WORCESTER POLYTECHNIC INSTITUTE

MECHANICAL ENGINEERING DEPARTMENT

Engineering Experimentation
ME-3901, D'2010

Laboratory #4:

Vibration measurements:

Strain gauges + MEMS + Elastic Modulus

General information

Office hours

Instructors: **Cosme Furlong**

Office: HL-151

Everyday:

9:00 to 9:50 am

Christopher Scarpino

Office: HL-153

During laboratory sessions

Teaching Assistants: During laboratory sessions

General information

Please refer to posted handouts:

"Laboratory 4: vibration measurements"

Objectives

The objectives of this laboratory are to:

- use <u>two-different</u> types of motion transducers to measure the natural frequencies, damping characteristics, and elastic modulus of a cantilever

For each motion transducer, vibration data will be analyzed to:

1. Determine the vibration amplitude, velocity, and acceleration in various units of measure;
2. Determine natural frequencies;
3. Measure and express damping characteristics as logarithmic decrement and percentage of critical damping;
4. Compare measurements with analytical and/or computational models of a cantilever;
5. Determine elastic modulus of a cantilever via vibration measurements

Amplitude (position), velocity, acceleration functions

- Recall that for a position function given by:

$$y(t) = A\sin(\omega t + \phi)$$

$[\omega, \text{rad/sec}]$

$[f = \frac{\omega}{2\pi}, \text{Hz}]$

- Velocity function is:

$$\dot{y}(t) = \omega A \cdot \cos(\omega t + \phi)$$

- Acceleration function is:

$$\ddot{y}(t) = \underbrace{-\omega^2 A}\cdot \sin(\omega t + \phi)$$

Magnitude of acceleration

Frequency-time domain: dynamic signal analyzer

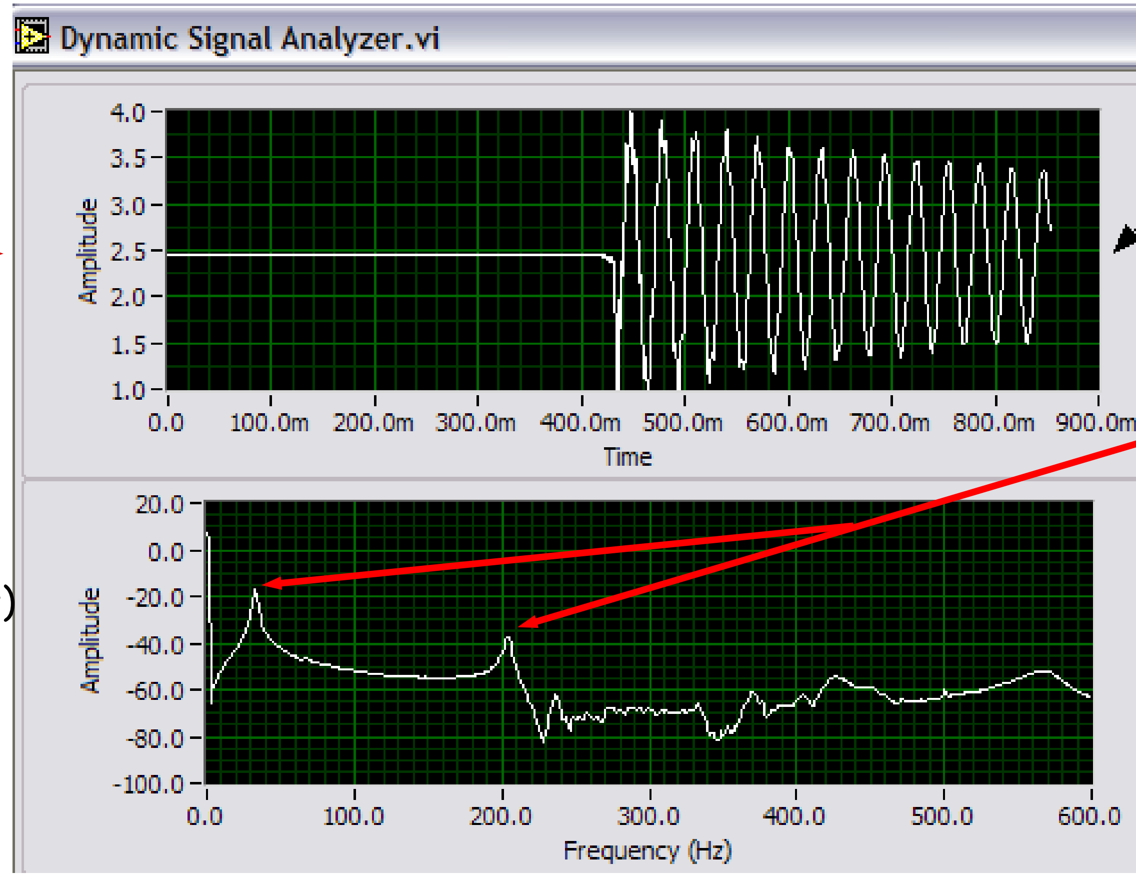
Sample: single pluck (two frequencies are identified)

Time-domain:

$y(t)$ or $\ddot{y}(t)$

(Uses 2^n discrete points)

Frequency-domain:
(discrete evaluation of)



Note "peaks," which correspond to natural frequencies of vibration

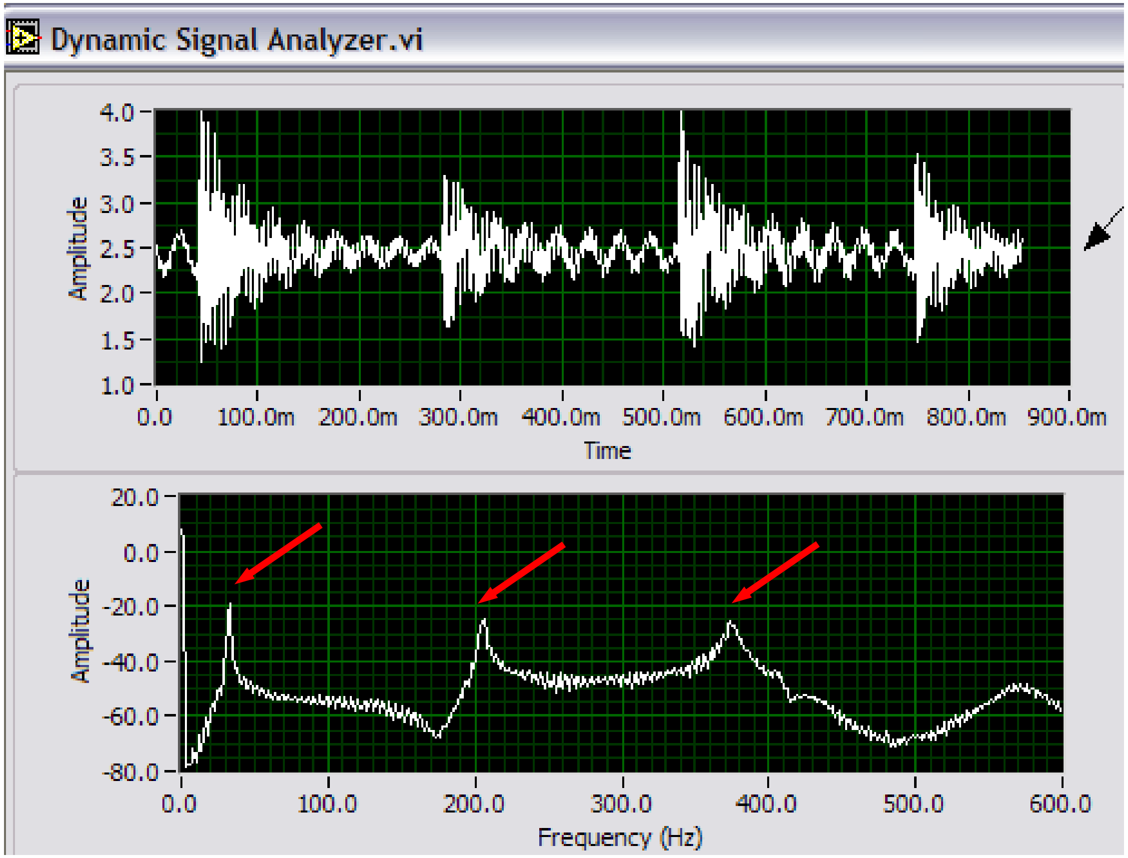
$$H(f) = \int_{-\infty}^{\infty} y(t)\exp(-j2\pi f t)dt$$

Amplitude or Acceleration (depending on the transducer used)

Frequency-time domain: dynamic signal analyzer

Sample: rapid plucking

(increases frequency resolution, three frequencies are identified)



Do these frequency "peaks" correspond to bending, torsion, or in-plane modes of vibration?

Please identify, if possible.

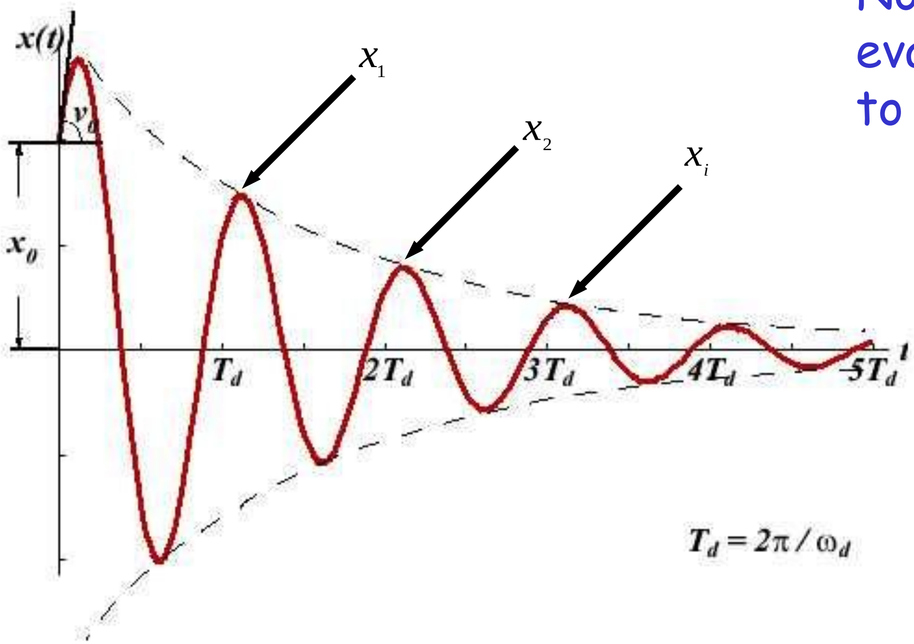
Hint: see equations that predict bending frequencies

Analysis of a single degree of freedom system

First case: $F(t) = 0$ - Free vibrations

$$\lambda^2 - \omega^2 < 0$$ Under-damped system

Logarithmic decrement: $$\delta = \ln\frac{x_1}{x_2} = \ln\frac{x_i}{x_{i+1}}$$



Note that you need to evaluate δ multiple times to obtain: $\bar{\delta} \pm \Delta\delta$

(including statistics and evaluated uncertainty)

Analysis of a single degree of freedom system

First case: $F(t) = 0$ - Free vibrations

$$\lambda^2 - \omega^2 < 0$$ Under-damped system

Logarithmic decrement:

$$\delta = \ln \frac{x_1}{x_2} = \ln \frac{e^{-\zeta \omega_n t_1} \sin(\sqrt{1-\zeta^2}\,\omega_n t_1 + \phi)}{e^{-\zeta \omega_n (t_1+T_d)} \sin[\sqrt{1-\zeta^2}\,\omega_n (t_1 + T_d) + \phi]}$$

$$\delta = \ln \frac{x_1}{x_2} = \ln \frac{e^{-\zeta \omega_n t_1}}{e^{-\zeta \omega_n (t_1+T_d)}} = \ln e^{\zeta \omega_n T_d} = \zeta \omega_n T_d$$

Recall that: $T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n \sqrt{1-\zeta^2}} \Rightarrow$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}}$$

(account for measured uncertainty in δ)

You can determine damping ratio with knowledge of δ

$$\zeta = \frac{b}{b_c}$$ (Damping ratio – percentage of critical damping)

Background: cantilevers

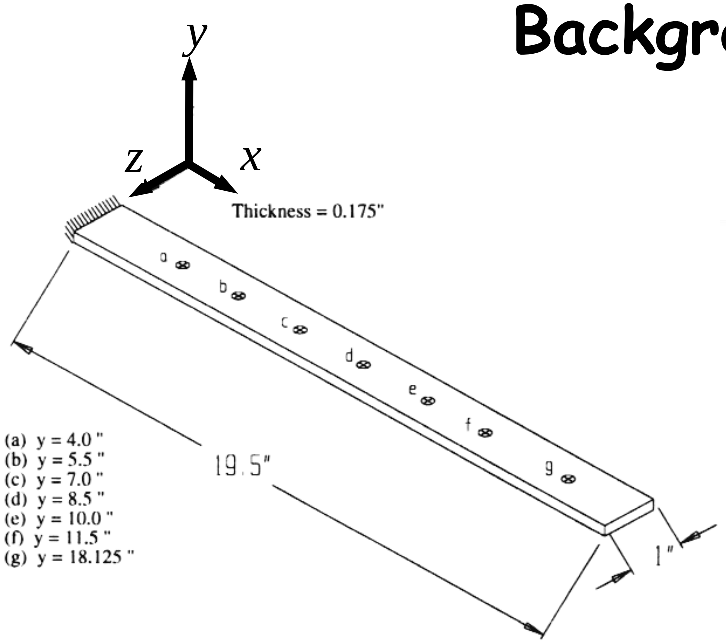


Fig. 3 Low carbon steel beam showing locations of interest

Note: first frequency can also be expressed as

$$\omega_n = 1.015\sqrt{\frac{E}{\rho}}\left(\frac{t}{L^2}\right), \text{ rad/s}$$

Natural frequencies

First: $\omega_n = 3.5160\sqrt{\frac{EI}{\overline{m}L^4}}$, rad/s (3)

where,

E = Young's modulus = 30×10^6 psi,

I = Moment of inertia of the beam section = 4.46614×10^{-4} in^4,

$\overline{m}$ = mass per unit of length = 1.26805×10^{-4} lbf·s^2/in^2, and

L = length of the beam = 19.5 in,

Second: $\omega_n = 22.0345\sqrt{\frac{EI}{\overline{m}L^4}}$, (4)

Third: $\omega_n = 61.6972\sqrt{\frac{EI}{\overline{m}L^4}}$, (5)

Fourth: $\omega_n = 120.0902\sqrt{\frac{EI}{\overline{m}L^4}}$, (6)

Fifth: $\omega_n = 199.8600\sqrt{\frac{EI}{\overline{m}L^4}}$, (7)

$$EI\frac{\partial^4 y}{\partial x^4} + \overline{m}\frac{\partial^2 y}{\partial t^2} = 0 . \quad (8)$$

Table 4. Natural Frequencies obtained using Eq. 3 to 7

Natural Frequency	Magnitude, Hz
1	15.1270
2	94.8010
3	265.446
4	516.675
5	859.876

Recovery of elastic modulus from vibration measurements

Assume the use of the fundamental frequency (higher frequencies can also be used):

$$\omega_n = 1.015\sqrt{\frac{E}{\rho}}\left(\frac{t}{L^2}\right), \ \text{rad/s}$$

Solve for the elastic modulus, E, at a measured ω_n :

$$E = E(\omega_n, \rho, t, L)$$

Note: take multiple measurements at different beam lengths

Note that uncertainty analysis is necessary:

$$\delta E = \delta E(\omega_n, \rho, t, L)$$

Make note on:

Measuring resolutions/sensitivities obtained with MEMS

&

Strain gauges