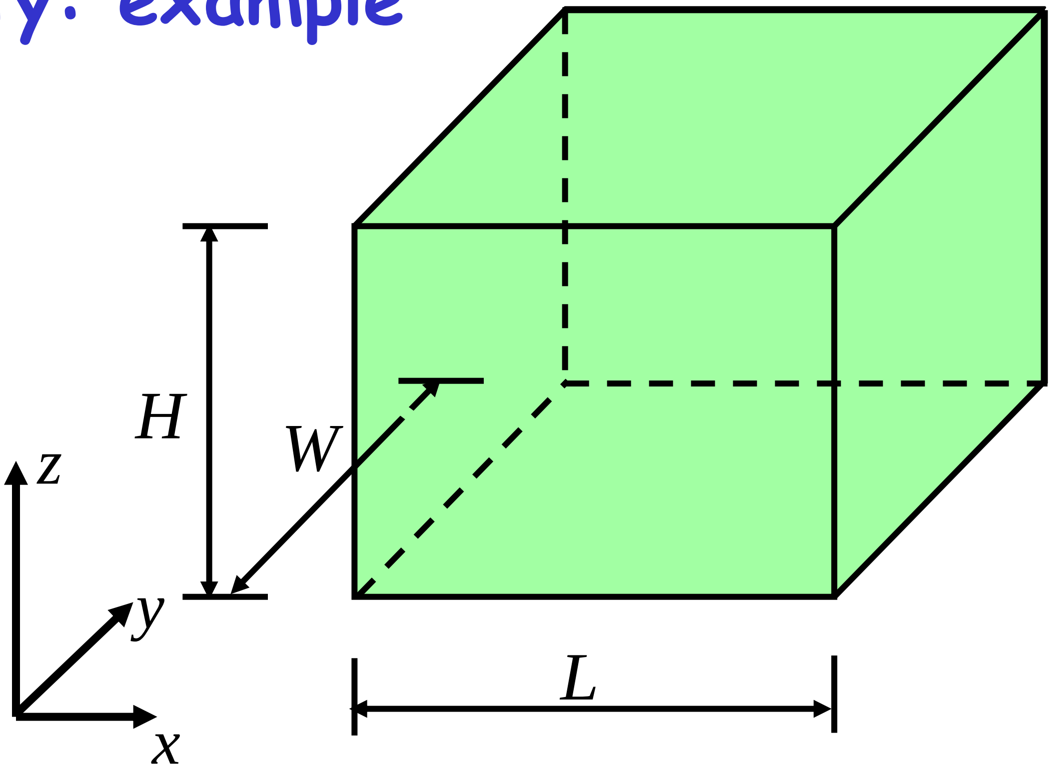
Lab #1 results

Resistors in Series



RSS-type uncertainty: example

Consider a solid block with dimensions of L, W, and H, as shown in the figure. The block is placed in the environment at temperature of 20°C where the convective heat transfer coefficient is 15±3 W/m^2-°C. The block is at the temperature of 300 ±5°C. Determine the magnitude of the heat transfer by convection, Q_c, *from the top surface of the block and the corresponding overall uncertainty in this magnitude*. Also, list in descending order percentage contributions to the overall uncertainty in Q_c due to the individual uncertainties.



$L = 140 \pm 3\,cm \longrightarrow \delta L = \pm 3\,cm$

$W = 25 \pm 1\,cm \longrightarrow \delta W = \pm 1\,cm$

$H = 9\,cm \xrightarrow{\frac{1}{2}\text{LSD}} \delta H = \pm 0.5\,cm$

LSD = Least Significant Digit

RSS-type uncertainty

Consider *explicit equation* for convective heat transfer:

$$Q_c = hA_c\left(T_s - T_e\right)$$

This equation can be represented, in the most general way, by the *fundamental equation*, also known as *phenomenological equation*, as follows:

$$Q_c = Q_c\left(h, A_c, T_s, T_e\right)$$

The *RSS-type uncertainty*, based on the above equation, can be expressed as

$$\delta Q_c = \left[\left(\frac{\partial Q_c}{\partial h}\delta h\right)^2 + \left(\frac{\partial Q_c}{\partial A_c}\delta A_c\right)^2 + \left(\frac{\partial Q_c}{\partial T_s}\delta T_s\right)^2 + \left(\frac{\partial Q_c}{\partial T_e}\delta T_e\right)^2\right]^{\frac{1}{2}}$$

where the symbol δ denotes the uncertainty

Example, cont'd

$$A_c = LW = (1.4\ m)(0.25\ m) = 0.35\ m^2$$

$$Q_c = hA_c(T_s - T_e) = \left(15\ \frac{W}{m^2\,°C}\right)(0.35\ m^2)(300-20)°C = 1{,}470\ W$$

The fundamental equation is

$$Q_c = Q_c(h, A_c, T_s, T_e)$$

The RSS-type overall uncertainty can be determined from

$$\delta Q_c = \left[\left(\frac{\partial Q_c}{\partial h}\delta h\right)^2 + \left(\frac{\partial Q_c}{\partial A_c}\delta A_c\right)^2 + \left(\frac{\partial Q_c}{\partial T_s}\delta T_s\right)^2 + \left(\frac{\partial Q_c}{\partial T_e}\delta T_e\right)^2\right]^{\frac{1}{2}}$$

or

$$\delta Q_c = \left(\partial Q_c \delta h^2 + \partial Q_c \delta A_c^2 + \partial Q_c \delta T_s^2 + \partial Q_c \delta T_s^2\right)^{\frac{1}{2}}$$

Example, cont'd

$$\partial Q_c \delta h^2 = \left(\frac{\partial Q_c}{\partial h} \delta h \right)^2 = \left[A_c \left(T_s - T_e \right) \delta h \right]^2 =$$

$$\delta h = \pm 3 \frac{W}{m^2 \, {}^\circ C}$$

$$= \left\{ \left(0.35\, m^2 \right) \left[\left(300 - 20 \right) {}^\circ C \right] \left(3 \frac{W}{m^2 \, {}^\circ C} \right) \right\}^2 = 8.64360 \times 10^4 \, W^2$$

$$\partial Q_c \delta A_c{}^2 = \left(\frac{\partial Q_c}{\partial A_c} \delta A_c \right)^2 = \left[h \left(T_s - T_e \right) \delta A_c \right]^2$$

$$A_c = LW \longrightarrow A_c = A_c \left(L, W \right)$$

$$\delta A_c = \left[\left(\frac{\partial A_c}{\partial L} \delta L \right)^2 + \left(\frac{\partial A_c}{\partial W} \delta W \right)^2 \right]^{\frac{1}{2}}$$

Example, cont'd

$$\delta A_c = \left[\left(\frac{\partial A_c}{\partial L} \delta L \right)^2 + \left(\frac{\partial A_c}{\partial W} \delta W \right)^2 \right]^{\frac{1}{2}}$$

$$\delta A_c = \left[(W \delta L)^2 + (L \delta W)^2 \right]^{\frac{1}{2}} = \left\{ [(0.25\ m)(0.03\ m)]^2 + [(1.4\ m)(0.01\ m)]^2 \right\}^{\frac{1}{2}} =$$

$$= \left(5.62500 \times 10^{-5}\ m^2 + 1.96000 \times 10^{-4}\ m^2 \right)^{\frac{1}{2}} = 1.58824 \times 10^{-2}\ m^2$$

$$\partial Q_c \delta A_c{}^2 = [h(T_s - T_e)\delta A_c]^2 =$$

$$= \left\{ \left(15 \frac{W}{m^2 \circ C} \right) [(300 - 20)^\circ C] \left(1.58824 \times 10^{-2}\ m^2 \right) \right\}^2 = 4.44970 \times 10^3\ W^2$$

Example, cont'd

$$\partial Q_c \delta T_s{}^2 = \left(\frac{\partial Q_c}{\partial T_s} \delta T_s \right)^2 = \left(hA_c \delta T_s \right)^2 =$$

$\delta T_s = \pm 5\,°C$

$$= \left[\left(15 \frac{W}{m^2\,°C} \right) \left(0.35\, m^2 \right) \left(5\,°C \right) \right]^2 = 6.89063 \times 10^2\ W^2$$

$$\partial Q_c \delta T_e{}^2 = \left(\frac{\partial Q_c}{\partial T_e} \delta T_e \right)^2 = \left(- hA_c \delta T_e \right)^2 =$$

$\delta T_e = \pm 0.5\,°C$

$\frac{1}{2}$LSD

$$= \left[- \left(15 \frac{W}{m^2\,°C} \right) \left(0.35\, m^2 \right) \left(0.5\,°C \right) \right]^2 = 6.89063 \times 10^0\ W^2$$

Example, cont'd

$$\delta Q_c = \left(\partial Q_c \delta h^2 + \partial Q_c \delta A_c^2 + \partial Q_c \delta T_s^2 + \partial Q_c \delta T_s^2\right)^{\frac{1}{2}} =$$

$$= \left[\left(8.64360\times 10^4 + 4.44970\times 10^3 + 6.89063\times 10^2 + 6.89063\times 10^0\right) W^2\right]^{\frac{1}{2}} =$$

$$= 3.02625\times 10^2\ W = 302.6\ W$$

This is the overall uncertainty

The percentage overall uncertainty is

$$\%\delta Q_c = \frac{\delta Q_c}{Q_c}\times 100\% = \frac{302.6\ W}{1{,}470\ W}\times 100\% = 20.6\%$$

The percentage contributions of the individual uncertainties to the overall uncertainty can be computed as

$$\%\partial Q_c \delta h = \frac{\partial Q_c \delta h^2}{\delta Q_c^2}\times 100\% = \frac{8.64360\times 10^4\ W^2}{\left(3.02625\times 10^2\ W\right)^2}\times 100\% = 94.3811\%$$

Example, cont'd

$$\%\partial Q_c \delta h = \frac{\partial Q_c \delta h^2}{\delta Q_c^2} \times 100\% = \frac{8.64360 \times 10^4 \ W^2}{\left(3.02625 \times 10^2 \ W\right)^2} \times 100\% = 94.3811\%$$

Highest contribution

$$\%\partial Q_c \delta A_c = \frac{\partial Q_c \delta A_c^2}{\delta Q_c^2} \times 100\% = \frac{4.44970 \times 10^3 \ W^2}{\left(3.02625 \times 10^2 \ W\right)^2} \times 100\% = 4.8587\%$$

Contribution No. 2

$$\%\partial Q_c \delta T_s = \frac{\partial Q_c \delta T_s^2}{\delta Q_c^2} \times 100\% = \frac{6.89063 \times 10^2 \ W^2}{\left(3.02625 \times 10^2 \ W\right)^2} \times 100\% = 0.7524\%$$

Contribution No. 3

$$\%\partial Q_c \delta T_e = \frac{\partial Q_c \delta T_e^2}{\delta Q_c^2} \times 100\% = \frac{6.89063 \times 10^0 \ W^2}{\left(3.02625 \times 10^2 \ W\right)^2} \times 100\% = 0.0075\%$$

Lowest contribution

$$CHECK = \%\partial Q_c \delta h + \%\partial Q_c \delta A_c + \%\partial Q_c \delta T_s + \%\partial Q_c \delta T_e =$$
$$= 94.3811\% + 4.8587\% + 0.7524\% + 0.0075\% = 99.9997\% \approx 100.0\%$$