WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS ME-3320, B'2023

Lecture 15 November 2023





Fatigue failure Notches and stress concentrations

□ Master Examples 6-1 and 6-2: estimating S-N diagrams

□ Master Example 6-3: determining fatigue stress-concentration factors





Notches and stress concentrations

□ Notches introduce stress-concentrations. See <u>lectures 07-08, 13, and 14</u>



- \Box Correcting for stress-concentrations. Stress concentration factors in fatigue: K_f, K_{fs}
- $\hfill\square$ Use of stress concentration factors in fatigue:

$$\sigma = K_f \ \sigma_{\text{nominal}}$$

$$au = K_{fs} \ au_{nominal}$$



Notches and stress concentrations

 $\hfill\square$ Stress concentration factors in fatigue:

 $K_f = 1 + q(K_t - 1)$

 \Box Theoretical (static) stress-concentration factor: K_t



Neuber's constant (depends on the value of the ultimate tensile strength of the material used).
 See, for example, Tables 6-6, 6-7, and 6-8



Residual stresses: must be taken into account

- Residual stress are built-in or introduced (typically during manufacturing) to an unloaded part.
- Residual stresses can be the cause of crack initiation and, therefore, fatigue failure

Example: rotary dryer. Welding lifters to a rotary shell



Source: ASM International

Residual stresses introduced during welding caused crack initiation Lifter Weld Shell Crack





Fatigue failure Designing for HCF

□ Fatigue design situations







Review Example 6-4: under fully-reversed bending: parametric approach

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from a machine frame. Examples of cantilevered bracket configurations are shown in the figures. Task is to design a cantilever bracket to support a fully reversed bending load.



Review Example 6-4: under fully-reversed bending: parametric approach

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 10⁹ cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.







Review Example 6-4: under fully-reversed bending: iterative approach



□ **Comment**: use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)





Fatigue failure Designing for HCF

□ Review Example 6-4: under fully-reversed bending



Geometric Stress-Concentration Factor K_t for a Filleted Flat Bar in Bending

Fatigue failure Designing for HCF

□ Review Example 6-4: under fully-reversed bending

Table 6-6 Neuber's Constant for Steels			Table 6-7Neuber's Constantfor Annealed Aluminum			Table 6-8Neuber's Constantfor Hardened Aluminum			
S _{ut} (ksi)	√ a (in ^{0.5})		S _{ut} (kpsi)	a (in ^{0.5})		S _{ut} (kpsi)	√ a (in ^{0.5})		
50	0.130		10	0.500		15	0.475		
55	0.118		15	0.341		20	0.380		
60	0.108		20	0.264		30	0.278		
70	0.093		25	0.217		40	0.219		
80	0.080		30	0.180		50	0.186		
90	0.070		35	0.152		60	0.162		
100	0.062		40	0.126		70	0.144		
110	0.055		45	0.111		80	0.131		
120	0.049				L.	90	0.122		
130	0.044					-			
140	0.039		Max	need t	o do a	unvo fit	ting in or	dan ta	
160	0.031		Muy	determine Neuber's constant functions:					
180	0.024		аете						
200	0.018			$v = $ Neuber's constant = \sqrt{a}					
220	0.013		v	y = f(x)					
240	0.009			U V	Х	$\mathbf{S} = \mathbf{S}_{ut}$			

Designing for HCF: fluctuating uniaxial stresses



Fatigue failure Modified Goodman-diagram





Fatigue failure Augmented modified Goodman-diagram





Stress-concentration factors in fluctuating stresses

Note that component may "yield" locally







Stress-concentration factors in fluctuating stresses



Fatigue failure - Modified Goodman's diagram Safety factors in fluctuating stresses: Cases 1 and 2





Fatigue failure - Modified Goodman's diagram Safety factors in fluctuating stresses: Cases 3 and 4



Review Example 6-4: under fully-reversed bending: parametric approach

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 10⁹ cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.







Example 6-5: fatigue under fluctuating bending. Design bracket to support the load. <u>Verify for maximum deflections</u>



Design of a Cantilever Bracket for Fluctuating-Bending Loading



Fatigue failure Designing for HCF

□ Review Example 6-4: under fully-reversed bending



Geometric Stress-Concentration Factor K_t for a Filleted Flat Bar in Bending

□ For next lecture: Master Examples 6-5 and and 6-6; use and understand corresponding MathCad solutions (in the CD that came with your book and/or on Norton's Machine Design website)





Example 6-6: multiaxial fluctuating stresses. Verify the design against failure (e.g., by determining safety factors, other?)



Notch radius (wall) is 0.25", $K_t=1.70, K_{ts}=1.35$

- Applied load: sinusoidal [-200,340] lb
- □ <u>Finite life</u> of about 6x10⁷ cycles
- 🛛 Material: Al 2024-T4
- Operating conditions: room temp.

Initial dimensions:

$$ID = 1.5 in$$
$$OD = 2 in$$
$$a = 8.0 in$$
$$l = 6.0 in$$

□ **Comment**: use MathCad example to perform design iterations (file is included in the CD-ROM that came with your textbook)





Example 6-6: multiaxial fluctuating stresses.

Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.







Example 6-6: multiaxial fluctuating stresses.

Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.









Example 6-6: Multiaxial Fluctuating Stresses

Problem	Determine the safety fa	actors for the bracket tu	be shown in Figure 5-7.			
Units	ksi := $10^3 \cdot psi$					
Given	The material is 2024-T4 aluminum					
	Yield strength	$S_y := 47 ksi$				
	Tensile strength	$S_{ut} := 68 \ ksi$				
Assumptions	Tube length Arm length Tube OD Tube ID Load The load is dynamic and Consider shear due to t	$l := 6 \cdot in$ $a := 8 \cdot in$ $od := 2.0 \cdot in$ $id := 1.5 \cdot in$ $F_{min} := -200 \cdot lbf$ d the assembly is at room ransverse loading as we	F _{max} := 340∙lbf n temperature. Il as other stresses.			
A finite life design will be sought with a life of $N := 6 \cdot 10^7$ cycles. The notch radius at the wall is $r := 0.25 \cdot in$ and stress-concentration factors are for bending $K_t := 1.7$, and for shear, $K_{ts} := 1.35$.						
Solution	See Figure 5-7 and Math a more complete explana	cad file EX06-06. Also to the stress analys	see Example 4-9 for is for this problem.			
1 Aluminum do be estimated S' _{f@5E8} is	bes not have an endurance lir I from equation 6.5c. Since t	nit. Its endurance strength the S _{ut} is larger than 48 ksi	at 5E8 cycles can i, the uncorrected			



 $S'_{f5E8} := 19 ksi$

$$\frac{2 \text{ The correction factors are calculated from equations 6.7 and used to find a corrected indicator strength at the standard 5ES cycle.
$$C_{load} := 1.0 \qquad \text{for bending} \\
Ag_{5} = 0.0105 \text{ od}^{2} \qquad Ag_{5} = 0.042in^{2} \\
d_{eg} := \sqrt{\frac{Ag_{5}}{0.0766}} \qquad d_{eg} = 0.740in \\
C_{stze} := 0.869 \left(\frac{d_{eg}}{in}\right)^{-0.097} C_{stze} = 0.895 \\
Table 6-3 \text{ constants} \qquad A := 2.7 \qquad b := -0.265 \qquad \text{Note "negative"} \\
exponent \\
S_{ut} \text{ is used in } kpsi \qquad C_{surf} := A \left(\frac{S_{ut}}{ks}\right)^{b} \qquad C_{surf} = 0.883 \qquad (a) \\
C_{temp} := 1 \\
C_{reluab} := 0.753 \qquad \text{for } 99.9\% \\
Note that this is only \\
16.6\% \text{ of the } S_{ut} (\text{ and } 24\% \text{ of the } S_{y}) \\
\text{Note that the bending value of C_{load} is used despite the fact that there is both bending and torsion present. The torsional shear stress will be converted to an equivalent tensile stress will be converted to an equivalent tensil$$$$

= 5E8.



3 This problem calls for a life of 6E7 cycles, so a strength value at that life must be estimated from the S-N line of Figure 6-33b using the corrected fatigue strength at that life. Equation 6.10a for this line can be solved for the desired strength after we compute the values of its coefficients a and b from equation 6.10c.

 $S_m := 0.90 \cdot S_{ut}$ $S_m = 61.2 \, ksi$ From Table 6-5 for 5E8 z := 5.699 $b := -\frac{1}{z} \cdot log \left(\frac{S_m}{S_{f5E8}} \right)$ b = -0.1288 (c) $a := \frac{S_m}{10^{3 \cdot b}}$ $a = 148.9 \, ksi$ $S_n := a \cdot N^b$ $S_n = 14.84 \, ksi$

Note that S_m is calculated as 90% of S_{ut} because loading is bending rather than axial (see Eq. 6.9). The value of z is taken from Table 6-5 for N = 5E8 cycles. This is a corrected fatigue strength for the shorter life required in this case and so is larger than the corrected test value, which was calculated at a longer life.

4 The notch sensitivity of the material must be found to calculate the fatigue stress-concentration factors. Table 6-8 shows the Neuber factors for hardened aluminum. Interpolation gives a value of a := 0.147² in at the material's S_{ut}. Equation 6.13 gives the resulting notch sensitivity for the assumed notch radius.

$$q := \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$
 $q = 0.773$ (d)

5 The fatigue stress-concentration factors are found from equation 6.11b using the given geometric stress-concentration factors for bending and torsion, respectively.

$$K_f := 1 + q \cdot (K_t - 1)$$
 $K_f = 1.541$ (e)

$$K_{fs} := 1 + q \cdot (K_{ts} - 1)$$
 $K_{fs} = 1.270$ (f)

K_t and K_{ts} are given (we are lucky!)

This is calculated

materials with an S_{ρ}

differently for



6 The bracket tube is loaded in both bending (as a cantilever beam) and in torsion. The shapes of the shear, moment and torque distributions are shown in Figure 4-30. All are maximum at the wall. The alternating and mean components of the applied force, moment, and torque at the walls are

<u>Forces</u>: evaluated for amplitude and mean components

<u>Moments</u>: evaluated for amplitude, mean, and maximum components Loads $F_{\alpha} := \frac{F_{m\alpha x} - F_{min}}{2}$ $F_{\alpha} = 270 \, lbf$ (g) $F_m := \frac{F_{max} + F_{min}}{2} \qquad \qquad F_m = 70\,lbf$ $M_{\alpha} := F_{\alpha} \cdot l$ $M_{\alpha} = 1620 \, lbf \cdot in$ Moments (h) $M_m := F_m \cdot l$ $M_m = 420 \, lb f \cdot in$ $M_{max} := M_a + M_m$ $M_{max} = 2040 \, lbf \cdot in$ Torques $a := 8.0 \cdot in$ Evaluated for
 amplitude and mean
 components $T_{a} = 2160 \, lbf \, in$ $T_{\alpha} := F_{\alpha} \cdot \alpha$ $T_m = 560 \, lbf \cdot in$ $T_m := F_m \cdot a$

7 The fatigue stress-concentration factor for the mean stresses depends on the relationship between the maximum local stress in the notch and the yield strength as defined in equation 6.17, a portion of which is shown here.

Outer fiber $c := 0.5 \cdot od$ c = 1.000 inMoment of $I := \frac{\pi}{64} \cdot \left(od^4 - id^4\right)$ $I = 0.5369 in^4$ $J := 2 \cdot I$ $J = 1.0738 in^4$ If $K_f \cdot \left|\sigma_{max}\right| < S_y$ then $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$ $K_f \cdot \left|\frac{M_{max} \cdot c}{I}\right| = 5.86 ksi$

which is less than $S_y = 47 \text{ ksi so}$, $K_{fm} := K_f$ and $K_{fsm} := K_{fs}$

Compensate for local "yield," if any



In this case, there is no reduction in stress-concentration factors for the mean stress because there is no yielding at the notch to relieve the stress concentration.

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8 The largest tensile bending stress will be in the top or bottom outer fiber at points A or A'. The largest torsional shear stress will be all around the outer circumference of the tube. (See Example 4-9 for more details.) First take a differential element at point A or A' where both of these sresses combine. (See Figure 4-32.) Find the alternating and mean components of the normal bending stress and of the torsional shear stress on point A using equations 4.11b and 4.24b, respectively.

$$\begin{cases} \sigma_{\alpha} := K_{f} \cdot \frac{M_{\alpha} \cdot c}{I} & \sigma_{\alpha} = 4.65 \, ksi \\ \tau_{\alpha} := K_{fs} \cdot \frac{T_{\alpha} \cdot c}{J} & \tau_{\alpha} = 2.56 \, ksi \\ \sigma_{m} := K_{fm} \cdot \frac{M_{m} \cdot c}{I} & \sigma_{m} = 1.21 \, ksi \\ \tau_{m} := K_{fsm} \cdot \frac{T_{m} \cdot c}{J} & \tau_{m} = 0.66 \, ksi \end{cases}$$

$$(k)$$

Evaluate applied, amplitude and mean, stresses - point A is subjected to bending and shear

9 Find the alternating and mean von Mises effective stresses at point A from equation 6.22b.

$$\sigma_{\chi\alpha} := \sigma_{\alpha} \qquad \sigma_{\gamma\alpha} := 0 \cdot psi \qquad \tau_{\chi\gamma\alpha} := \tau_{\alpha}$$

$$\sigma'_{\alpha} := \sqrt{\sigma_{\chi\alpha}^{2} + \sigma_{\gamma\alpha}^{2} - \sigma_{\chi\alpha} \cdot \sigma_{\gamma\alpha} + 3 \cdot \tau_{\chi\gamma\alpha}^{2}} \qquad \sigma'_{\alpha} = 6.42 \, ksi$$

$$\sigma_{\chi m} := \sigma_{m} \qquad \sigma_{\gamma m} := 0 \cdot psi \qquad \tau_{\chi\gamma m} := \tau_{m} \qquad (m)$$

$$\sigma'_m := \sqrt{\sigma_{\chi m}^2 + \sigma_{\chi m}^2 - \sigma_{\chi m} \cdot \sigma_{\chi m} + 3 \cdot \tau_{\chi \gamma m}^2} \qquad \sigma'_m = 1.66 \, ksi$$

10 Because the moment and torque are both caused by the same applied force, they are synchronous and in-phase and any change in them will be in a constant ratio.

Evaluate for all cases, if unsure about which case

Evaluate equivalent

and mean, stresses

Mises, amplitude

$$N_{f} := \frac{S_{n} \cdot S_{ut}}{\sigma'_{\alpha} \cdot S_{ut} + \sigma'_{m} \cdot S_{n}}$$

 $N_f = 2.2$ At point A



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11 Since the tube is a short beam, we need to check the shear due to transverse loading at point *B* on the neutral axis where the torsional shear is also maximal. The maximum transverse shear stress at the neutral axis of a hollow, thin-walled, round tube was given as equation 4.15*d*.

Cross-section area
$$A := \frac{\pi}{4} \cdot \left(od^2 - id^2 \right) \qquad A = 1.374 in^2$$

$$\tau_{abend} := K_{fs} \cdot \frac{2 \cdot F_a}{A} \qquad \tau_{abend} = 499 \, psi$$

$$\tau_{mbend} := K_{fsm} \cdot \frac{2 \cdot F_m}{A} \qquad \tau_{mbend} = 129 \, psi$$
(0)

B is subjected to pure shear bis in pure shear. The total shear stress at point B is the sum of the transverse shear stress and the torsional shear stress which act on the same planes of the element.

 $\tau_{atotal} \coloneqq \tau_{abend} + \tau_a$ $\tau_{atotal} = 3055 \, psi$ (p) $\tau_{mtotal} \coloneqq \tau_{mbend} + \tau_m$ $\tau_{mtotal} = 792 \, psi$

12 Find the alternating and mean von Mises effective stresses at point B from equation 6.22b.

$$\sigma_{xa} := 0 \cdot psi \qquad \sigma_{ya} := 0 \cdot psi \qquad \tau_{xya} := \tau_{atotal}$$

$$\sigma_{a}' := \sqrt{\sigma_{xa}^{2} + \sigma_{ya}^{2} - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xya}^{2}} \qquad \sigma_{a}' = 5.29 \, ksi$$

$$\sigma_{xm} := 0 \cdot psi \qquad \sigma_{ym} := 0 \cdot psi \qquad \tau_{xym} := \tau_{mtotal} \qquad (q)$$

$$\sigma_{m}' := \sqrt{\sigma_{xm}^{2} + \sigma_{ym}^{2} - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^{2}} \qquad \sigma_{m}' = 1.37 \, ksi \qquad Note t$$

$$S \quad in the second se$$

 $N_{f} = 2.7$

At point B

13 The safety factor for point B is found using equation 6.18e.

$$N_f \coloneqq \frac{S_n \cdot S_{ut}}{\sigma'_{\alpha} \cdot S_{ut} + \sigma'_m \cdot S_n}$$

Both points A and B are safe against fatigue failure.

Note the use of S_n in this equation (finite life)

Evaluate equivalent Mises, amplitude and mean, stresses

Evaluate for all

cases, if unsure

about which case

1/|V

Evaluate equivalent

 $\begin{array}{c}
 a \quad \forall \quad na \\
 b \quad na \quad \forall \quad na \\
 c \quad \forall \quad na \quad \forall$

Account for transversal shear point B is subjected T_{mbend}

Reading

- Chapters 6 of textbook: Sections 6.5 to 6.8
- Review notes and text: ES2501, ES2502

Homework assignment

- Author's: as indicated in Website of our course
- Solve: as indicated in Website of our course



