WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation

Laboratory 4: Vibration Measurements

COURSE No.: ME-3901
DATE: Term D'12

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Objectives

The objectives of this laboratory are to use different types of motion transducers to measure the dynamic characteristic and the elastic material properties of a cantilever. The motion transducers will be calibrated to determine transfer function characteristics.

For each motion transducer, vibration data will be analyzed to:

1. Determine the vibration amplitude, velocity, and acceleration in various units of measure;
2. Determine natural frequencies;
3. Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
4. Compare measurements with analytical and/or computational models of a cantilever; and
5. Determine elastic modulus of a cantilever via vibration measurements.

Background

Health monitoring is the process of studying and assessing the integrity of structures, which is crucial for preventing failure and for achieving reliable designs. Health monitoring can be done by dynamic or static analysis, or a combination of both. For dynamic analysis, dynamic characteristics of the structures, including natural frequencies, modal shapes, and damping factors, are determined via modal analysis. In static analysis, deformations or changes in the orientation of structures, due to application of loads, or unexpected damages, are determined via comparisons with reference models.

In either static or dynamic health monitoring, the utilization of appropriate transducers is required to provide accurate measurement of structural responses in both frequency and time domains. Conventional devices utilized for health monitoring are based on piezoelectric transducers. These transducers are usually large in size, require high actuation power, and have narrow frequency bandwidths, which reduce their accuracy, versatility, and applicability to study smaller structures. The advanced developments of IC microfabrication and microelectromechanical systems (MEMS) have led to the progressive designs of small footprint, low dynamic mass and actuation power MEMS inertial sensors. Due to their high natural frequencies, these MEMS inertial sensors provide wide frequency bandwidths and high measuring accuracies.

Equipment

Motion transducers to be used (the number and type will be set by the instructor):

1. Strain gages;
2. Piezoelectric accelerometers & power sources; and
3. MEMS (Microelectromechanical Systems) accelerometers.

Structure of interest and fixtures:

1. Aluminum beam; and
2. C-clamp.

DAQ:

1. NI modules, screw terminals, and LabView software;
2. 2310 Signal Conditioning Amplifier.

Tasks

Strain gages:

Refer to Laboratory #3: Strain and Pressure Measurements. Similar calibration and measurement procedures will be utilized. Recommended procedure is:

1. Derive analytical solutions to determine:
    a. Stresses and strains of a cantilever subjected to a concentrated load;
    b. Fundamental natural frequencies and damping factors of a cantilever;

2. Using a C-clamp, mount cantilever to the edge of the bench and attach one strain gage. Make sure to attach gage at a location on the beam which has as maximum strain level as possible; check with your analytical solutions;

3. Calculate the necessary output gain for the gage; use a 10 V bridge excitation; recommended filter level is 100; setup an analog input channel to the range of [-2,2] Volts.

4. To determine an appropriate output gain consider loading the beam under static situation and with a known load;

5. Pluck the end of the cantilever and record the vibration decay curve;

6. Use and modify suitable LabView VIs to record time & frequency data;

7. Using the recorded data, determine modal parameters (i.e., natural frequencies and damping factors) and material properties of the cantilever;

8. Compare your results with analytical/computational models of a cantilever; and

9. Do uncertainty analysis on the determined modal parameters and measured material properties.

Piezoelectric accelerometers:

Piezoelectric accelerometers require a constant current source to power their internal electronics. This creates a DC bias voltage upon which the acceleration signal rides. Hence the accelerometers cannot be used to measure static acceleration because when placed in a static acceleration environment the output will gradually drift to zero. Recommended procedure is:

1. Using a C-clamp, mount cantilever to the edge of the bench and attach the piezoelectric accelerometer;

2. Connect the accelerometer to a suitable power supply and the output signal to an appropriate analog input channel for the DAQ and VI;

3. Make sure to identify and verify output level of the accelerometer, given in **milliVolts/g** (g = 9.81 m/s^2), and setup appropriate voltage range for the analog input channel for the DAQ;

4. Pluck the end of the cantilever and record the vibration decay curve;

5. Use and modify suitable LabView VIs to record time-frequency data;

6. Using the recorded data, determine modal parameters (i.e., natural frequencies and damping factors) and material properties of the cantilever;

7. Compare your results with analytical/computational models of a cantilever; and

8. Do uncertainty analysis on the determined modal parameters and measured material properties.

MEMS accelerometers:

In this laboratory, the Analog Devices ADXL276 (AD22237) dual-axes accelerometer will be utilized. The ADXL276 MEMS accelerometer is currently being used in a wide range of consumer, industrial, military, and automotive applications. The ADXL276 accelerometer has a measuring range of ±35 g (g = 9.81 m/s^2) and it provides analog output signals with sensitivity on the order of 55 mV/g. It requires 3 V to 5.25 V for operation and is capable of operating in a wide range of temperatures. Recommended procedure is:

1. Using a C-clamp, mount cantilever to the edge of the bench and attach a MEMS accelerometer; make sure to take into account sensitivity axes of the device;

2. Connect the accelerometer to a suitable power supply and the signal output to an appropriate analog input channel for the DAQ and VI;

3. Make sure to identify and verify output level of the accelerometer, given in **milliVolts/g** (g = 9.81 m/s^2), and setup appropriate voltage range for the analog input channel for the DAQ;

4. Pluck the end of the cantilever and record the vibration decay curve;

5. Use and modify suitable LabView VIs to record time-frequency data;

6. Using the recorded data, determine modal parameters (i.e., natural frequencies and damping factors) and material properties of the cantilever;

7. Compare your results with analytical/computational models of a cantilever; and

8. Do uncertainty analysis on the determined modal parameters and measured material properties.

Report

Your report must include:

1. For each motion transducer, computation of the natural frequency and damping of the cantilever/accelerometer system from the measured vibration decay curve. Provide the results in units of Hz, circular frequency, log decrement, and percent of critical damping;

2. Tables of results giving the maximum acceleration, velocity, and displacement of the cantilever using the data from the test. Give your answers for acceleration, velocity and displacement in both English (use inches or mils) and SI units (millimeters). In addition, report maximum acceleration in units of g's. Finally, specify the values in terms of RMS, peak, and peak-to-peak (1 mil = 0.001 inches);

3. Table providing summary of the modal parameters and elastic material properties obtained with each motion transducer. In the same table, also provide analytical and/or computational results. Make sure to indicate percentage differences.

4. Uncertainty analyses on the determined modal parameters and measured material properties. Make sure to identify uncertainty parameters in order of importance.

References:

R. Kok, *Development of a wireless MEMS inertial system for health monitoring of structures*, MS Thesis, Mechanical Engineering Department, Worcester Polytechnic Institute, 2004

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J. P. Holman, *Experimental Methods for Engineers*, McGraw-Hill, 2012

T. G. Beckwith, R. D. Marangoni, and J. H. Lienhard, *Mechanical Measurements*, 6th ed., Prentice-Hall, 2007

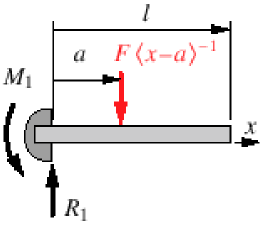
R. L. Norton, *Machine Design*, 3ed Ed., Prentice-Hall, 2007

Analog Devices, Application Note, "ADXL202 / ADXL210 - low cost ±2 g / ±10 g dual axis iMEMS® accelerometers with digital output," Analog Devices, Inc., Norwood, MA, 2006

Analog Devices, Application Note, "±150°/s single chip yaw rate gyro evaluation board," Analog Devices, Inc., Norwood, MA, 2006

J. Hall, *Laboratory 4: Vibration Measurements, Natural Frequency, and Damping*, Engineering Experimentation, Worcester Polytechnic Institute, 2010

(a) Cantilever beam with concentrated loading

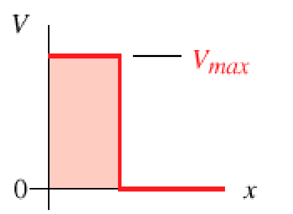


$$R_1 = F$$

$$M_1 = Fa$$

Loading

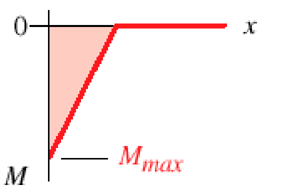
$$q = M_1\langle x\rangle^{-2} + R_1\langle x\rangle^{-1} - F\langle x-a\rangle^{-1}$$



$$V_{max} = R_1 = F$$

$$V = M_1\langle x\rangle^{-1} + R_1 - F\langle x-a\rangle^{0} = F\left(1-\langle x-a\rangle^{0}\right)$$

Shear

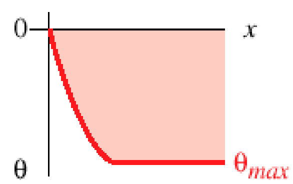


$$M_{max} = -Fa$$

when $a = l$: $M_{max} = -Fl$

$$M = -M_1 + R_1 x - F\langle x-a\rangle^{1} = F\left(-a + x - \langle x-a\rangle^{1}\right)$$

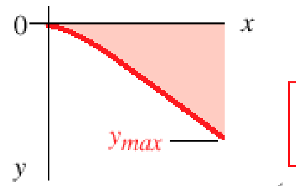
Moment



$$\theta = \frac{1}{EI}\left(-M_1 x + \frac{R_1}{2}x^2 - \frac{F}{2}\langle x-a\rangle^2\right)$$

$$\theta = \frac{F}{2EI}\left(-2ax + x^2 - \langle x-a\rangle^2\right)$$

Slope



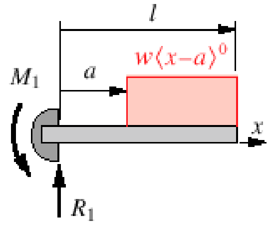
$$y_{max} = \frac{Fa^2}{6EI}(a-3l)$$

when $a = l$: $y_{max} = -\dfrac{Fl^3}{3EI}$

$$y = \frac{1}{EI}\left(-\frac{M_1}{2}x^2 + \frac{R_1}{6}x^3 - \frac{F}{6}\langle x-a\rangle^3\right) = \frac{F}{6EI}\left(x^3 - 3ax^2 - \langle x-a\rangle^3\right)$$

Deflection

(b) Cantilever beam with uniformly distributed loading

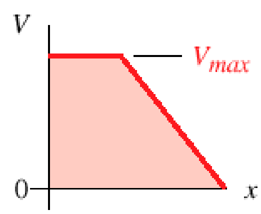


$$R_1 = w(l-a)$$

$$M_1 = \frac{w}{2}\left(l^2 - a^2\right)$$

Loading

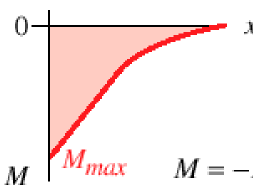
$$q = M_1\langle x\rangle^{-2} + R_1\langle x\rangle^{-1} - w\langle x-a\rangle^{0}$$



$$V_{max} = R_1 = w(l-a)$$

$$V = M_1\langle x\rangle^{-1} + R_1 - w\langle x-a\rangle^{1} = w\left[(l-a) - \langle x-a\rangle^{1}\right]$$

Shear

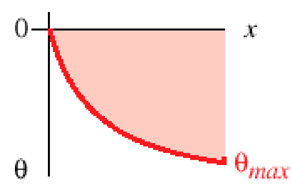


$$M_{max} = M_1 = \frac{w}{2}\left(l^2 - a^2\right)$$

when $a = 0$: $M_{max} = \dfrac{wl^2}{2}$

$$M = -M_1 + R_1 x - w\langle x-a\rangle^{2} = \frac{w}{2}\left[2(l-a)x - \left(l^2 - a^2\right) - 2\langle x-a\rangle^2\right]$$

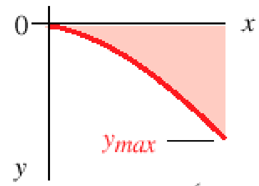
Moment



$$\theta = \frac{1}{EI}\left(-M_1 x + \frac{R_1}{2}x^2 - \frac{w}{6}\langle x-a\rangle^3\right)$$

$$\theta = \frac{w}{6EI}\left(3(l-a)x^2 - 3\left(l^2 - a^2\right)x - \langle x-a\rangle^3\right)$$

Slope



$$y_{max} = \frac{w}{24EI}\left(-3l^4 + 4a^3 l - a^4\right)$$

when $a = 0$: $y_{max} = -\dfrac{wl^4}{8EI}$

$$y = \frac{1}{EI}\left(-\frac{M_1}{2}x^2 + \frac{R_1}{6}x^3 - \frac{w}{24}\langle x-a\rangle^4\right) = \frac{w}{24EI}\left(4(l-a)x^3 - 6\left(l^2 - a^2\right)x^2 - \langle x-a\rangle^4\right)$$

Deflection

FIGURE D-1

Cantilever Beams with Concentrated or Distributed Loading. Note: < > Denotes a Singularity Function

Sample calculations for a low-carbon steel cantilever

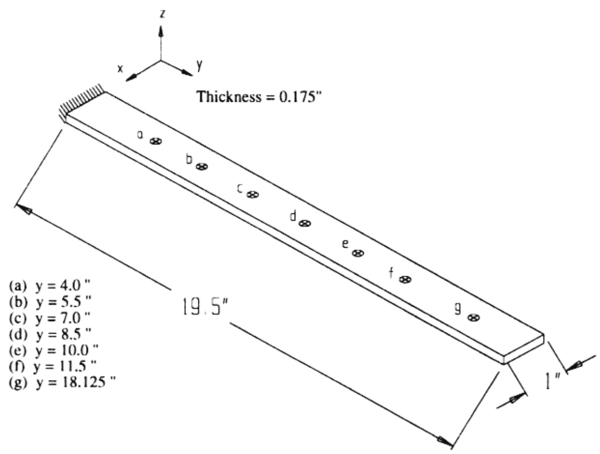


Fig. 3 Low carbon steel beam showing locations of interest

In order to verify the results obtained experimentally, the analytical and numerical solutions of the modal analysis of a cantilever beam were reviewed. The first five natural frequencies are found using Eq. 3 to 7 (see [8]). These equation are found by solving the partial differential equation given in Eq. 8 for zero displacement and slope at the clamped end and zero bending moment and shear at the free end. The evaluated frequencies are shown in Table 4.

$$\omega_n = 3.5160\sqrt{\frac{EI}{\overline{m}L^4}}\ , \qquad (3)$$

where,

E = Young's modulus = 30×10^6 psi,
I = Moment of inertia of the beam section = 4.46614×10^{-4} in^4 ,
$\overline{m}$ = mass per unit of length = 1.26805×10^{-4} lbf·s^2/in^2, and
L = length of the beam = 19.5 in,

$$\omega_n = 22.0345\sqrt{\frac{EI}{\overline{m}L^4}}\ , \qquad (4)$$

$$\omega_n = 61.6972\sqrt{\frac{EI}{\overline{m}L^4}}\ , \qquad (5)$$

$$\omega_n = 120.0902\sqrt{\frac{EI}{\overline{m}L^4}}\ , \qquad (6)$$

$$\omega_n = 199.8600\sqrt{\frac{EI}{\overline{m}L^4}}\ , \qquad (7)$$

$$EI\frac{\partial^4 y}{\partial x^4} + \overline{m}\frac{\partial^2 y}{\partial t^2} = 0\ . \qquad (8)$$

Table 4. Natural Frequencies obtained using Eq. 3 to 7

Natural Frequency	Magnitude, Hz
1	15.1270
2	94.8010
3	265.446
4	516.675
5	859.876

Basics about MEMS accelerometers

THEORY OF OPERATION

The ADXL76, ADXL269, and ADXL276 are fabricated using a proprietary *i*MEMS surface micromachining process that has been in high-volume production since 1993. The fabrication technique uses standard integrated circuit manufacturing methods enabling all the signal processing circuitry to be combined on the same chip with the sensor. The higher levels of integration and economies of volume production that exist for other ICs exist for ADI's surface micromachined process as well. This can be seen by comparing prior generation ADXL50 and ADXL75 to this third generation ADXL76 product family.

The ADXL76 family requires no external components, other than a standard power supply bypass capacitor. A 2-pole switched-capacitor Bessel filter is included on the chip.

The surface micromachined sensor element is made by depositing polysilicon on a sacrificial oxide layer that is then etched away leaving the suspended sensor element. Figure 1 is a simplified view of the ADXL76 sensor structures. In the ADXL276 there are two sensing elements orthogonal to each other and in the ADXL269 there two sensing elements of opposite polarity.

Each sensor has 42 unit sensing cells to sense acceleration. This differential capacitor sensor is composed of fixed plates and movable plates attached to the beam that moves in response to acceleration. Movement of the beam changes the differential capacitance which is measured with the on-chip circuitry.

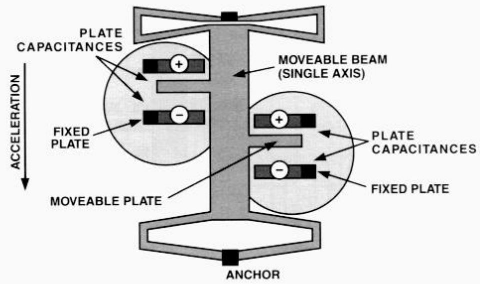


Figure 1. Simplified View of Sensor Under Acceleration

The fixed plates are driven by complementary 100 kHz clocks. When no acceleration is applied the signal output is zero. When the beam is deflected due to an acceleration the resulting signal is proportional to the acceleration (Figure 2). After amplification the signal is synchronously demodulated. This removes the oscillator frequency from the signal and provides a dc output signal. In the final stage of the accelerometer IC the dc signal is further amplified and filtered (Figure 3).

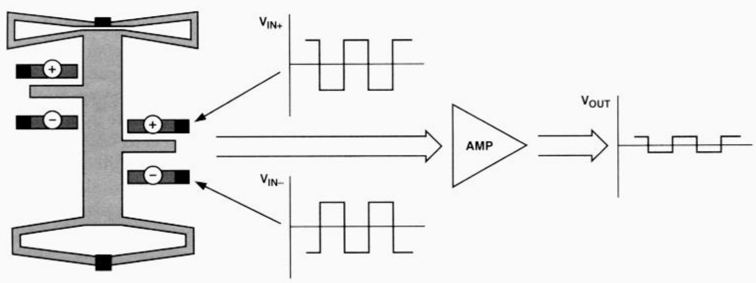


Figure 2. Drive signals to the differential capacitors are 180 degrees out of phase. When the differential capacitors are unequal, there is a signal out of the sensor which is proportional to the force on the beam. This modulated output from the beam is then demodulated to create a dc output (not shown).

ADXL76, ADXL269, ADXL276

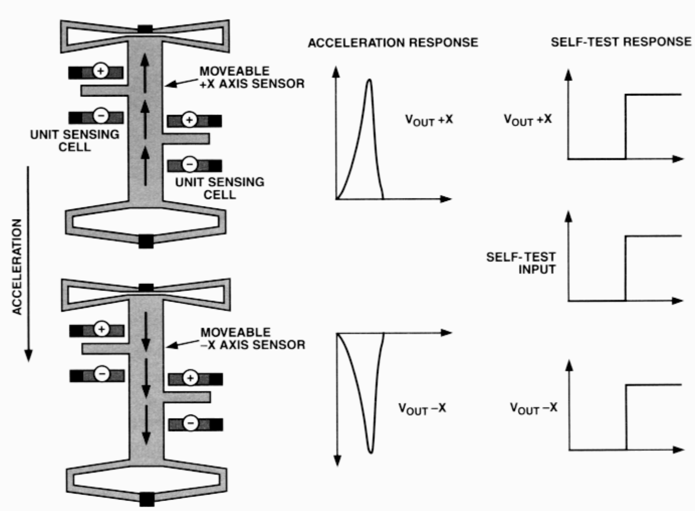


Figure 3. Simplified Top-Down View of +X and –X Sensors Under Acceleration in the ADXL269

Mounting specifications for a typical piezoelectric accelerometer