WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2012

Lecture 03

19 March 2012

General information

Office hours

Instructors: **Cosme Furlong**

Office: HL-151

Everyday:

9:00 to 9:50 am

Christopher Scarpino

Office: HL-153

During laboratory sessions

Teaching Assistants: **During laboratory sessions**

Minimization of errors

Error, ε, in measuring a quantity, x, is defined as:

$$Error = \varepsilon \equiv \underbrace{x_m}_{\text{Measured quantity}} - \underbrace{x_{true}}_{\text{True value}}$$

A primary objective in designing and executing an experiment is to *minimize* this error.

Uncertainty and probability

When an experiment is completed, we must determine:

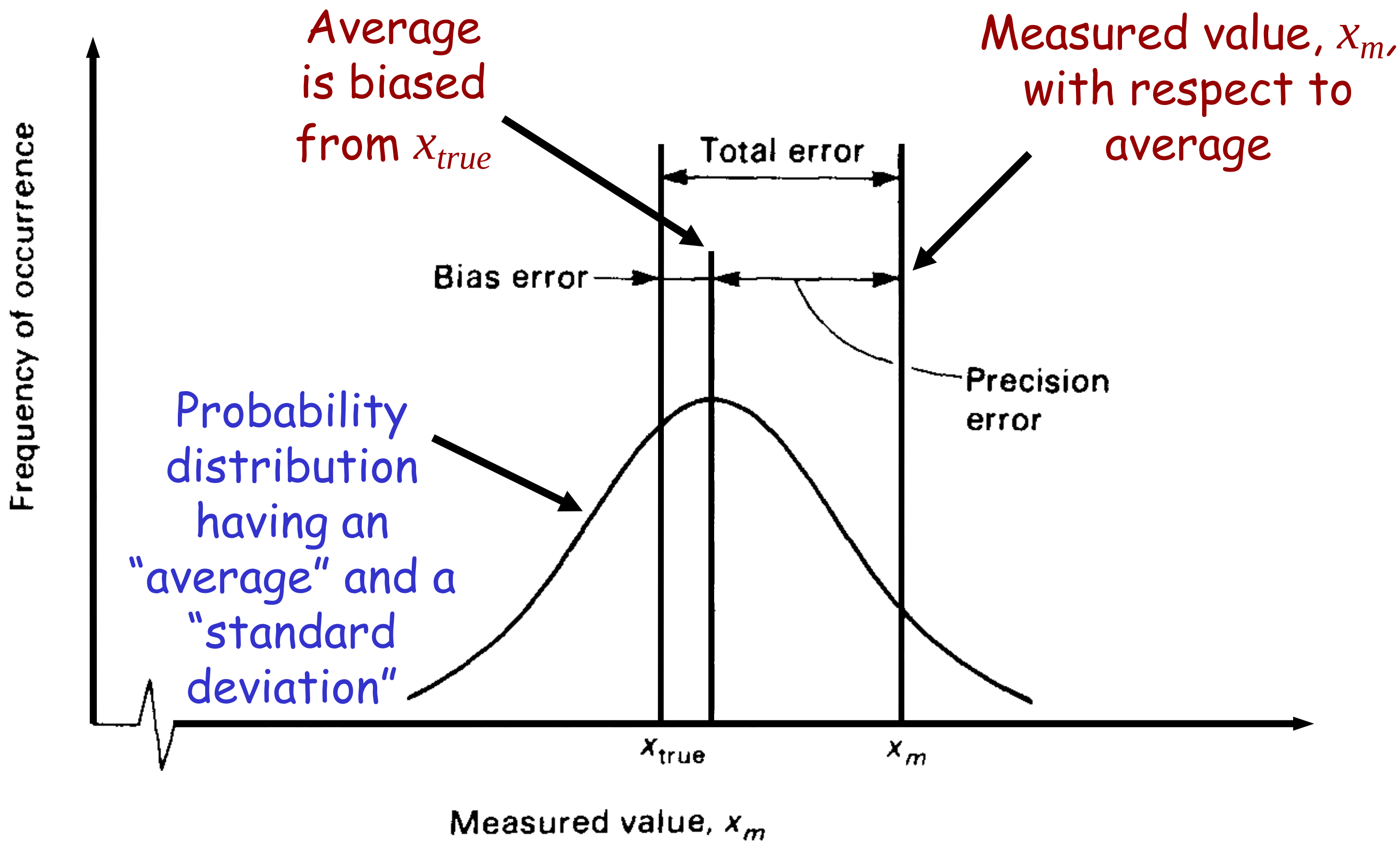
1) Measurement uncertainty; and
2) Probability (odds of obtaining "a given number of measurements in n having errors outside the uncertainty limits");

In other words, it is necessary to determine:

$$\boxed{x_m - u \le x_{true} \le x_m + u\,; \quad (n:1)}$$

where "u" is the "uncertainty" estimated at odds of "n:1"

Bias and precision errors



Classification of errors

1) Bias or systematic errors

- Calibration errors
- Consistently recurring human errors
- Defective equipment
- Loading errors
- Limitations of system resolution

Classification of errors

2) Precision or random errors

- Human errors
- Caused by disturbances to equipment
- Caused by fluctuating experimental conditions
- Derived from insufficient measurement-system resolution

Classification of errors

3) Illegitimate errors

- Blunders and mistakes during an experiment
- Computational errors after an experiment

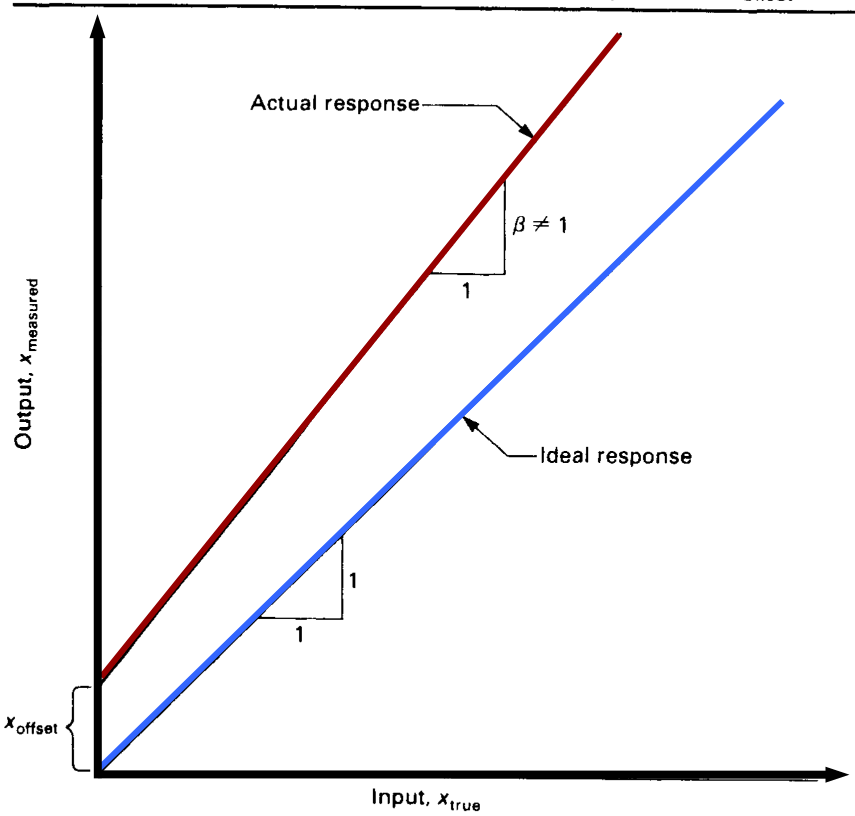
Classification of errors

4) Errors that can be bias errors or precision errors

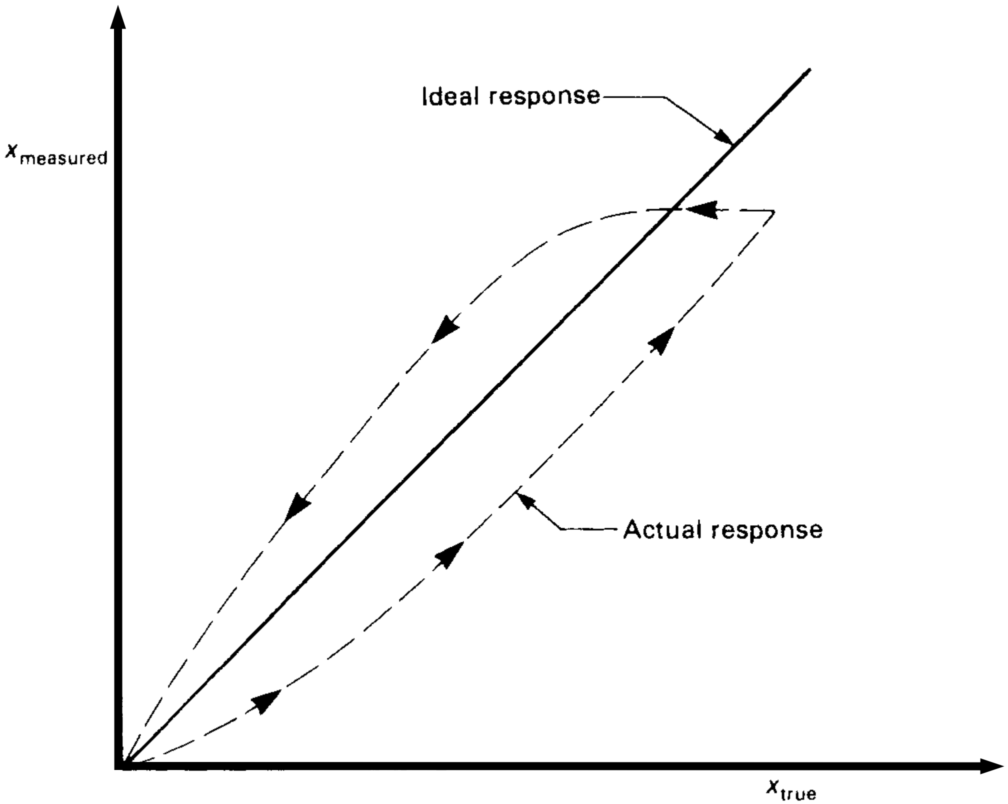
- From instrumentation errors: backlash, friction, hysteresis
- From calibration drift and variation in test or environmental conditions
- From variations in procedure or definition among experimenters

Calibration errors

Figure 3.2 Calibration errors. For ideal response, $x_{measured} = x_{true}$. Actual response may include zero-offset error (x_{offset}) and scale error ($\beta \neq 1$) so that $x_{measured} = \beta x_{true} + x_{offset}$.



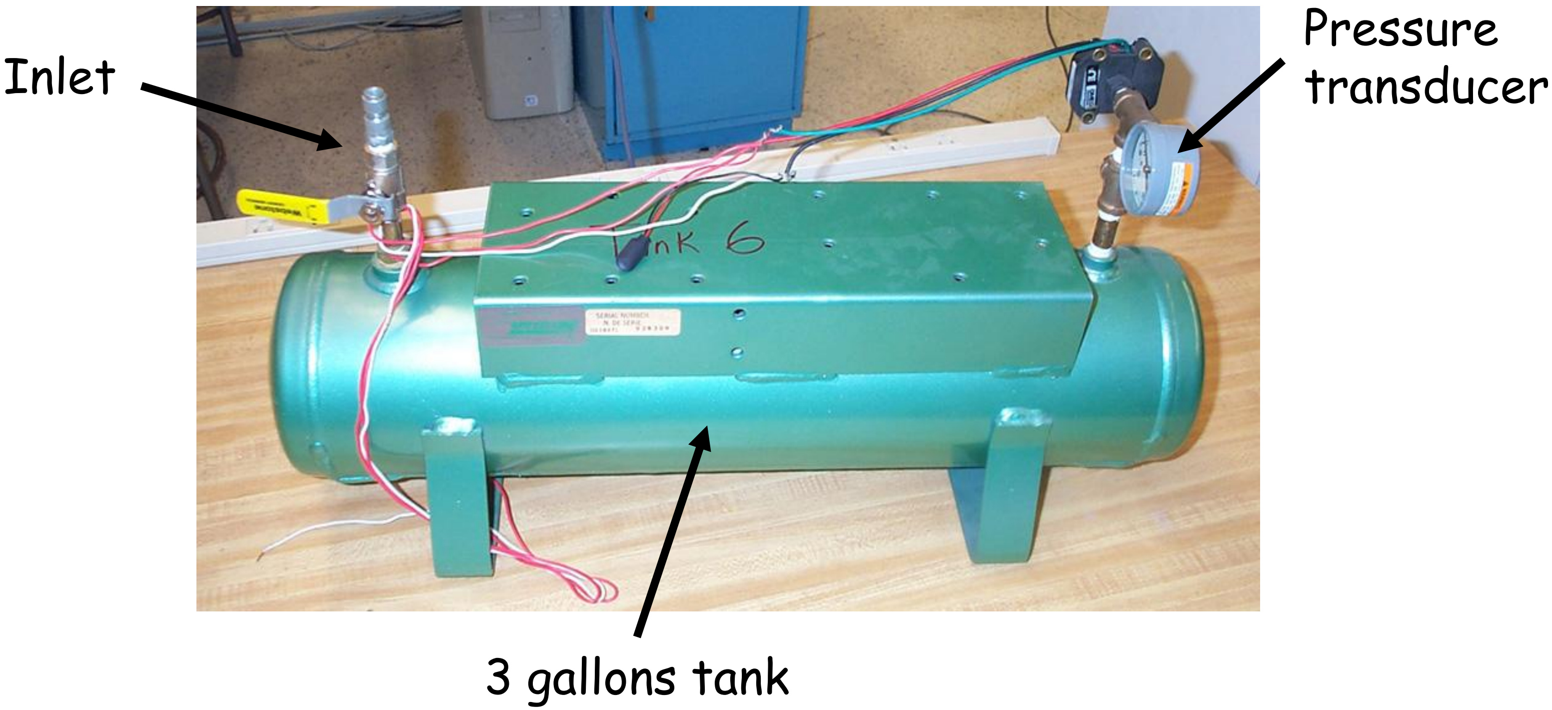
Hysteresis errors



Mean, Standard deviation, Variance, and Chauvenet's criterion

Actual measurements

Consider pressure measurements in a low-pressure cylinder



Mean, Standard deviation, Variance, and Chauvenet's criterion

Actual measurements

Results of repeated measurements of pressure are shown in the following table:

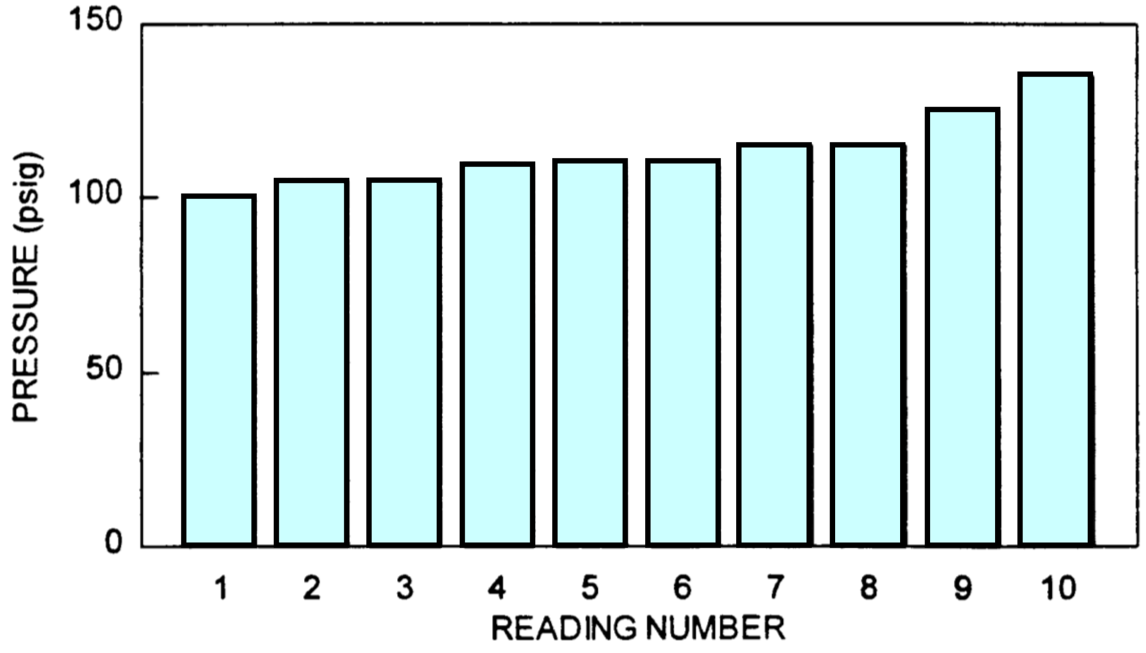
Pressure measurements on a 3 gallons tank

READING No.	PRESSURE (psig)
1	100
2	105
3	105
4	110
5	110
6	110
7	115
8	115
9	125
10	135

Mean, Standard deviation, Variance, and Chauvenet's criterion

Actual measurements

Bar graph of measured pressure data (review procedure to construct a histogram):



Mean, Standard deviation, Variance, and Chauvenet's criterion

Problem statement

From the measured data, compute their:

- Mean value
- Standard deviation and Variance
- Apply Chauvenet's criterion for rejection of "**not**" representative data

Mean, Standard deviation, Variance, and Chauvenet's criterion

Solution

Present the computations and the intermediate results in Tabular form. See Table 1.

Table 1. Procedure for computation of statistics

i READING No.	x_i PRESSURE (psig)	BEFORE DATA REJ $d_i = x_i - x_m$ (psig)	BEFORE DATA REJ d_i^2 ($psig^2$)	AFTER DATA REJ d_i (psig)	AFTER DATA REJ d_i^2 ($psig^2$)
1	100	-13	169	-10.56	111.51
2	105	-8	64	-5.56	30.91
3	105	-8	64	-5.56	30.91
4	110	-3	9	-0.56	0.31
5	110	-3	9	-0.56	0.31
6	110	-3	9	-0.56	0.31
7	115	2	4	4.44	19.71
8	115	2	4	4.44	19.71
9	125	12	144	14.44	208.51
10	135	22	484	**Rejected**	**Rejected**
	$\boxed{\Sigma x_i = 1130}$		$\boxed{\Sigma d_i^2 = 960}$		$\boxed{\Sigma d_i^2 = 422.19}$

Solution

The mean of the original data is computed to be:

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1130}{10} = 113.0\, psig$$

Next, individual deviations of the original data with respect to the mean are computed:

$$d_i = x_i - x_m$$

Solution

Using these individual deviations, the standard deviation of the original data is computed:

$$\sigma = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - x_m)^2\right]^{\frac{1}{2}} = \left(\frac{960}{10-1}\right)^{\frac{1}{2}} = 10.33\ psig \quad ,$$

where it should be noted that because the number of readings, n, is < 20, the denominator in the previous equation is $n-1$;

use value of n when number of readings, $n \geq 20$.

Variance of original data is:

$$\sigma^2 = (10.33\ psig)^2 = 106.71\ psig^2 \quad .$$

Solution: Chauvenet's criterion

Table 2. Chauvenet's criterion for data rejection

Number of readings, n	Ratio of maximum acceptable deviation to standard deviation, d_{max}/σ
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
10	1.96
15	2.13
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1000	3.48

Solution: Chauvenet's criterion

Referring to Table 2, the ratio of maximum acceptable deviation to standard deviation for $n = 10$ is:

$$\frac{d_{\max}}{\sigma} = 1.96$$

which indicates that the maximum deviation allowed in the data is:

$$d_{\max} = 1.96 \times \sigma = 1.96 \times 10.33 = 20.25 \quad psig\ ;$$

compare the maximum deviation, $d_{\max}$, with the individual deviations of the original readings (Table 1, third column).

Solution: Chauvenet's criterion

After doing comparisons, it is clear that:

- the only reading (i.e., No. 10) with $d_{10} = 22 \geq d_{\max}$, must be rejected to satisfy Chauvenet's criterion.

PLEASE NOTE: THE DATA REJECTION CAN BE APPLIED ONLY ONCE DURING A GIVEN ANALYSIS. HOWEVER, IT MAY LEAD TO REJECTION OF MORE THAN ONE DATA POINT AT THAT TIME.

Solution: Chauvenet's criterion

After data rejection, new statistics must be calculated.

New mean is therefore:

$$x_m = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \frac{995}{9} = 110.56\, psig \quad .$$

With this value, you must also compute new individual deviations, their squares, and the sum of the squares of the deviations (see: last two columns of Table 1).

Solution: Chauvenet's criterion

New standard deviation is:

$$\sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - x_m)^2 \right]^{\frac{1}{2}} = \left(\frac{422.19}{9-1} \right)^{\frac{1}{2}} = 7.26\ psig$$

New variance is:

$$\sigma^2 = (7.26\ psig)^2 = 52.71\ psig^2 \quad .$$

Note that $n = 9$ (after data rejection), and standard deviation was computed with $n - 1$ samples.

Examination of the above results indicates that the mean, standard deviation, and variance decrease following data reduction.

Let's study implications of these results…

Mean, Standard deviation, Variance, and Chauvenet's criterion

Solution

Present the computations and the intermediate results in Tabular form. See Table 1.

Table 1. Procedure for computation of statistics

i READING No.	x_i PRESSURE (psig)	BEFORE DATA REJ $d_i = x_i - x_m$ (psig)	d_i^2 (psig^2)	AFTER DATA REJ d_i (psig)	d_i^2 (psig^2)
1	100	-13	169	-10.56	111.51
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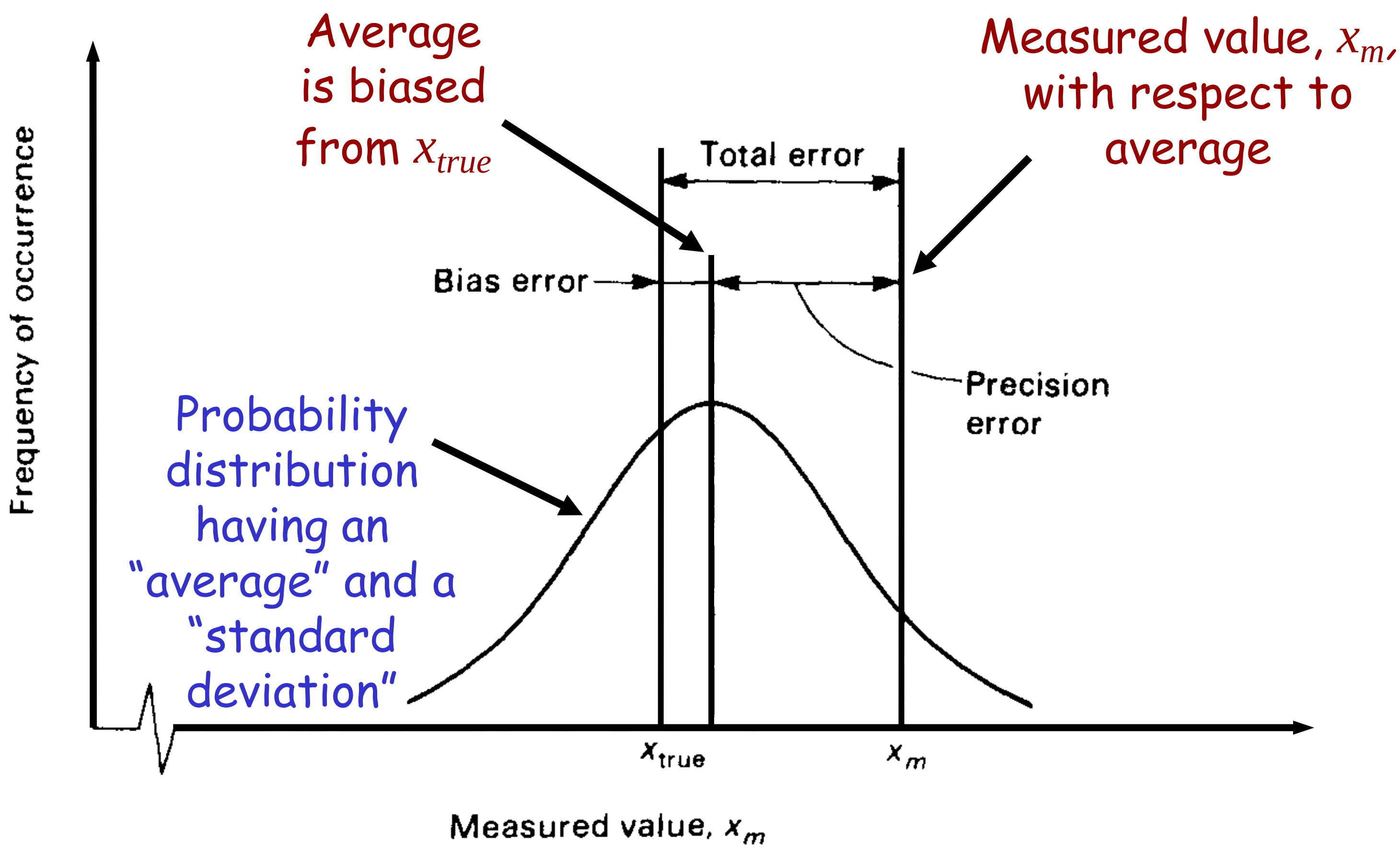
8	115	2	4	4.44	19.71
9	125	12	144	14.44	208.51
10	135	22	484	**Rejected**	**Rejected**
	$\Sigma x_i = 1130$		$\Sigma d_i^2 = 960$		$\Sigma d_i^2 = 422.19$

Bias and precision errors

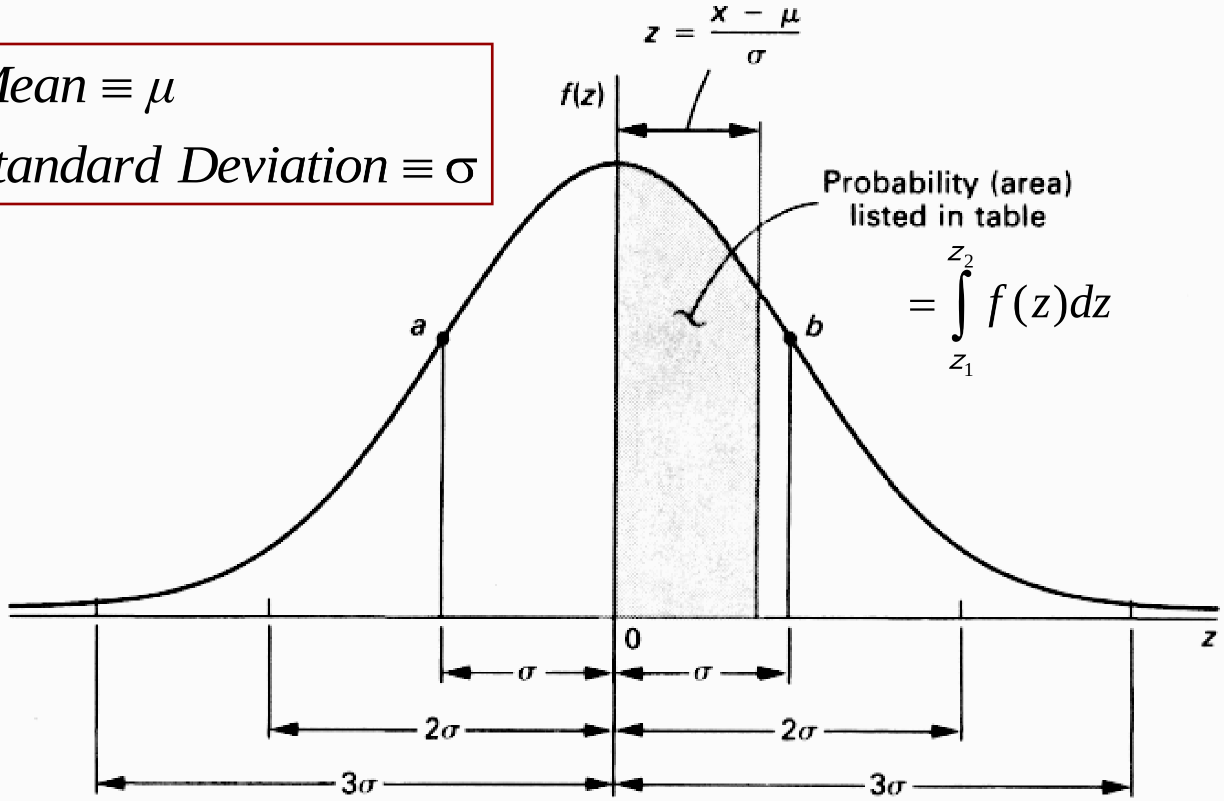


Standard normal probability distribution function (PDF)
(aka. normalized Gaussian function)

$Mean \equiv \mu$

$Standard\ Deviation \equiv \sigma$

Figure 3.8 Standard normal distribution curve. Note that a and b are inflection points.



Gauss function:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

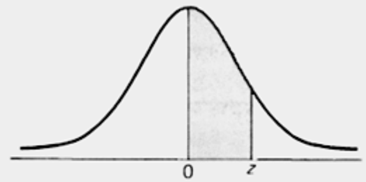
Normalized Gauss function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

From: Beckwith, et. al.

Standard normal PDF: tabulated area

Table 3.2 Areas under the standard normal curve



Second Decimal Place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000*									

* For $z \geq 3.90$, the areas are 0.5000 to four decimal places.

Recall that:

$$z = \frac{x-\mu}{\sigma}$$

From: Beckwith, et. al.

Use of the standard normal PDF

Example 3.2 (Beckwith):

- What is the area under the curve between $z = -1.43$ and $z = 1.43$
- What is the significance of this area?

Solution:

- From Table 3.2 (Beckwith), ½ area is 0.4236. Total area is: 0.8472;
- Significance: 84.72 % of the population falls between the "normal" values of $z = -1.43$ and $z = 1.43$

Use of the standard normal PDF

Example 3.3 (Beckwith):

- What range of x will contain 90 % of the data

Solution:

- Find z such that 90/2 = 45 % of the data lie between zero and z;
- Second 45 % will lie between $-z$ and zero;
- Using interpolation: $z_{0.45} \approx 1.645$.
- Since $z = (x - \mu)/\sigma$:

$$(\mu - z_{0.45}\ \sigma) < x < (\mu + z_{0.45}\ \sigma)$$

$$(\mu - 1.645\,\sigma) < x < (\mu + 1.645\,\sigma)$$

Precision uncertainty

Consider the problem of measuring the electrical resistance of a component (as done in Lab #1); governing equation to use is:

$$R = \frac{V}{I}$$

Recall that parameters are determined as: $V \pm \delta V$,

$I \pm \delta I$

Therefore, R will be recovered as: $R \pm \delta R$

How do we determine δR ?

Precision uncertainty

Governing equation to use is:

$$R = \frac{V}{I}$$

Phenomenological equation is, therefore, $R = R(V, I)$

Square-root of the sum-of-the squares (RSS) approach indicates that the uncertainty, δR, in R, can be determined as

$$\delta R = \left[\left(\frac{\partial R}{\partial V} \delta V \right)^2 + \left(\frac{\partial R}{\partial I} \delta I \right)^2 \right]^{1/2}$$

Uncertainty in measured voltage is: δV

Uncertainty in provided voltage is: δI

Precision uncertainty

Individual partial derivates are: $$\frac{\delta R}{\partial V} = \frac{1}{I}; \quad \frac{\delta R}{\partial I} = -\frac{V}{I^2}$$

Substitute: $$\delta R = \left[\left(\frac{1}{I} \delta V \right)^2 + \left(-\frac{V}{I^2} \delta I \right)^2 \right]^{1/2}$$

Normalized uncertainty is: $$\frac{\delta R}{R} = \left[\left(\frac{1}{V} \delta V \right)^2 + \left(-\frac{1}{I} \delta I \right)^2 \right]^{1/2}$$

Problem: estimate uncertainty and normalized uncertainty in measurements of electrical resistance (as done in Lab #1).

Assume: (a) measuring range $[-10,10]$ V; (b) 12-bit digitization; (c) measured voltage of 0.11 V; (d) $I = 1$ mA; half the least significant digit for δI; (e) determine $R \pm \delta R$

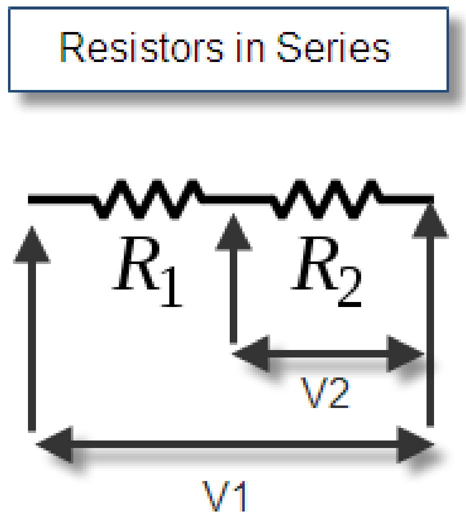
Discuss your results.

Precision uncertainty

Consider the problem of measuring the electrical resistance of a component (as done in Lab #1); governing equation used to determine R_2 is:

$$R_2 = \frac{V_2 \cdot R_1}{V_1 - V_2}$$

Determine: $R_2 \pm \delta R_2$



Part of Lab #1 results