WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

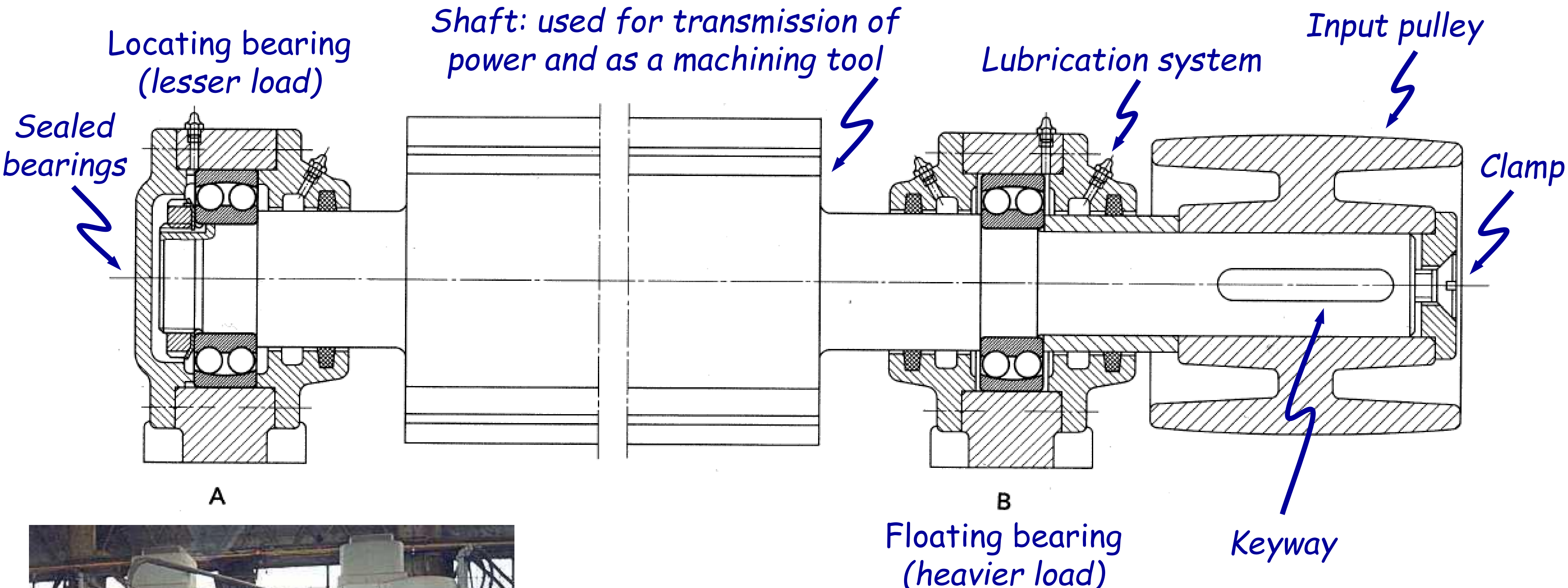
DESIGN OF MACHINE ELEMENTS
ME-3320, B'2022

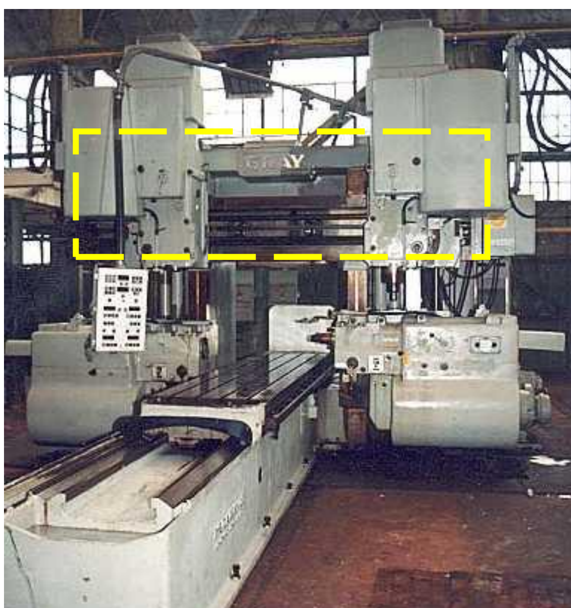
Lecture 20

December 2022

Shaft design

Example of rotating machinery: self-aligning ball bearings





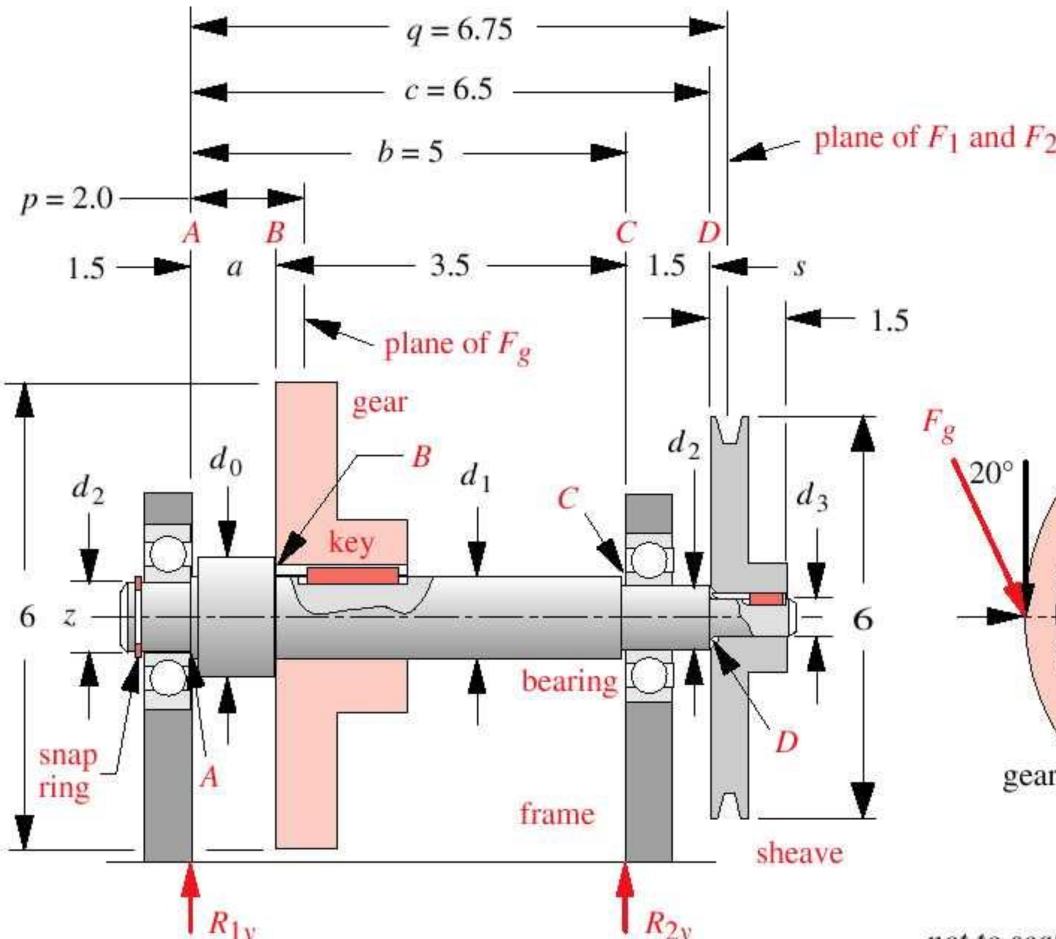
- **Cutter shaft of a planer:** shaft diameter (at bearings locations) is 40 mm. Input power is 12 HP at maximum speed of 4,500 rpm

Shaft design

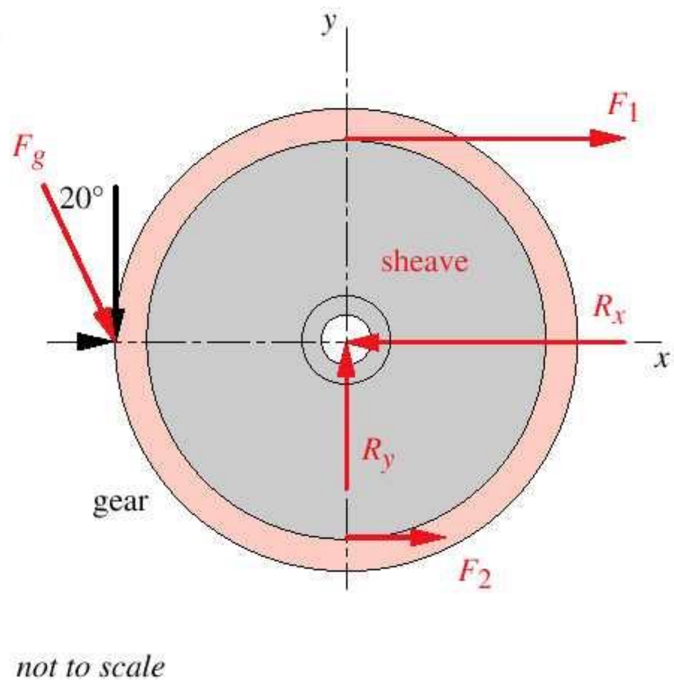
Fully-reversed bending and constant torsion

- **Review and Master: Example 10-1**

Design shaft to support attachments



- Safety factor: 2.5
- Infinite life
- Material: SAE 1020 (good notch sensitivity)
- Operating conditions: room temperature
- Power: 2 HP at 1,750 rpm
- SCF of 3.5 for radii in bending, 2 in torsion, and 4 at the keyway
- Assume notch radius of 0.01 in

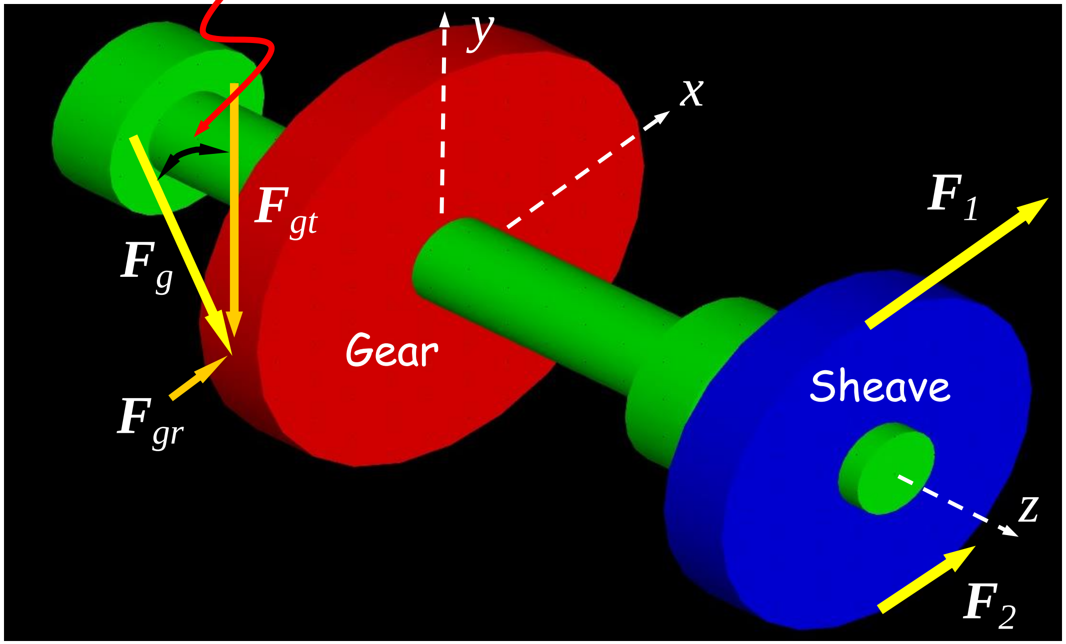


not to scale

Shaft design
Fully-reversed bending and constant torsion

- **Review and Master: Example 10-1**

Pressure angle

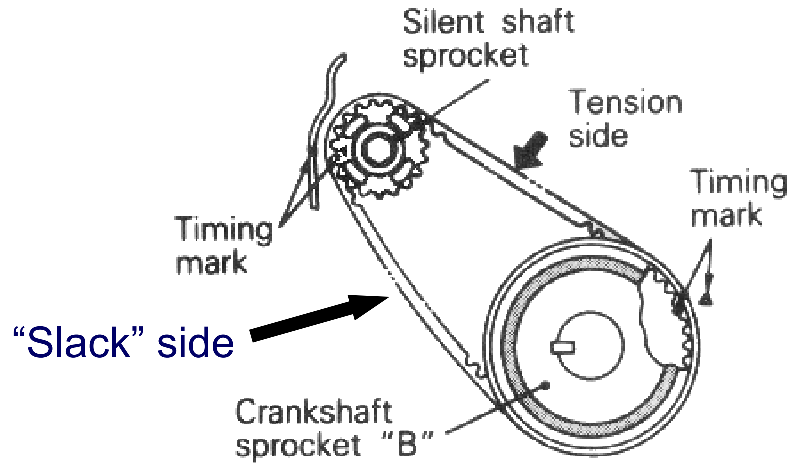


- Safety factor: 2.5
- Infinite life
- Material: SAE 1020 (good notch sensitivity)
- Operating conditions: room temperature
- Power: 2 HP at 1,750 rpm
- SCF of 3.5 for radii in bending, 2 in torsion, and 4 at the keyway
- Assume notch radius of 0.01 in

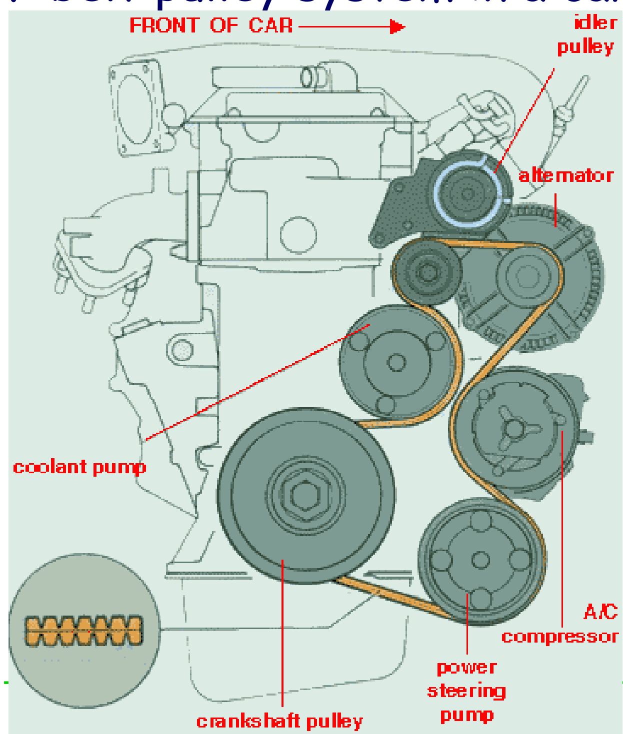
Tension & slack side components of the net force

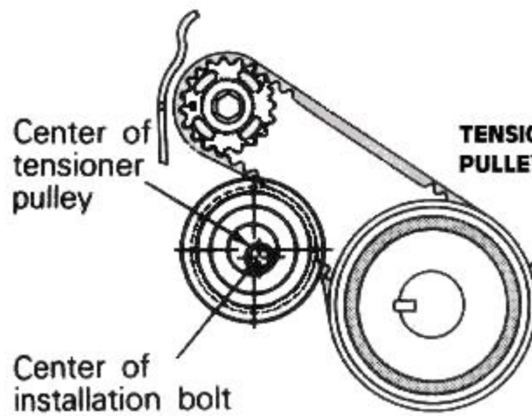
Shaft design: components. Examples 10-1 and 10-2

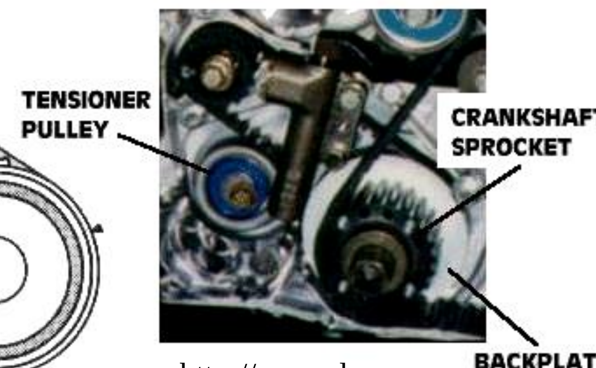
V-belt pulley



V-belt pulley system in a car





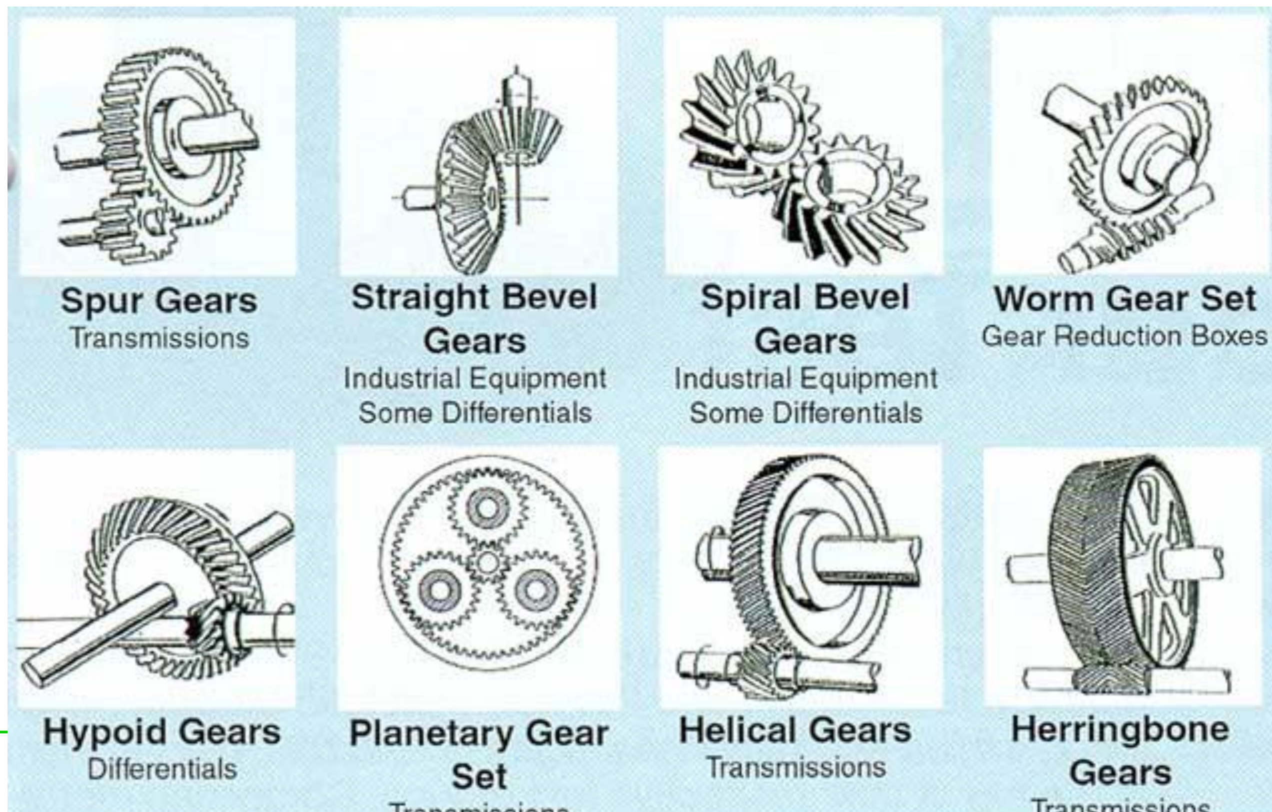


http://www.dsm.org

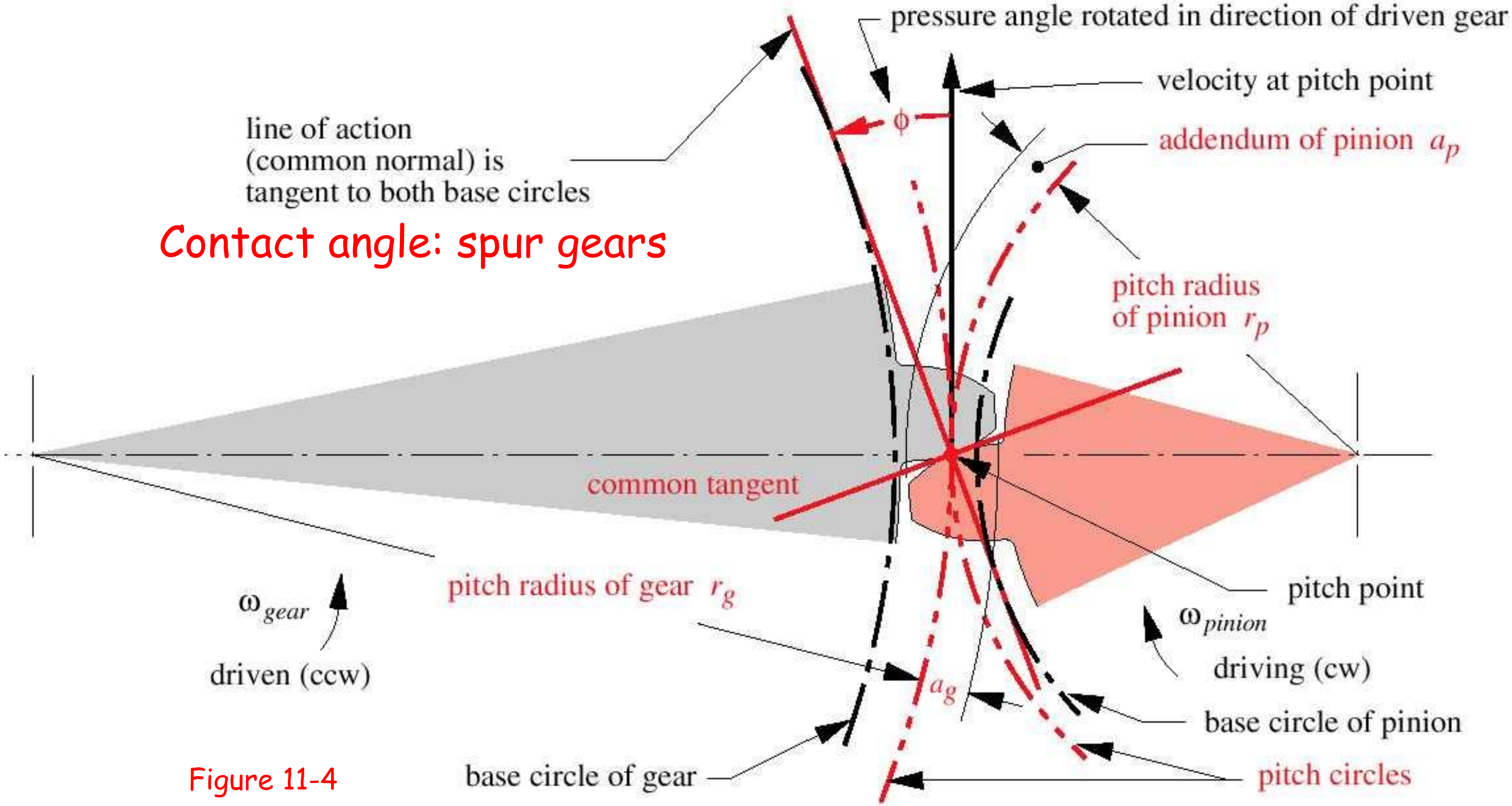
Shaft design: components. Examples 10-1 and 10-2

Spur

Helical



Shaft design: components. Examples 10-1 and 10-2

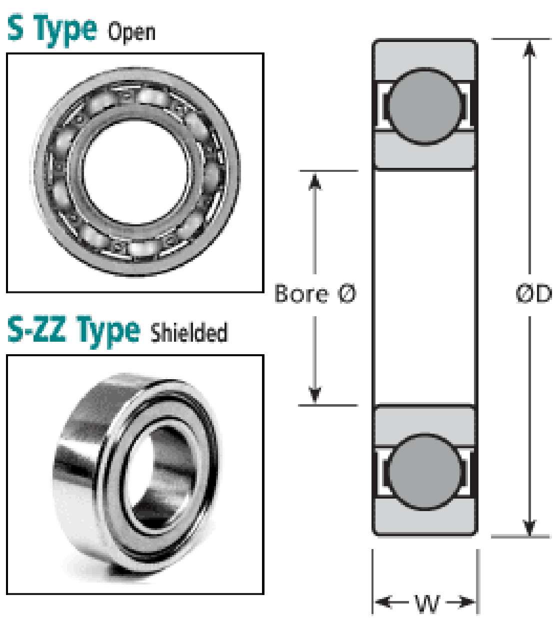


Contact angle: spur gears

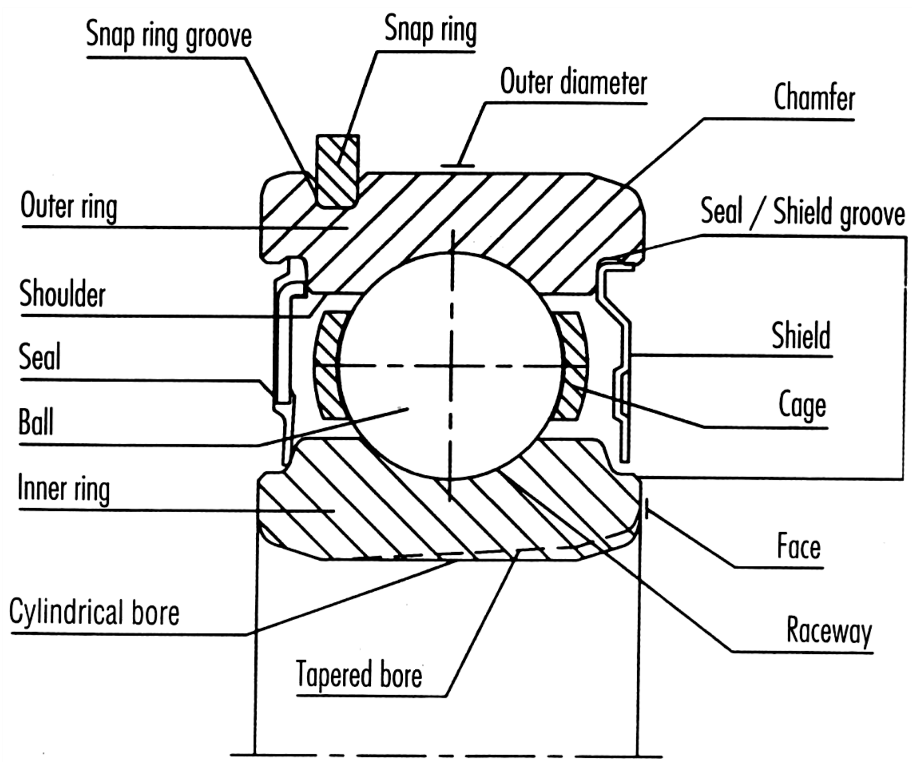
Figure 11-4

Shaft design: components. Examples 10-1 and 10-2

Radial ball bearings



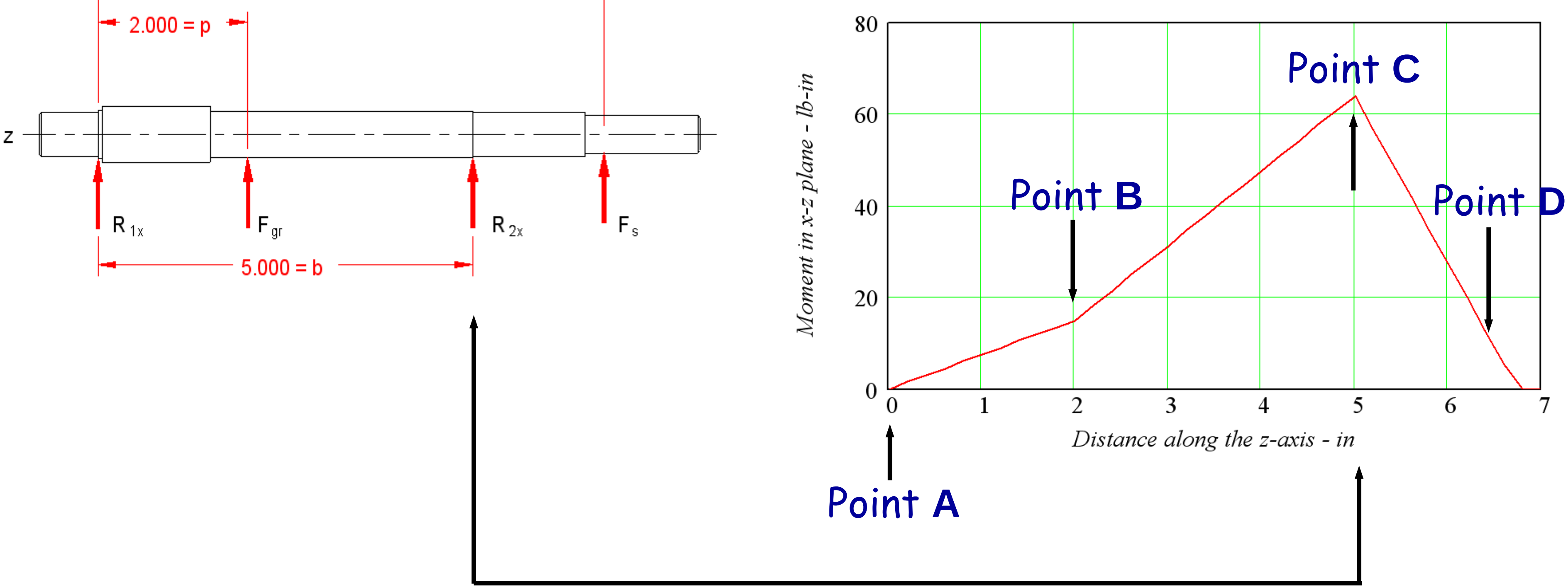
Cross-section of a radial ball-bearing



Shaft design: components. Examples 10-1 and 10-2

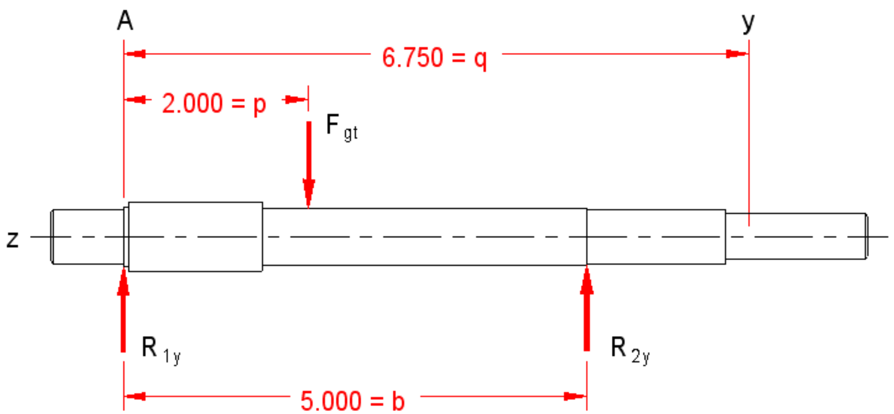
- **Load diagram in the X-Z plane**

Moments diagram, X-Z plane

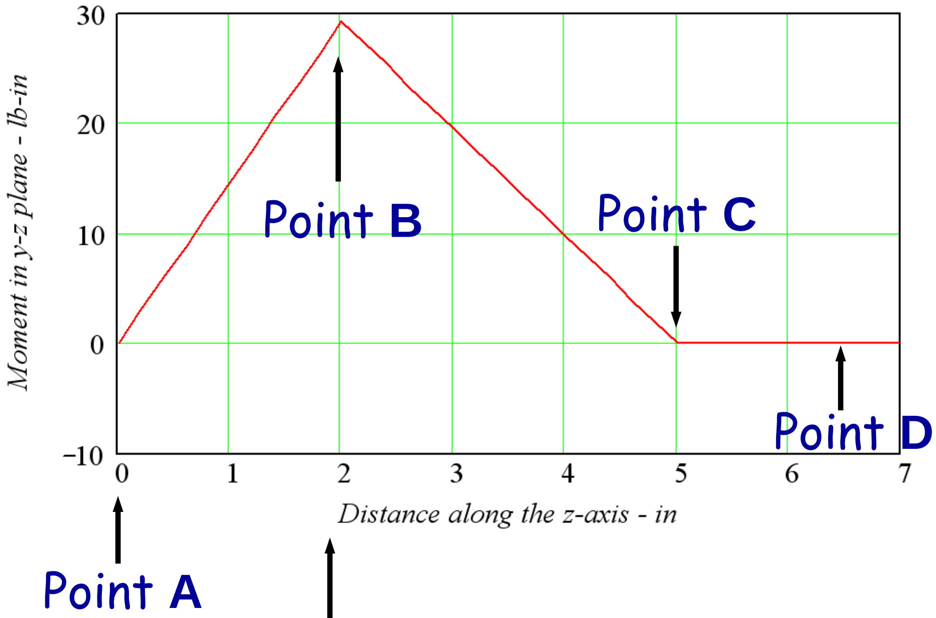


Shaft design: components. Examples 10-1 and 10-2

- **Load diagram in the Y-Z plane**



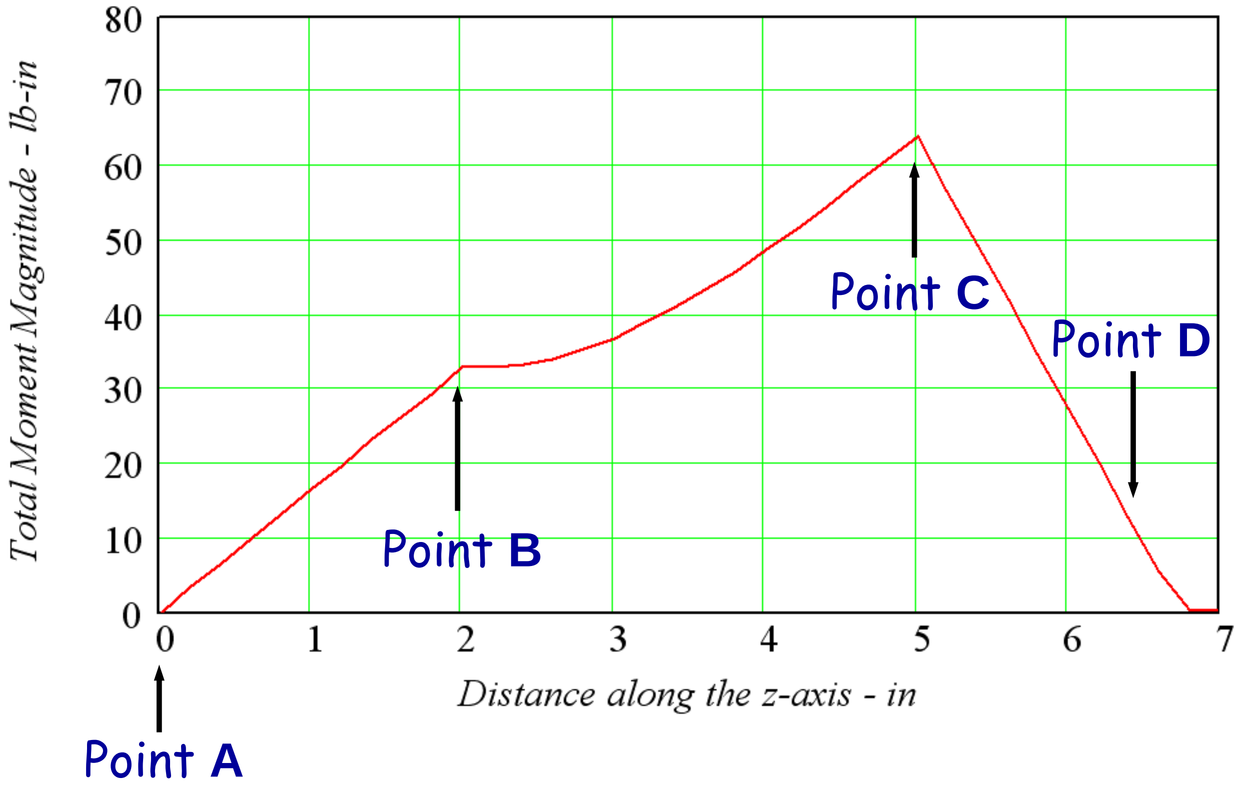
Moments diagram, Y-Z plane

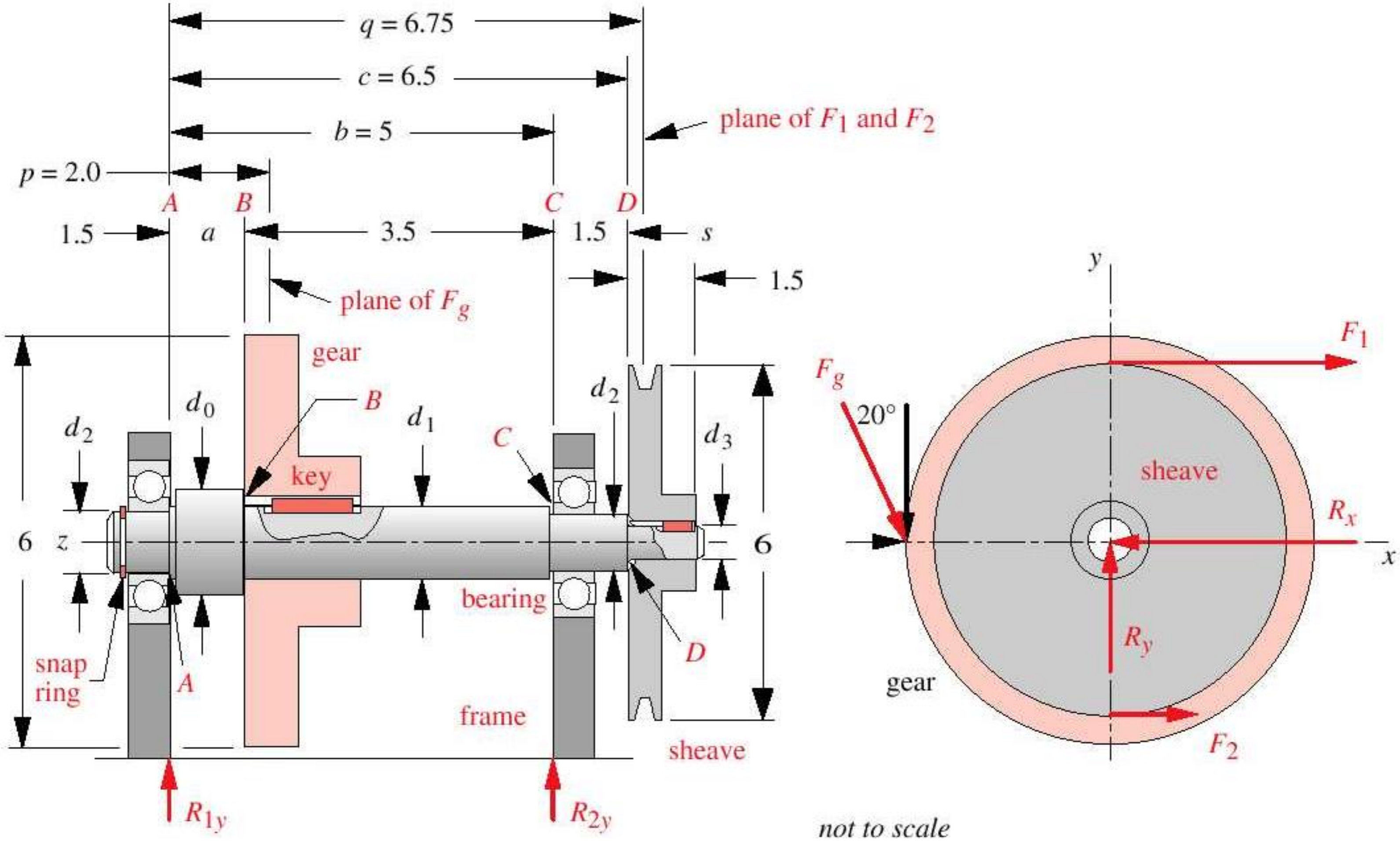


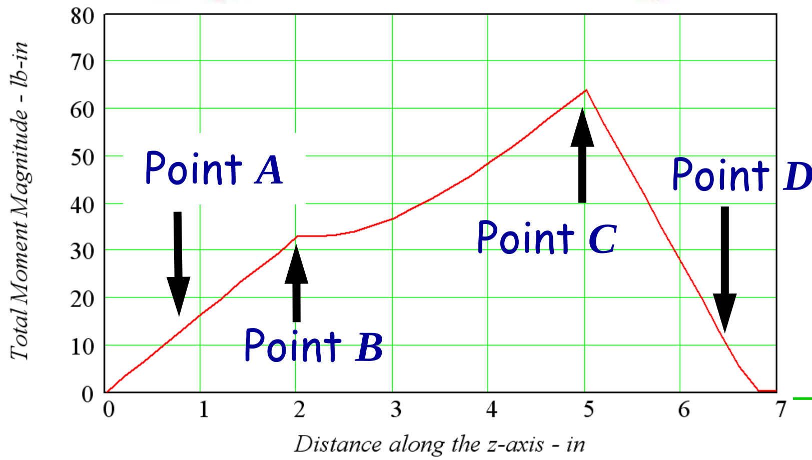
Shaft design: components. Examples 10-1 and 10-2

- **Total moment diagram:** note that the Amplitude & Mean components of the moments (and torque) require evaluation

$$M_T(z) = \left[M_{X-Z}^2 + M_{Y-Z}^2 \right]^{1/2}$$







Total moment:

$$M_T(z) = \left[M_{X-Z}^2 + M_{Y-Z}^2 \right]^{1/2}$$

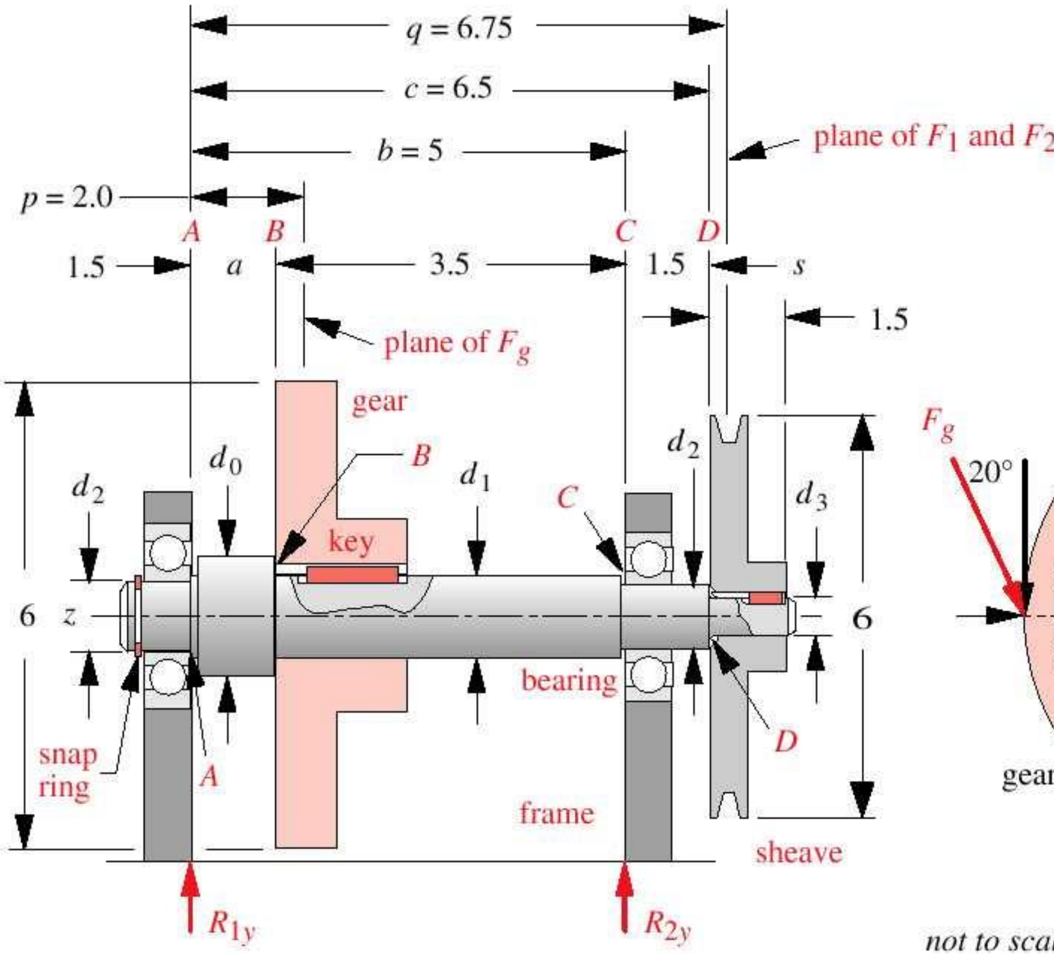
Shaft design

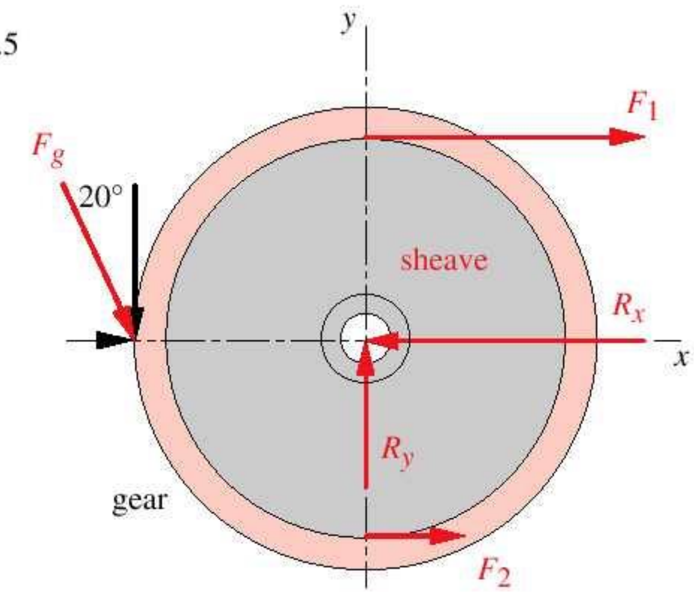
Fluctuating bending and torsion

- **Review and Master: Example 10-2**

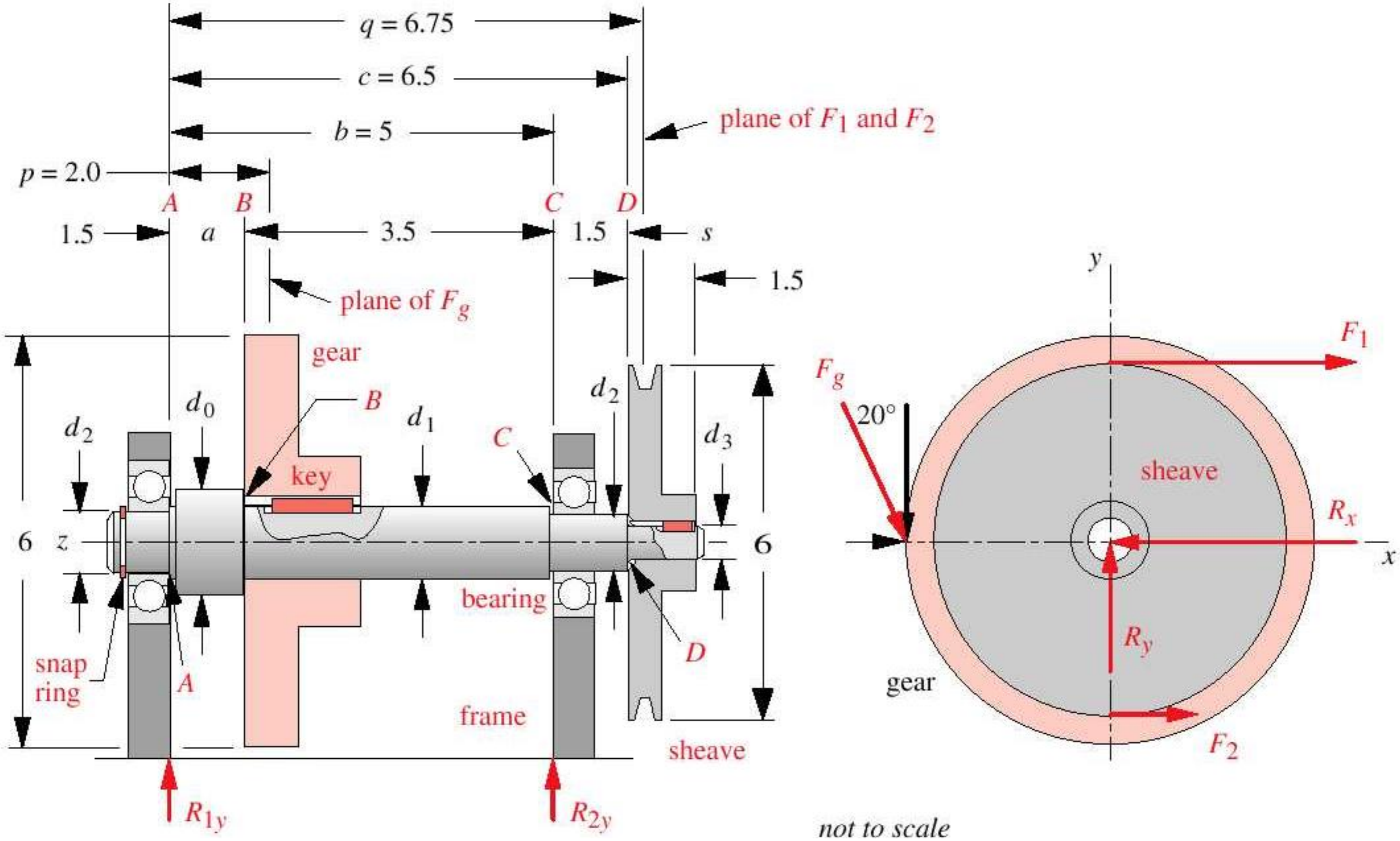
Design shaft to support attachments

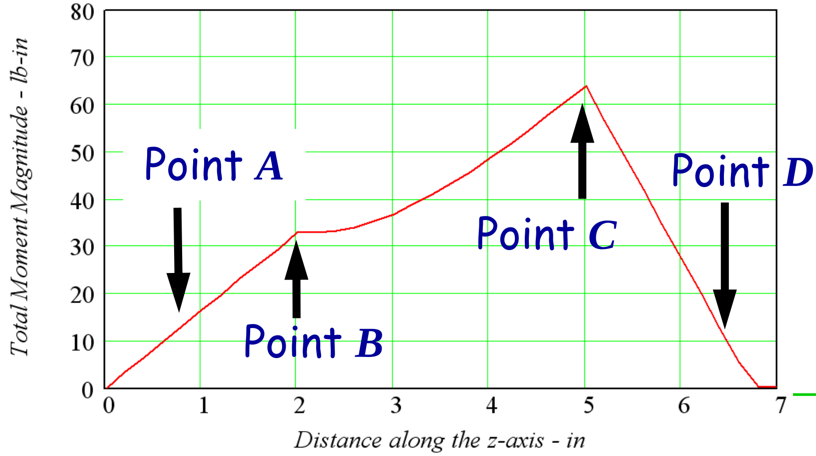
- Mean and alternating torque are both 74 lb-in
- Safety factor: 2.5
- Infinite life
- Material: SAE 1020 (good notch sensitivity)
- Operating conditions: room temperature
- Power: 2 HP at 1,750 rpm
- SCF of 3.5 for radii in bending, 2 in torsion, and 4 at the keyway





not to scale





Total moment:

$$M_T(z) = \left[M_{X-Z}^2 + M_{Y-Z}^2 \right]^{1/2}$$

Shaft design

ASME method: fully-reversed bending and constant torsion

- Based on failure envelope (shown before):

$$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\tau_m}{S_{ys}}\right)^2 = 1$$

- Safety factor: N_f

- von Mises stress in shear (strain-energy theory): $S_{ys} = \frac{S_y}{\sqrt{3}}$

- Amplitude stress in bending and mean torsional stresses: σ_a, τ_m (corrected for fatigue stress-concentration factors)

- Shaft diameter is calculated as:

$$d = \left\{ \frac{32 N_f}{\pi} \left[\left(K_f \frac{M_a}{S_f} \right)^2 + \frac{3}{4} \left(K_{fsm} \frac{T_m}{S_y} \right)^2 \right]^{1/2} \right\}^{1/3}$$

Shaft design

ASME method: __fluctuating bending and torsion__

- **Based on von Mises stresses (amplitude and mean):** σ'_a, σ'_m
- **Failure envelope given as:**

$$\frac{1}{N_f} = \frac{\sigma'_a}{S_f} + \frac{\sigma'_m}{S_{ut}}$$

- **von Mises stress in shear (strain-energy theory):** $S_{ys} = \frac{S_y}{\sqrt{3}}$
- **Amplitude and mean stress components in bending and shear: (corrected for fatigue stress-concentration factors)** $\sigma_a, \sigma_m, \tau_a, \tau_m$
- **Shaft diameter is calculated as:**

$$d = \left\{ \frac{32 N_f}{\pi} \left[\frac{\sqrt{(K_f M_a)^2 + \frac{3}{4}(K_{fs} T_a)^2}}{S_f} + \frac{\sqrt{(K_{fm} M_m)^2 + \frac{3}{4}(K_{fsm} T_m)^2}}{S_{ut}} \right] \right\}^{1/3}$$

Shaft design

Deflection: bending

- Simple shafts: refer to notes from previous lectures
- Stepped shafts: *calculations become more involved*

$$\frac{M}{EI} = \frac{\text{Moment function determined using singularity functions}}{EI}$$

It is required the integration of equations:

$$\theta_{slope} = \int \frac{M}{EI}\, dz + C_3$$

$$\delta_{deflection} = \int \theta_{slope}\, dz + zC_3 + C_4$$

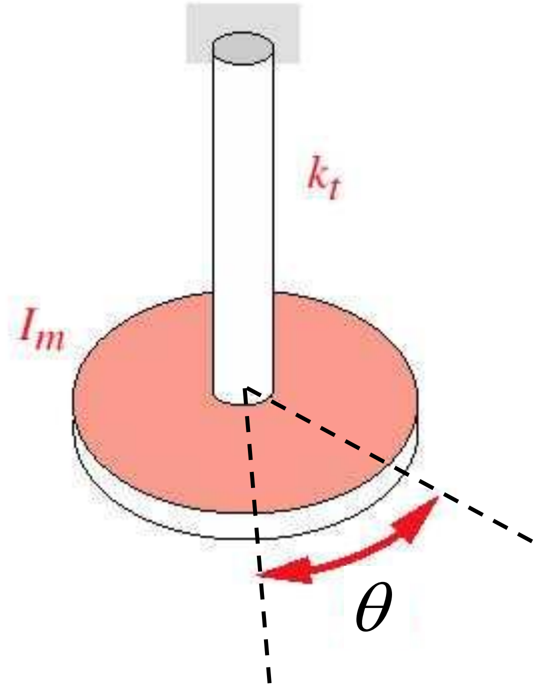
Numerical integration is preferred:

see MathCad Examples EX10-03 and EX10-09

Shaft design

Deflection: torsion

Single disk



Angular deflection (simple shaft):

$$\theta = \frac{T\,l}{GJ}$$

Torsional spring constant (simple shaft):

$$k_t = \frac{T}{\theta} = \frac{G\,J}{l}$$

Angular deflection (stepped shaft):

$$\theta = \theta_1 + \theta_2 + \theta_3 = \frac{T}{G}\left(\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right)$$

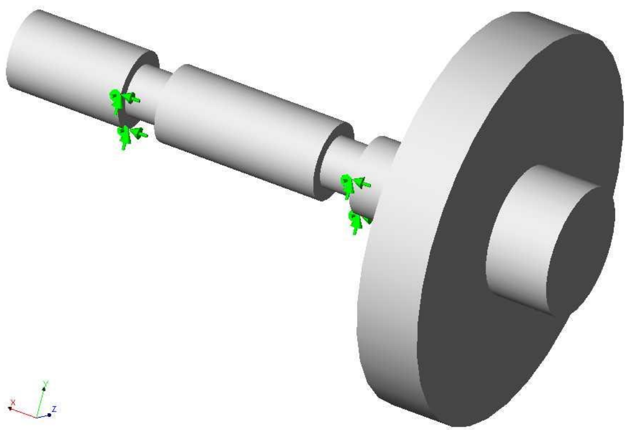
Effective torsional spring constant (stepped shaft):

$$\frac{1}{k_{t_{effective}}} = \frac{1}{k_{t_1}} + \frac{1}{k_{t_2}} + \frac{1}{k_{t_3}}$$

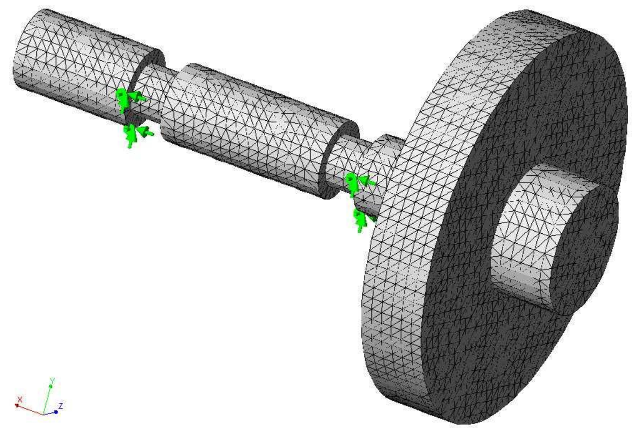
Shaft design

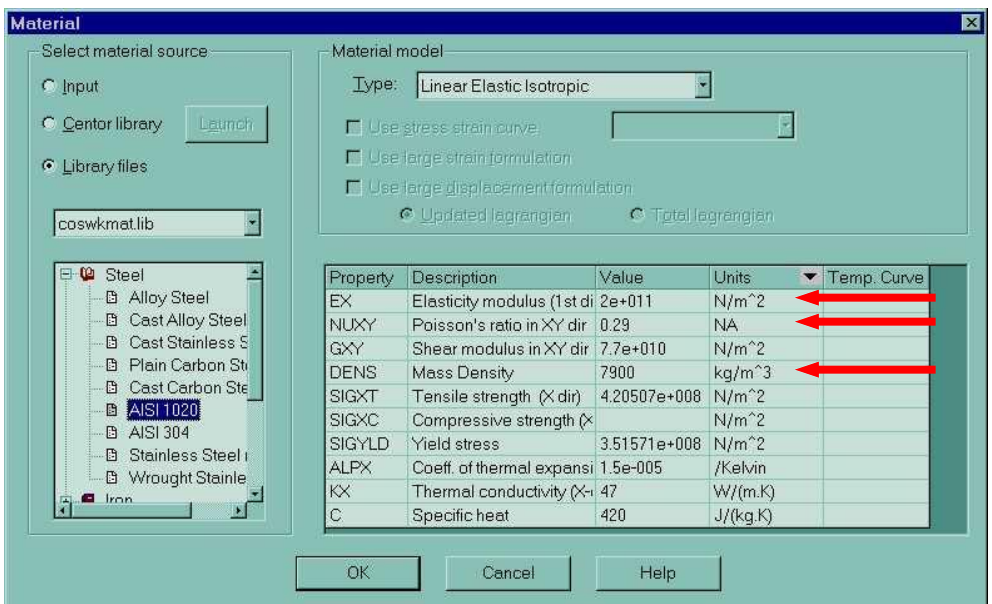
Natural frequencies of vibration: FEM model of gear-shaft

CAD model



Domain discretization





Material

Select material source

- Input
- Centor library (Launch)
- Library files

coswkmat.lib

Steel
- Alloy Steel
- Cast Alloy Steel
- Cast Stainless S
- Plain Carbon St
- Cast Carbon Ste
- AISI 1020
- AISI 304
- Stainless Steel
- Wrought Stainle

Iron

Material model

Type: Linear Elastic Isotropic

- Use stress strain curve
- Use large strain formulation
- Use large displacement formulation
- Updated lagrangian
- Total lagrangian

Property	Description	Value	Units	Temp. Curve
EX	Elasticity modulus (1st di	2e+011	N/m^2	
NUXY	Poisson's ratio in XY dir	0.29	NA	
GXY	Shear modulus in XY dir	7.7e+010	N/m^2	
DENS	Mass Density	7900	kg/m^3	
SIGXT	Tensile strength (X dir)	4.20507e+008	N/m^2	
SIGXC	Compressive strength (X		N/m^2	
SIGYLD	Yield stress	3.51571e+008	N/m^2	
ALPX	Coeff. of thermal expansi	1.5e-005	/Kelvin	
KX	Thermal conductivity (X-i	47	W/(m.K)	
C	Specific heat	420	J/(kg.K)	

OK Cancel Help

FEM model solves the discrete version of equation:

$$\sum F = m\,\ddot{x}$$

Also written as

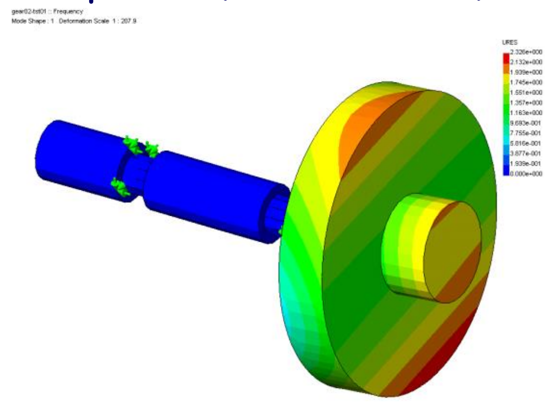
$$m\,\ddot{\boldsymbol{x}} + k\,\boldsymbol{x} = \boldsymbol{0}$$

Shaft design

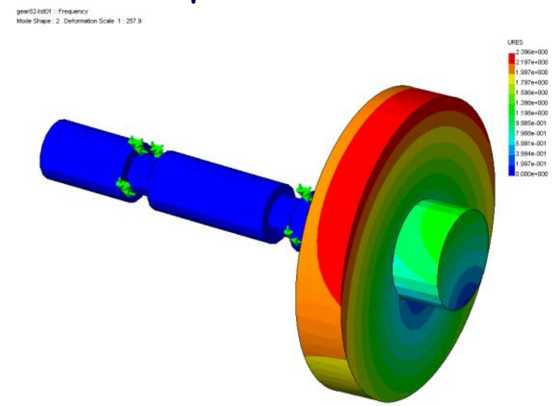
Natural frequencies of vibration: FEM model of gear-shaft

Representative results

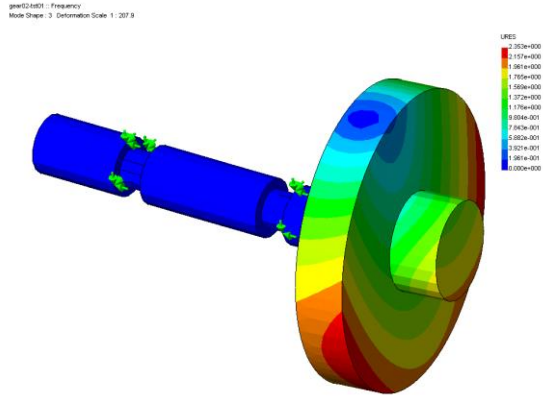
Mode shape #1 (fundamental) ~ 1040 Hz



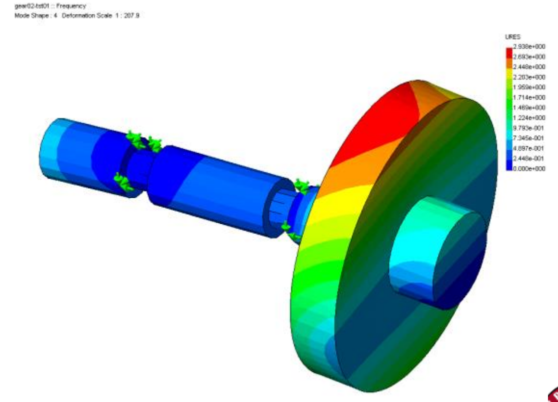
Mode shape #2 ~ 1240 Hz



Mode shape #3 ~ 1340 Hz

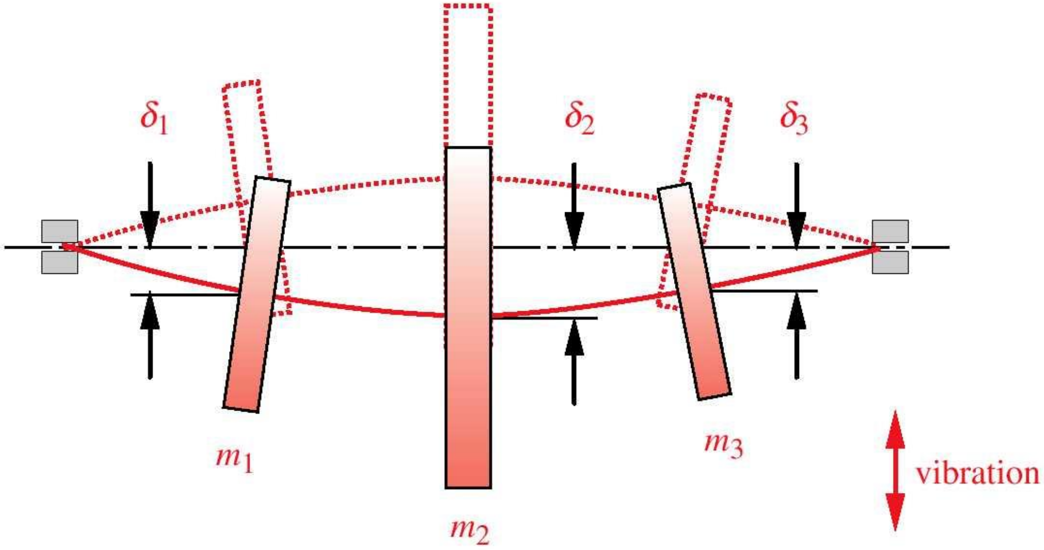


Mode shape #4 ~ 5320 Hz



Shaft design: transversal modes of vibration

Estimation of fundamental ***transversal*** *first natural frequency*

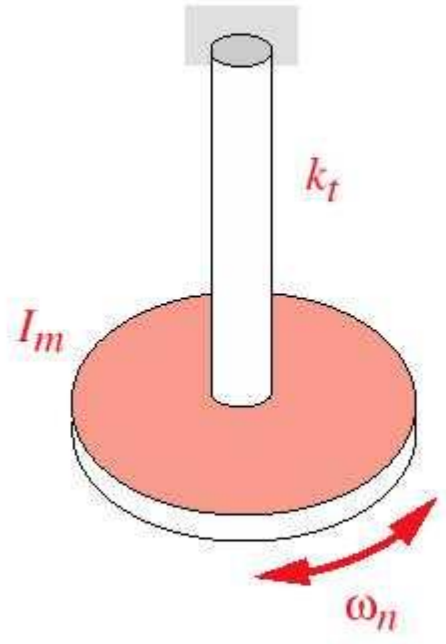


Using Rayleigh's method: $$\omega_n = \sqrt{g \frac{\sum_i m_i \, \delta_i}{\sum_i m_i \, \delta_i^2}} \; ; \; [rad/sec]$$

Shaft design: torsional modes of vibration

Estimation of fundamental torsional first natural frequency

Single disk



Simple shaft: $$\omega_n = \sqrt{\frac{k_t}{I_m}}$$

With: $k_t = \dfrac{GJ}{l}$, and $I_m = \dfrac{m\,r^2}{2}$

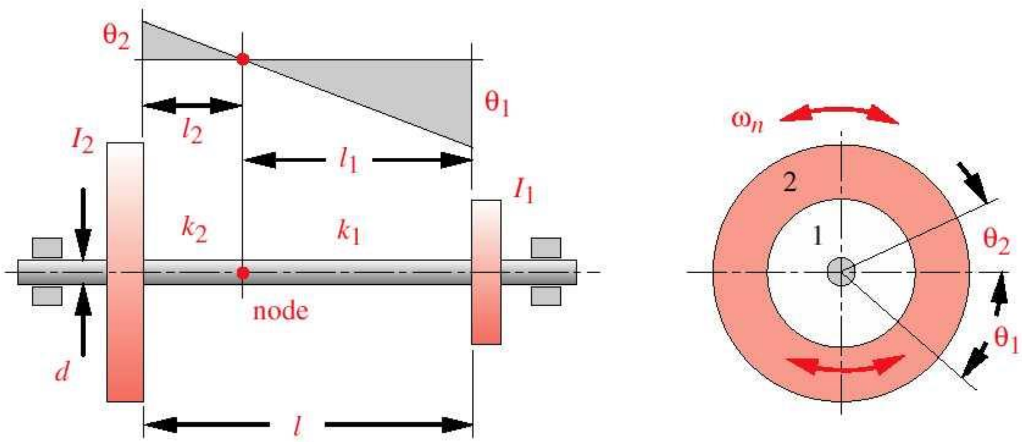
For stepped shafts:

$$J = J_{eff} = \frac{l}{\sum_i \frac{l_i}{J_i}}$$

Shaft design: torsional modes of vibration

*Estimation of fundamental **torsional** first natural frequency*

Two disks configuration



Natural frequency:

$$\omega_n = \sqrt{k_{t_{effective}} \frac{I_1 + I_2}{I_1 I_2}}$$

Note that:

1) I_1, and I_2 are mass moment of inertia

2) Effective torsional stiffness is

$$\frac{1}{k_{t_{effective}}} = \frac{1}{k_{t_1}} + \frac{1}{k_{t_2}}$$

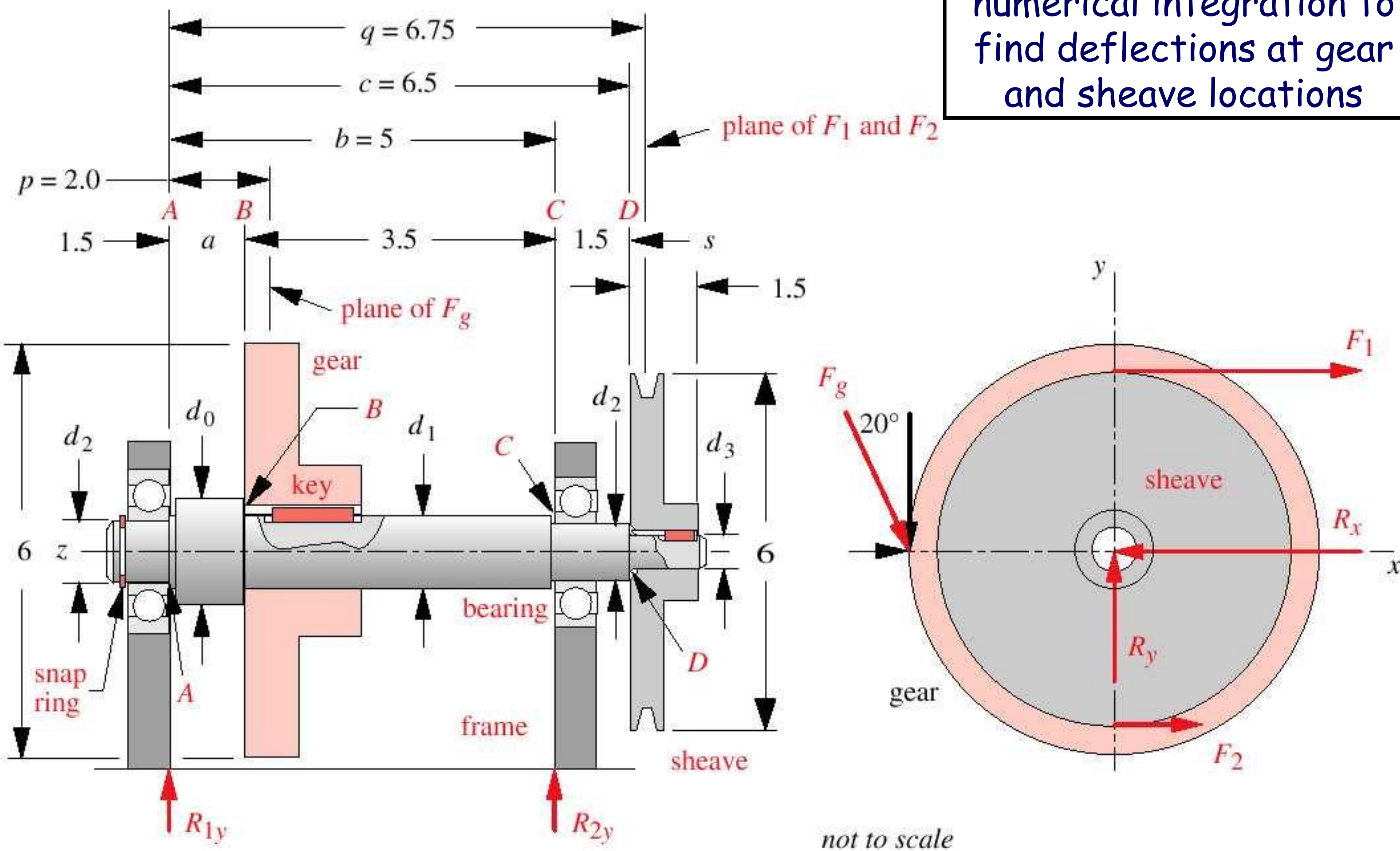
(when two ϕ steps define shaft geometry)

Shaft design: transversal & torsional modes of vibration

Example

- **Review and Master:** **Example 10-8**

Requires use of numerical integration to find deflections at gear and sheave locations



Reading

- Chapters 10 of textbook: Sections 10.9 to 10.16
- Review notes and text: ES2501, ES2502, ES2503

Homework assignment

- Author's: as indicated in website of our course
- Solve: as indicated in website of our course