WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, A'2010

Lecture 05
13 September 2010
General information

Office hours

**Instructor:** Cosme Furlong; cfurlong@wpi.edu
Everyday from 11:00 to 11:50 am
or by appointment

**Teaching Assistants:** During laboratory sessions
Pressure measurements: definitions

Absolute pressure (e.g., psia)

Positive gage pressure

Atmospheric pressure

Negative gage pressure

Zero absolute pressure

0

(e.g., psig)
Pressure measurements: definitions

1 atmosphere (atm) = 14.696 pounds per square inch absolute

\[ = 1.01325 \times 10^5 \text{ newtons per square meter (Pa)} \]

\[ = 2116 \text{ pounds-force per square foot (lbf/ft}^2) \]

\[ 1 \text{ N/m}^2 = 1 \text{ pascal (Pa)} \]

1 atmosphere (atm) = 760 millimeters of mercury (mmHg)

\[ 1 \text{ bar} = 10^5 \text{ newtons per square meter (100 kPa)} \]

1 microbar = 1 dyne per square centimeter

\[ = 2.089 \text{ pounds-force per square foot} \]

\[ = 0.1 \text{ newtons per square meter (0.1 Pa)} \]

1 millimeter of mercury (mmHg) = 133.322 microbar

\[ = 133.322 \text{ newtons per square meter (133.3 Pa)} \]

1 micrometer = \(10^{-6}\) meters of mercury (\(\mu\text{m, microns}\))

\[ = 10^{-3} \text{ millimeters of mercury (mmHg)} \]

\[ = 0.133322 \text{ newtons per square meter (0.133 Pa)} \]

1 torr = 1 millimeter of mercury (mmHg)

1 inch of mercury = 70.73 pounds-force per square foot

1 inch of water = 6.203 pounds-force per square foot

1 pound per square inch absolute = 6894.76 newtons per square meter (6.894 kPa)

\[ = 0.070307 \text{ kilograms-force per square centimeter (kgf/cm}^2) [\text{kilopounds per square centimeter (kp/cm}^2)] \]
Mechanical pressure measurement devices: U-manometer

Pressure balance indicates:

\[ p_a + \frac{g}{g_c} h \rho_m = p + \frac{g}{g_c} h \rho_f \]

\[ p - p_a = \frac{g}{g_c} h (\rho_m - \rho_f) \]

Recall:

1. \( g_c = 32.174 \text{ lbm} \cdot \text{ft/lbf} \cdot \text{s}^2 \)
2. \( g_c = 1 \text{ slug} \cdot \text{ft/lbf} \cdot \text{s}^2 \)
3. \( g_c = 1 \text{ g} \cdot \text{cm/dyn} \cdot \text{s}^2 \)
4. \( g_c = 1 \text{ kg} \cdot \text{m/N} \cdot \text{s}^2 \)
5. \( g_c = 9.80665 \text{ kgm} \cdot \text{m/kgf} \cdot \text{s}^2 \)
Static calibration of gages: dead-weight tester

Accuracy limited by: (1) friction between cylinder and piston; and (2) uncertainty in the area of the piston.

Percentage error due to cylinder-piston clearance:

\[
\text{Percent error} \sim \frac{(\rho \Delta p)^{1/2} b^3}{\mu D L}
\]

where \( \rho \) = density of the oil
\( \Delta p \) = pressure differential on the cylinder
\( b \) = clearance spacing
\( \mu \) = viscosity
\( D \) = piston diameter
\( L \) = piston length
Bourdon-tube pressure gage

For consistent and inexpensive measurements of static pressure

Linear, uniform, deformations of the tube are desired

Unknown pressure

Elliptical section
Diaphragm gages

Differential pressure

Deflection characteristics of three diaphragm arrangements:

(a) \( y_{\text{max}} = \frac{3p}{16Et^3} a^4 (1 - \mu^2) \)
\( y(r) = \frac{3p(1 - \mu^2)}{16Et^3} \left( a^2 - r^2 \right)^2 \)

(b) \( y_{\text{max}} = \frac{3p(1 - \mu^2)}{16Et^3} \left[ a^4 + 3b^4 - 4a^2 b^2 - 4a^2 b^2 \ln(a/b) \right. \\
\left. + \frac{16a^2 b^2}{a^2 - b^2} \left( \ln \frac{a}{b} \right)^2 \right] \)

(c) \( y_{\text{max}} = \frac{3W(1 - \mu^2)}{4\pi Et^3} \left[ a^2 - b^2 - \frac{4a^2 b^2}{a^2 - b^2} \left( \ln \frac{a}{b} \right)^2 \right] \)
Diaphragm gages: capacitive sensors

Capacitance:

\[ C = \varepsilon_r \varepsilon_0 \frac{A}{d} \]

Wheatstone bridge:

\[ V_o = V_{in} \frac{\Delta C}{2(2C + \Delta C)} \]
Diaphragm gages: resistive sensors

Make sure to know the meaning of this term

Typical piezoresistive pressure sensor die and package

http://www.novasensor.com
Diaphragm gages: resistive sensors

Cross section of an electronic package

Top view of silicon die

Wheatstone bridge:

\[
V_o = V_{in} \left( \frac{R_1}{R_1 + R_4} - \frac{R_2}{R_2 + R_3} \right)
\]
### NovaSensor's NPH solid state sensor

#### Sensor characteristics

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>UNITS</th>
<th>NOTES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GENERAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure Range</td>
<td>2.5</td>
<td>kPa</td>
<td>= 0–10&quot; H2O</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>kPa</td>
<td>= 0–1 psi</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>kPa</td>
<td>= 0–5 psi</td>
</tr>
<tr>
<td>Maximum Pressure</td>
<td>5x</td>
<td></td>
<td>rated pressure</td>
</tr>
<tr>
<td><strong>ELECTRICAL @ 25°C (77°F) unless otherwise stated</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input Excitation</td>
<td>1.5</td>
<td>mA</td>
<td>2mA max.</td>
</tr>
<tr>
<td>Insulation Resistance</td>
<td>$10^7$</td>
<td>Ω</td>
<td>@ 50 V&lt;sub&gt;DC&lt;/sub&gt;</td>
</tr>
<tr>
<td>Input Impedance (10&quot;, 1PSI)</td>
<td>3,200</td>
<td>Ω</td>
<td>± 25%</td>
</tr>
<tr>
<td>Input Impedance (30kPa)</td>
<td>4,000</td>
<td>Ω</td>
<td>± 20%</td>
</tr>
<tr>
<td>Output Impedance</td>
<td>5,000</td>
<td>Ω</td>
<td>± 20%</td>
</tr>
<tr>
<td>Bridge Impedance</td>
<td>5,000</td>
<td>Ω</td>
<td>± 20%</td>
</tr>
<tr>
<td><strong>ENVIRONMENTAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature Range</td>
<td></td>
<td>°C</td>
<td>-40° to +125 °C</td>
</tr>
<tr>
<td>Operating&lt;sup&gt;(9)&lt;/sup&gt;</td>
<td></td>
<td>°C</td>
<td>-40° to +257 °F</td>
</tr>
<tr>
<td>Compensation Range</td>
<td>0 to +70</td>
<td>°C</td>
<td>+32° to +158 °F</td>
</tr>
<tr>
<td>Vibration</td>
<td>10</td>
<td>g&lt;sub&gt;RMS&lt;/sub&gt;</td>
<td>20 to 2000Hz</td>
</tr>
<tr>
<td>Shock</td>
<td>100</td>
<td>g</td>
<td>11 milliseconds</td>
</tr>
<tr>
<td>Life (Dynamic Pressure Cycle)</td>
<td>$1 \times 10^6$</td>
<td>cycles</td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>&lt;5</td>
<td>grams</td>
<td>&lt;0.2 oz.</td>
</tr>
<tr>
<td>Media Compatibility</td>
<td></td>
<td></td>
<td>Noncorrosive gases and dry air</td>
</tr>
<tr>
<td>Wetted Materials</td>
<td>Top Port: Nickel, gold plated Kovar, silicone gel, gold wire, RTV, silicon &amp; glass.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom Port: Gold plated Kovar, silicon, glass and RTV.&lt;sup&gt;(10)&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
NovaSensor’s NPH solid state sensor
Sensor characteristics
NovaSensor’s NPH solid state sensor

Sensor characteristics

Resistors (strain gages):
- Single crystal silicon substrate (N-type)
- Ion-implantation (or diffused) Boron into silicon (P-type)
- Al contacts
- SiO₂ and Si₃N₄ as insulating layers
NovaSensor’s NPH solid state sensor
Computational (Finite Element) simulations

Shell formulation: constant thickness of 3 \( \mu \text{m} \)
1000 \( \mu \text{m} \times 1000 \mu \text{m} \) diaphragm

Material properties utilized:
- 160 GPa
- 2.33 gr/cm\(^3\)

Fully constrained boundary conditions

Uniform pressure
NovaSensor’s NPH solid state sensor
Computational (Finite Element) simulations

Out-of-plane displacements

Note locations where maximum stresses, as related to maximum strains, appear – those locations are used for placing of strain gauges

Equivalent stresses (von Mises)
NovaSensor’s NPH solid state sensor
Interferometric measurements of displacements

Interferometric microscope

Interferogram of top surface of the diaphragm

Diaphragm as fabricated: 0.0 psi

Shape information: 3D surface
NovaSensor’s NPH solid state sensor
Interferometric measurements of displacements

-1.0 psi

0.0 psi
NovaSensor’s NPH solid state sensor
Interferometric measurements of displacements

+1.0 psi

+2.0 psi
Dynamic response considerations

Transient response considerations

Transient response depends on: (1) response of the transducer (sensor); and (2) response of the pressure-transmitting fluid.

Schematic of a pressure-transmitting system
Dynamic response considerations

Transient response considerations

Pressure amplitude ratio: 
\[
\left| \frac{p}{p_0} \right| = \frac{1}{\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right]^{1/2} + 4h^2 \left( \frac{\omega}{\omega_n} \right)^2} 
\]

(6-4)

In this equation, \( p \) is the amplitude of the pressure signal impressed on the transducer. The natural frequency \( \omega_n \) is given by

\[
\omega_n = \sqrt{\frac{3\pi r^2 c^2}{4LV}}
\]

(6-5)

and the damping ratio \( h \) is

\[
h = \frac{2\mu}{\rho cr^3} \sqrt{\frac{3LV}{\pi}}
\]

(6-6)

In the above formulas, \( c \) represents the velocity of sound in the fluid, \( \mu \) is the dynamic viscosity of the fluid, and \( \rho \) is the fluid density. The phase angle for the pressure signal is

\[
\phi = \tan^{-1} \left( -\frac{2h(\omega/\omega_n)}{1 - (\omega/\omega_n)^2} \right)
\]

(6-7)

The velocity of sound for air may be calculated from

- \( c = 49.1 T^{1/2} \text{ ft/s} \) with \( T \) in °R
- \( c = 20.04 T^{1/2} \text{ m/s} \) with \( T \) in K

When the tube diameter is very small, as in a capillary, it is possible to produce a very large damping ratio, so that Eq. (6-4) will reduce to the following for frequencies below the natural frequencies:

\[
\left| \frac{p}{p_0} \right| = \frac{1}{\left[ 1 + 4h^2 \left( \frac{\omega}{\omega_n} \right)^2 \right]^{1/2}}
\]

(6-8)

If the transmitting fluid is a gas, the entire system can act as a Helmholtz resonator with a resonant frequency of

\[
\omega_n = \left[ \frac{\pi r^2 c^2}{V(L + \frac{1}{2}\sqrt{\pi^2 r^2})} \right]^{1/2}
\]

(6-9)
Reading assignment

- Beckwith: Ch. 3, 14
- Bishop: Ch. 6

References:

Homework assignment

- Bishop: P6.6

Handout-B

B1.- Derive complete uncertainty equation (i.e., RSS uncertainty) for the unknown pressure $p$ in a $U$-manometer. Discuss your observations.

B2.- Derive complete uncertainty equation (i.e., RSS uncertainty) for the “percent error” in the dead-weight tester described in this notes. Discuss your observations.