Bearings and lubrication

Journal bearings (short/long) -- sliding type

Journal bearings (short/long) and bushings

Thick-wall journal bearing

Typically used in low-speed applications

$l/d$: $[0.25–2]$

Hard material

Soft material
Bearings and lubrication: friction

Journal bearings (short/long) -- sliding type

Friction as a function of velocity

Friction

Relative velocity

(a) Shaft stationary - metal contact
   - forces and centers in line

(b) Shaft rotating slowly
   - boundary lubrication
   - contact point leads centerline

(c) Shaft rotating rapidly
   - hydrodynamic lubrication
   - no metal contact
   - fluid is pumped by shaft
   - shaft lags bearing centerline
Bearings and lubrication: materials

**Journal bearings (short/long) -- sliding type**

*Note hardness difference between shaft and bearing material*

---

**Table 10-3  Recommended Bearing Materials for Sliding Against Steel or Cast Iron**

<table>
<thead>
<tr>
<th>Bearing Material</th>
<th>Hardness kg/mm²</th>
<th>Minimum Shaft Hardness kg/mm²</th>
<th>Hardness Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead-base babbitt</td>
<td>15-20</td>
<td>150</td>
<td>8</td>
</tr>
<tr>
<td>Tin-base babbitt</td>
<td>20-30</td>
<td>150</td>
<td>6</td>
</tr>
<tr>
<td>Alkali-hardened lead</td>
<td>22-26</td>
<td>200-250</td>
<td>9</td>
</tr>
<tr>
<td>Copper-lead</td>
<td>20-23</td>
<td>300</td>
<td>14</td>
</tr>
<tr>
<td>Silver (overplated)</td>
<td>25-50</td>
<td>300</td>
<td>8</td>
</tr>
<tr>
<td>Cadmium base</td>
<td>30-40</td>
<td>200-250</td>
<td>6</td>
</tr>
<tr>
<td>Aluminum alloy</td>
<td>45-50</td>
<td>300</td>
<td>6</td>
</tr>
<tr>
<td>Lead bronze</td>
<td>40-80</td>
<td>300</td>
<td>5</td>
</tr>
<tr>
<td>Tin bronze</td>
<td>60-80</td>
<td>300-400</td>
<td>5</td>
</tr>
</tbody>
</table>


Babbitt (Isaac Babbitt, 1839, Taunton, MA). Common compositions: 90% Tin, 10% Copper; 89% tin, 7% antimony, 4% copper
Bearsings and lubrication: shear stresses in lubricant

**Journal bearings (short/long) -- sliding type**

Shear stresses: \[ \tau_{xy} = \eta \frac{du}{dy} \]

and for constant thickness, \( h \), \[ \tau_{xy} = \eta \frac{U}{h} \]

where, \( \eta \) - is absolute viscosity
\( U \) - is linear (tangential) velocity
\( h \) - is film (lubricant) thickness

**Linear (tangential) velocity:**

Parallel plates shearing an oil film
(Analogy)

Differential element in shear

Film (lubricant) thickness
Bearings and lubrication: viscosity

- **Absolute viscosity:** $\eta$
- **Units of absolute viscosity in English system:** lb-sec/in$^2$ (reyn)
- **Units of absolute viscosity in SI system:** Pa-sec
- **Typical values are expressed in $\mu$reyn and mPa-sec** (1 centipoise (cP) = 1mPa-sec)
- **Kinematic viscosity ($\nu$) and absolute viscosity are related as:** $\eta = \rho \nu$
- **Units of kinematic viscosity are:** in$^2$/sec (English) and cm$^2$/sec (stoke - SI)

The term "viscosity" without modifiers refers to absolute viscosity.
Bearings and lubrication: shear stresses in lubricant

Reynold’s equation for eccentric journal bearings. Definitions:

Diametral clearance: $C_d$

Radial clearance: $C_r = C_d / 2$

Eccentricity: $e$ (Maximum value of eccentricity is: $C_r$)

Eccentricity ratio: $\varepsilon = \frac{e}{C_r}$

Film thickness as a function of angular position ($\theta$): $h \approx C_r (1 + \varepsilon \cos \theta)$

Minimum and maximum film thickness: $h_{\text{min}} = C_r (1 - \varepsilon), \ h_{\text{max}} = C_r (1 + \varepsilon)$

(Analogy: used to derive mathematical models)

(a) Nonparallel plates shearing an oil film

(b) An eccentric journal is equivalent to nonparallel plates
Bearings and lubrication: shear stresses in lubricant

Reynold’s equation for eccentric journal bearings

Reynold’s equation:
\[ \frac{1}{6\eta} \left[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) \right] = U \frac{\partial h}{\partial x} \]

Close-form solutions for Reynold’s equation do not exist, however, it can be solved numerically (e.g., with finite element methods).

Particular close-form solutions for simplified Reynold’s equation do exist

Neglecting pressure gradient in the axial direction (assumption):
\[ \frac{\partial p}{\partial z} = 0 \]

Involves solution of simplified Reynold’s equation:
\[ \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = 6\eta U \frac{\partial h}{\partial x} \]
Bearings and lubrication: Reynold’s equation

Long bearing assumption: Sommerfeld solution

No-leakage of lubricant at both ends of journal

Pressure distribution, \( p \), is:

\[
p = \frac{\eta U r}{C_r^2} \left[ \frac{6\varepsilon (\sin \theta)(2 + \varepsilon \cos \theta)}{(2 + \varepsilon^2)(1 + \varepsilon \cos \theta)^2} \right] + p_o
\]

Total load, \( P \), is:

\[
P = \frac{\eta U l r^2}{C_r^2} \frac{12\pi \varepsilon}{(2 + \varepsilon^2)(1 + \varepsilon \cos \theta)^{1/2}}
\]

Average pressure, \( P_{avg} \), is:

\[
P_{avg} = \frac{P}{ld}
\]

Important dimensionless quantity: Sommerfeld number, \( S \)

\[
S = \frac{(2 + \varepsilon^2)(1 + \varepsilon^2)^{1/2}}{12\pi \varepsilon} = \eta \left( \frac{\pi n'}{P_{avg}} \right) \left( \frac{d}{C_d} \right)^2
\]

(\( n' \) is rotational speed in revolutions per second)
Bearings and lubrication: Reynold’s equation

Short bearing assumption: Ocvirk solution (l/d: [0.25-2])

Incorporates leakage of lubricant at both ends of journal

Pressure distribution, $p$, is:
$$p = \frac{\eta U l^2}{r C_r^2 \left(\frac{l^2}{4} - z^2\right)} \frac{3\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3}$$

Total load, $P$, is:
$$P = K_\varepsilon \frac{\eta Ul^3}{C_r^2} = K_\varepsilon \frac{4\pi \eta d n' l^3}{C_d^2}$$

Coefficient, $K_\varepsilon$ (dimensionless), is:
$$K_\varepsilon = \frac{\varepsilon \left[ \pi^2 (1 - \varepsilon^2) + 16\varepsilon^2 \right]^{1/2}}{4(1 - \varepsilon^2)^2}$$

Important dimensionless quantity: Ocvirk number, $O_N$

$$O_N = 4\pi K_\varepsilon = \left(\frac{p_{avg}}{\eta n'}\right) \left(\frac{d}{l}\right)^2 \left(\frac{C_d}{d}\right)^2 = \frac{\pi \varepsilon \left[ \pi^2 (1 - \varepsilon^2) + 16\varepsilon^2 \right]^{1/2}}{(1 - \varepsilon^2)^2}$$
Bearings and lubrication: Reynold’s equation

**Short bearing assumption: Ocvirk solution**

Incorporates leakage of lubricant at both ends of journal

Correcting eccentricity ratio (correction based on experimental observations):

$$\varepsilon_{x(corrected)} = 0.21394 + 0.38517 \log(O_N) - 0.0008(O_N - 60)$$

**FIGURE 10-10**

Analytical and Experimental Relationship Between Eccentricity Ratio $\varepsilon$ and Ocvirk Number $O_N$

Bearings and lubrication

Pressure distribution in journal bearings

Power loss: \[ \Phi = 2\pi T_r (n'_2 - n'_1) \]

with \[ T_r = T_s + Pe \sin \phi \]

\(T_s = \eta \frac{d^3 l (n'_2 - n'_1)}{C_d} \frac{\pi^2}{(1 - \varepsilon^2)^{1/2}}\)

\[ \theta_{\text{max}} = \cos^{-1} \left( \frac{1 - \sqrt{1 + 24\varepsilon^2}}{4\varepsilon} \right) \]

\[ \phi = \tan^{-1} \left( \frac{\pi \sqrt{1 - \varepsilon^2}}{4\varepsilon} \right) \]
Bearsings and lubrication

Pressure distribution in short journal bearings

%P-θ distributions (approximations) in short and long bearings:

Ocvirk solution for \( l/d = 1.0 \)
(Short-bearing solution)

Sommerfeld solution
(Long-bearing solution)

\( l/d = 0.75 \)
\( l/d = 0.50 \)
\( l/d = 0.25 \)
Bearings and lubrication: design considerations

**Journal bearings (short/long) -- sliding type**

**If shaft has been designed**
(for stress and/or deflection)

- Chose ratio l/d of bearing
  (based on packaging considerations)

- Larger l/d ratios give lower film pressures

- Clearance ratio \( \left( \frac{C_d}{d} \right) \) are typically in the range of 0.001 to 0.002

- Larger clearance ratios decrease load number \( O_N \)

- Higher \( O_N \) numbers give larger eccentricity, pressure, and torque

- Larger clearance ratios used for better lubricant flow (increasing cooling conditions)

- An Ocvik number is chosen and lubricant is then selected

**If shaft has not been designed**

- Diameter and length of bearing can be found from iteration of bearing equations with an assumed Ocvik number

- A trial lubricant must be chosen and its viscosity defined

- After bearing has been designed, heat and momentum (flow) analysis can be done... not in this course...

**Note:**

\[ O_N \approx 30 \ (\varepsilon=0.82) \text{ moderate loading (recommended)} \]

\[ O_N \approx 60 \ (\varepsilon=0.90) \text{ heavy loading} \]

\[ O_N \approx 90 \ (\varepsilon=0.93) \text{ severe loading} \]
Examples

Review and Master: Example 11-1. Sleeve bearing design for a defined shaft diameter
Reading

- Chapters 10 of textbook: Sections 11.8 to 11.10
- Review notes and text: ES2501, ES2502, ES2503

Homework assignment

- Author's: 11-1
- Solve: 11-1e, 11-3