# **WORCESTER POLYTECHNIC INSTITUTE MECHANICAL ENGINEERING DEPARTMENT**

#### **DESIGN OF MACHINE ELEMENTS ME-3320, B'2023**

**Lecture 15 November 2023**





#### *Notches and stress concentrations*

**Master Examples 6-1 and 6-2:** *estimating S-N diagrams*

**Master Example 6-3:** *determining fatigue stress-concentration factors*





#### *Notches and stress concentrations*

**Notches introduce stress-concentrations. See lectures 07-08, 13, and 14**



- **Correcting for stress-concentrations.**   $\Box$ **Stress concentration factors in fatigue:**  $K_f$  ,  $K_{f s}$
- **Use of stress concentration factors in fatigue:**

$$
\sigma = K_f \; \sigma_{\text{nominal}}
$$

$$
\tau = K_{fs} \tau_{\text{nominal}}
$$



*Notches and stress concentrations*

**Stress concentration factors in fatigue:**

 $K_f = 1 + q(K_f - 1)$ 

**Theoretical (static) stress-concentration factor:** *K<sup>t</sup>*  $\Box$ 

**Notch sensitivity factor:** *r a q* + = 1 1 *a = Neuber's constant*

**Neuber's constant (depends on the value of the ultimate tensile strength of the material used). See, for example, Tables 6-6, 6-7, and 6-8**





#### *Residual stresses: must be taken into account*

- **Residual stress are built-in or introduced (typically during manufacturing) to an unloaded part.**
- $\mathbb{R}^n$ **Residual stresses can be the cause of crack initiation and, therefore, fatigue failure**

*Example: rotary dryer. Welding lifters to a rotary shell*



*Source: ASM International*

*Residual stresses introduced during welding caused crack initiation*Lifter Weld Shell Crack





#### **Fatigue failure** *Designing for HCF*

#### **Fatigue design situations**  $\Box$







#### **Review Example 6-4: under fully-reversed bending:** *parametric approach*  $\Box$

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from a machine frame. Examples of cantilevered bracket configurations are shown in the figures. Task is to design a cantilever bracket to support a fully reversed bending load.



#### **Review Example 6-4: under fully-reversed bending:** *parametric approach* Ш

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 10<sup>9</sup> cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.







**Review Example 6-4: under fully-reversed bending:** *iterative approach* Ш



**Comment:** *use MathCad example to perform design iterations*  Ш *(file is included in the CD-ROM that came with your textbook)*





#### **Fatigue failure** *Designing for HCF*

#### **Review Example 6-4: under fully-reversed bending**



Geometric Stress-Concentration Factor  $K_t$  for a Filleted Flat Bar in Bending

#### **Fatigue failure** *Designing for HCF*

#### **Review Example 6-4: under fully-reversed bending**



#### *Designing for HCF: fluctuating uniaxial stresses*



## **Fatigue failure** *Modified Goodman-diagram*

![](_page_12_Figure_1.jpeg)

![](_page_12_Picture_2.jpeg)

![](_page_12_Picture_5.jpeg)

*Augmented modified Goodman-diagram*

![](_page_13_Figure_2.jpeg)

![](_page_13_Picture_3.jpeg)

#### *Stress-concentration factors in fluctuating stresses*

*Note that component may "yield" locally*

![](_page_14_Figure_3.jpeg)

![](_page_14_Picture_4.jpeg)

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![](_page_14_Picture_5.jpeg)

#### *Stress-concentration factors in fluctuating stresses*

![](_page_15_Figure_2.jpeg)

![](_page_15_Picture_3.jpeg)

#### **Fatigue failure – Modified Goodman's diagram** *Safety factors in fluctuating stresses: Cases 1 and 2*

![](_page_16_Figure_1.jpeg)

![](_page_16_Picture_2.jpeg)

#### **Fatigue failure – Modified Goodman's diagram** *Safety factors in fluctuating stresses: Cases 3 and 4*

![](_page_17_Figure_1.jpeg)

#### **Review Example 6-4: under fully-reversed bending:** *parametric approach* Ш

A feed-roll assembly is to be mounted at each end on support brackets cantilevered from the machine frame as shown in the Figure. The feed rolls experience a fully reversed load of 1000 lb amplitude, split equally between the two support brackets. Design a cantilever bracket to support a fully reversed bending load of 500 lb amplitude for 10<sup>9</sup> cycles with no failure. Its dynamic deflection cannot exceed 0.01 in.

![](_page_18_Figure_3.jpeg)

![](_page_18_Picture_4.jpeg)

![](_page_18_Figure_5.jpeg)

**Example 6-5: fatigue under fluctuating bending. Design**   $\Box$ **bracket to support the load. Verify for maximum deflections**

![](_page_19_Figure_2.jpeg)

Design of a Cantilever Bracket for Fluctuating-Bending Loading

![](_page_19_Picture_4.jpeg)

#### **Fatigue failure** *Designing for HCF*

#### **Review Example 6-4: under fully-reversed bending**

![](_page_20_Figure_2.jpeg)

Geometric Stress-Concentration Factor  $K_t$  for a Filleted Flat Bar in Bending

**For next lecture: Master** *Examples 6-5 and and 6-6***; use and understand corresponding MathCad solutions (in the CD that came with your book and/or on Norton's Machine Design website)**

![](_page_21_Picture_2.jpeg)

![](_page_21_Picture_3.jpeg)

**Example 6-6: multiaxial fluctuating stresses. Verify the design**  Ш **against failure (e.g., by determining safety factors, other?)**

![](_page_22_Figure_2.jpeg)

*Notch radius (wall) is 0.25", Kt=1.70, Kts=1.35*

*Applied load: sinusoidal [-200,340] lb*

*Finite life of about 6x10 <sup>7</sup> cycles*

- *Material: Al 2024-T4*
- *Operating conditions: room temp.*

#### *Initial dimensions:*

 $l = 6.0$  in *a* = 8.0 *in*  $OD = 2$  in  $ID = 1.5$  in

**Comment:** *use MathCad example to perform design iterations*   $\Box$ *(file is included in the CD-ROM that came with your textbook)*

![](_page_22_Picture_11.jpeg)

![](_page_22_Picture_12.jpeg)

**Example 6-6: multiaxial fluctuating stresses.**   $\Box$ 

**Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.**

![](_page_23_Figure_3.jpeg)

![](_page_23_Picture_4.jpeg)

![](_page_23_Picture_5.jpeg)

**Example 6-6: multiaxial fluctuating stresses.** 

**Example of mechanical configurations that can be idealized with arrangement shown in Example 6-6.**

![](_page_24_Picture_3.jpeg)

![](_page_24_Figure_4.jpeg)

![](_page_24_Picture_5.jpeg)

#### **Example 6-6: Multiaxial Fluctuating Stresses**

 $\Box$ 

![](_page_25_Picture_19.jpeg)

![](_page_25_Picture_3.jpeg)

| 2 The correction factors are calculated from equations 6.7 and used to find a corrected  |  |                                 |
|--|--|---------------------------------|
| 6.2 and 10   |  | for bending                     |
| $A_{95} := 0.0105 \text{ od}^2$  | $A_{95} = 0.042 \text{ in}^2$                                  | Make <b>sure to</b>             |
| $A_{95} = 0.0105 \text{ od}^2$   | $A_{95} = 0.042 \text{ in}^2$                                  | Make <b>sure to</b>             |
| $A_{95} = 0.0105 \text{ od}^2$   | $A_{95} = 0.042 \text{ in}^2$                                  | Make <b>sure to</b>             |
| $A_{95} = 0.042 \text{ in}^2$  | Make <b>Figure 10</b>  |                                 |
| $C_{size} = 0.869 \left( \frac{d_{eq}}{in} \right)^{-0.097}$   | $C_{size} = 0.895$   | Note "negative" <b>exponent</b> |
| $S_{ui}$ is <b>useed in</b> kpsi = 6.3 constants   | $A := 2.7$ $b := -0.265$                                       | Note "negative" <b>exponent</b> |
| $S_{ui}$ is <b>useed in</b> kpsi = 6.3 times   | $C_{siab} = 0.753$ for 99.9%                                   |                                 |
| <b>Note that this is only</b> $S_{f5B8} = C_{load}$ is used despite the fact that there is both bending and torsion present. The torsional shear stress will be converted to an equivalent tensile |  |                                 |
| <b>24 % of the</b> S <sub>vi</sub> .)  | Note that the bending value of $C_{tag}$ is used from equation |                                 |

![](_page_26_Picture_1.jpeg)

3 This problem calls for a life of 6E7 cycles, so a strength value at that life must be estimated from the  $S<sub>1</sub>N$  line of Figure 6-33b using the corrected fatigue strength at that life. Equation  $6.10a$  for this line can be solved for the desired strength after we compute the values of its coefficients  $a$  and  $b$  from equation 6.10 $c$ .

 $S_m = 61.2$ ksi  $S_m := 0.90 S_{tt}$ From Table 6-5 for 5E8  $z = 5.699$  $b := -\frac{1}{z} \cdot log\left(\frac{S_m}{S_f 5g8}\right)$  $b = -0.1288$  $(c)$  $a := \frac{S_m}{10^{3-b}}$  $a = 148.9$ ksi  $S_n := a \cdot N^b$  $S_{22} = 14.84$  ksi

Note that  $S_m$  is calculated as 90% of  $S_{\mathcal{U}}$  because loading is bending rather than axial (see Eq. 6.9). The value of z is taken from Table 6-5 for  $N = 5E8$  cycles. This is a corrected fatigue strength for the shorter life required in this case and so is larger than the corrected test value, which was calculated at a longer life.

4 The notch sensitivity of the material must be found to calculate the fatigue stress-concentration factors. Table 6-8 shows the Neuber factors for hardened aluminum. Interpolation gives a value of  $a = 0.147^2$  in at the material's  $S_{\mu\nu}$ . Equation 6.13 gives the resulting notch sensitivity for the assumed notch radius.

$$
q := \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}
$$
 
$$
q = 0.773
$$
 (d)

5 The fatigue stress-concentration factors are found from equation  $6.11b$  using the given geometric stress-concentration factors for bending and torsion, respectively.

$$
K_f := 1 + q \cdot (K_t - 1)
$$
  $K_f = 1.541$  (e)

$$
K_{fS} := 1 + q \cdot (K_{ts} - 1)
$$
  $K_{fS} = 1.270$  (f)

 $K_t$  and  $K_{ts}$  are given *(we are lucky!)*

*This is calculated* 

*materials with an S<sup>e</sup>*

*differently for* 

![](_page_27_Picture_9.jpeg)

6 The bracket tube is loaded in both bending (as a cantilever beam) and in torsion. The shapes of the shear, moment and torque distributions are shown in Figure 4-30. All are maximum at the wall. The alternating and mean components of the applied force, moment, and torque at the walls are

*Forces: evaluated for amplitude and mean components*

*Moments: evaluated for amplitude, mean, and maximum components* Loads  $F_a := \frac{F_{max} - F_{min}}{2}$   $F_a = 270 \text{ lbf}$ <br> $F_m := \frac{F_{max} + F_{min}}{2}$   $F_m = 70 \text{ lbf}$  $F_a := \frac{F_{max} - F_{min}}{2} \qquad F_a = 270 \, lbf$  $(g)$  $M_a := F_a \cdot l$  $M_a = 1620$ lbf·in Moments  $(h)$  $M_m := F_m \cdot l$  $M_m = 420$ lbf·in  $M_{max} = M_a + M_m$  $M_{max} = 2040 \, lbf \cdot in$ Torques  $a = 8.0 \cdot in$  $T_a = 2160$ lbf·in  $T_a = F_{a} \cdot a$  $T_m = 560 lbf \cdot in$  $T_m := F_m \cdot a$ 

 $c = 1.000$ in

*Evaluated for amplitude and mean components*

7 The fatigue stress-concentration factor for the mean stresses depends on the relationship between the maximum local stress in the notch and the yield strength as defined in equation 6.17, a portion of which is shown here.

inertia

Outer fiber

Moment of  $I := \frac{\pi}{64} \cdot \left( od^4 - id^4 \right)$   $I = 0.5369i n^4$  $J\coloneqq 2\cdot\! I$  $J = 1.0738$ in<sup>4</sup>  $\text{If } K_f \cdot \Big | \, \sigma_{max} \Big | \, < \, S_{\mathcal{Y}} \text{ then } \, K_{fm} := \, K_f \text{ and } \, K_{f \text{Sm}} := \, K_{f \text{s}}$ 

$$
K_f \left| \frac{M_{max} c}{I} \right| = 5.86 \text{ksi}
$$

which is less than  $S_y = 47$  ksi so,  $K_{fm} := K_f$  and  $K_{fsm} := K_{fS}$ 

 $c = 0.5$  od

*Compensate for local "yield," if any*

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![](_page_28_Picture_11.jpeg)

In this case, there is no reduction in stress-concentration factors for the mean stress because there is no yielding at the notch to relieve the stress concentration.

![](_page_28_Picture_13.jpeg)

8 The largest tensile bending stress will be in the top or bottom outer fiber at points A or A! The largest torsional shear stress will be all around the outer circumference of the tube. (See Example 4-9 for more details.) First take a differential element at point A or A' where both of these sresses combine. (See Figure 4-32.) Find the alternating and mean components of the normal bending stress and of the torsional shear stress on point A using equations 4.11b and 4.24b, respectively.

$$
\sigma_a := K_f \cdot \frac{M_a \cdot c}{I} \qquad \sigma_a = 4.65 k s i
$$
\n
$$
\tau_a := K_{fs} \cdot \frac{T_a \cdot c}{J} \qquad \tau_a = 2.56 k s i
$$
\n
$$
\sigma_m := K_{fm} \cdot \frac{M_m \cdot c}{I} \qquad \sigma_m = 1.21 k s i
$$
\n
$$
\tau_m := K_{fsm} \cdot \frac{T_m \cdot c}{I} \qquad \tau_m = 0.66 k s i
$$
\n(1)

*Evaluate applied, amplitude and mean, stresses – point A is subjected to bending and shear*

9 Find the alternating and mean von Mises effective stresses at point A from equation 6.22b.

| $\sigma_{xa} = \sigma_a$   | $\sigma_{ya} = 0 \cdot psi$   | $\tau_{xyz} = \tau_a$       |                       |
|----------------------------|---|-----------------------------|-----------------------|
| <b>Evaluate equivalent</b> | $\sigma'_a = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xyz}^2}$ | $\sigma'_a = 6.42 ksi$      |                       |
| <b>Miss, amplitude</b>     | $\sigma_{xm} = \sigma_m$  | $\sigma_{ym} = 0 \cdot psi$ | $\tau_{xym} = \tau_m$ |
| <b>and mean, stresses</b>  | $\sigma_{xxm} = \sigma_m$   | $\sigma_{ym} = 0 \cdot psi$ | $\tau_{xym} = \tau_m$ |

$$
\sigma'_m := \sqrt{\sigma_{xm}^2 + \sigma_{ym}^2 - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^2}
$$
\n
$$
\sigma'_m = 1.66 \text{ks}
$$

10 Because the moment and torque are both caused by the same applied force, they are synchronous and in-phase and any change in them will be in a constant ratio.

This is a Case 3 situation and the safety factor is found using equation 6.18e. *Evaluate for all* 

cases, if **unsure**  
about which case 
$$
N_f = \frac{S_n \cdot S_{ut}}{\sigma'_a \cdot S_{ut} + \sigma'_m \cdot S_n}
$$
  $N_f = 2.2$  At point A

*Note the use of Sn in this equation (finite life)*

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![](_page_29_Picture_11.jpeg)

*cases, if unsure* 

*Mises, amplitude* 

*and mean, stresses*

11 Since the tube is a short beam, we need to check the shear due to transverse loading at point  $B$  on the neutral axis where the torsional shear is also maximal. The maximum transverse shear stress at the neutral axis of a hollow, thin-walled, round tube was given as equation  $4.15d$ .

Cross-section area 
$$
A := \frac{\pi}{4} \cdot \left(\omega d^2 - id^2\right)
$$
  $A = 1.374i n^2$   
\n $\tau_{abend} := K_{fs} \cdot \frac{2 \cdot F_a}{A}$   $\tau_{abend} = 499 \text{ psi}$  (o)  
\n $\tau_{mbend} = K_{fsm} \cdot \frac{2 \cdot F_m}{A}$   $\tau_{mbend} = 129 \text{ psi}$ 

*Account for transversal shear – point B is subjected to pure shear*

Point  $B$  is in pure shear. The total shear stress at point  $B$  is the sum of the transverse shear stress and the torsional shear stress which act on the same planes of the element.

> $\tau_{atotal} = 3055 \,\text{psi}$  $\tau_{\alpha total} = \tau_{\alpha band} + \tau_{\alpha}$  $(p)$  $\tau_{\text{mtotal}} = 792 \,\text{psi}$  $\tau_{\text{mtotal}} = \tau_{\text{mbend}} + \tau_{\text{m}}$

12 Find the alternating and mean von Mises effective stresses at point B from equation 6.22b.

$$
\sigma_{xa} := 0 \cdot psi \qquad \sigma_{ya} := 0 \cdot psi \qquad \tau_{xya} := \tau_{atotal}
$$
\n
$$
\sigma'_a := \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa} \cdot \sigma_{ya} + 3 \cdot \tau_{xya}^2} \qquad \sigma'_a = 5.29 \text{ ksi}
$$
\n
$$
\sigma_{xm} := 0 \cdot psi \qquad \sigma_{ym} := 0 \cdot psi \qquad \tau_{xym} := \tau_{mtotal} \qquad (q)
$$
\n
$$
\sigma'_m := \sqrt{\sigma_{xm}^2 + \sigma_{ym}^2 - \sigma_{xm} \cdot \sigma_{ym} + 3 \cdot \tau_{xym}^2} \qquad \sigma'_m = 1.37 \text{ ksi}
$$

*Note the use of Sn in this equation (finite life)*

*Evaluate for all cases, if unsure about which case*  $7/1$ 

*Evaluate equivalent* 

*and mean, stresses*

*Mises, amplitude* 

Both points  $A$  and  $B$  are safe against fatigue failure.

13 The safety factor for point  $B$  is found using equation 6.18e.

 $N_f := \frac{S_n \cdot S_{ut}}{\sigma'_a \cdot S_{ut} + \sigma'_m \cdot S_n}$ 

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*At point B*

 $N_f = 2.7$ 

![](_page_30_Picture_11.jpeg)

## **Reading**

- **Chapters 6 of textbook: Sections 6.5 to 6.8**
- **Review notes and text: ES2501, ES2502**

#### **Homework assignment**

- **Author's: as indicated in Website of our course**
- **Solve: as indicated in Website of our course**

![](_page_31_Picture_6.jpeg)

![](_page_31_Picture_7.jpeg)