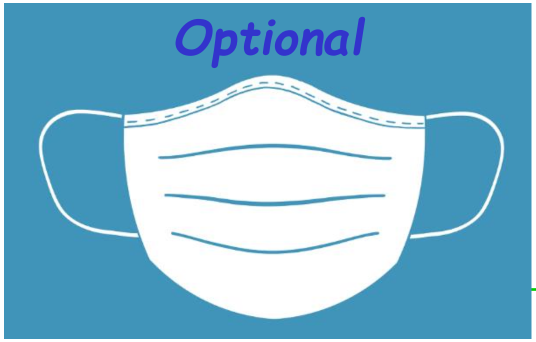
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, B'2024

Lecture 14

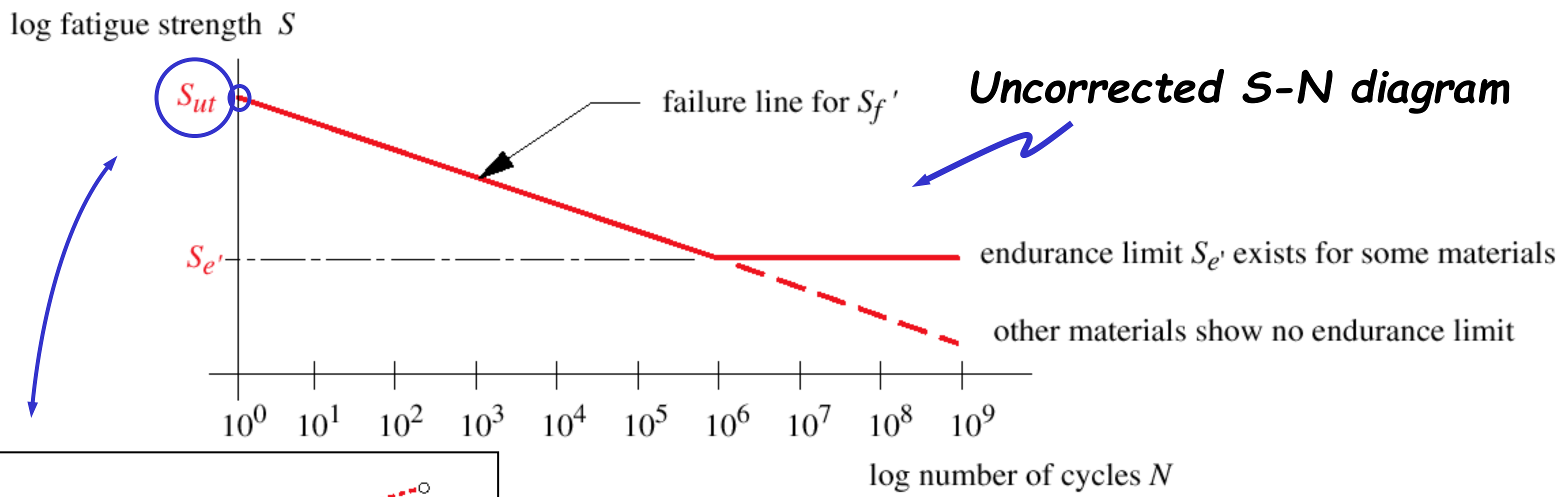
November 2024



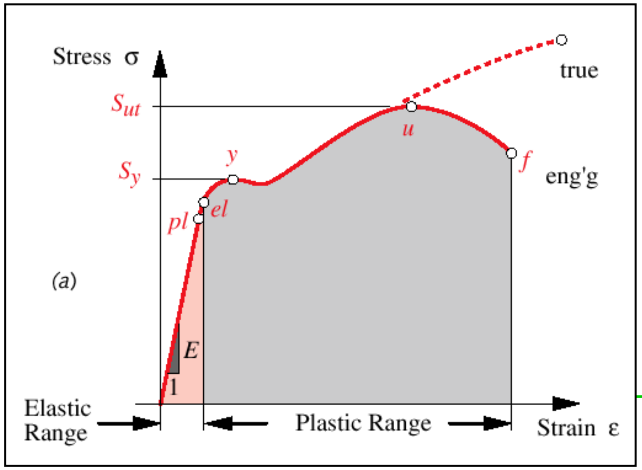
Fatigue failure

Theoretical or uncorrected fatigue data

- **Wohler strength-life or S-N diagram**
  - **Theoretical *or uncorrected* fatigue strength:** $S_{f'}$
  - **Theoretical *or uncorrected* endurance limit:** $S_{e'}$



Uncorrected S-N diagram



Estimating fatigue failure criteria

Based on experimental observations (bending, torsion, axial fatigue testing).

If data are not available… research, estimate, or… perform experiments...

Some materials with a "knee"

For steels:

$$S_{e'} = \begin{cases} 0.5\, S_{ut} & \text{for } S_{ut} < 200\ \text{ksi (1400 MPa)} \\ \cong 100\ \text{ksi (700 MPa)} & \text{for } S_{ut} \geq 200\ \text{ksi (1400 MPa)} \end{cases}$$

For irons:

$$S_{e'} = \begin{cases} 0.4\, S_{ut} & \text{for } S_{ut} < 60\ \text{ksi (400 MPa)} \\ \cong 24\ \text{ksi (160 MPa)} & \text{for } S_{ut} \geq 60\ \text{ksi (400 MPa)} \end{cases}$$

Some materials without a "knee"

For aluminums:

$$S_{f'@N=5\times10^8} = \begin{cases} 0.4\, S_{ut} & \text{for } S_{ut} < 48\ \text{ksi (330 MPa)} \\ \cong 19\ \text{ksi (130 MPa)} & \text{for } S_{ut} \geq 48\ \text{ksi (330 MPa)} \end{cases}$$

For copper alloys:

$$S_{f'@N=5\times10^8} = \begin{cases} 0.4\, S_{ut} & \text{for } S_{ut} < 40\ \text{ksi (280 MPa)} \\ \cong 14\ \text{ksi (100 MPa)} & \text{for } S_{ut} \geq 40\ \text{ksi (280 MPa)} \end{cases}$$

Fatigue failure

Correcting theoretical fatigue data

- **Materials with a "knee." Correcting function:**

$$S_e = C_{load} \cdot C_{size} \cdot C_{surface} \cdot C_{temperature} \cdot C_{reliability} \cdot S_{e'}$$

at 1 x 10 [6] cycles

- **Materials without a "knee." Correcting function:**

$$S_f = C_{load} \cdot C_{size} \cdot C_{surface} \cdot C_{temperature} \cdot C_{reliability} \cdot S_{f'}$$

at 5 x 10 [8] cycles

- **Correction factors:**

$$C_{load}, C_{size}, C_{surface}, C_{temperature}, C_{reliability}$$

- **Other factors may need to be added… depending on the type (& service) of components being designed...**

Fatigue failure
Correcting theoretical fatigue data

- **Loading effects (correction):**

$$C_{load} = \begin{cases} 1.0 & \text{Bending / Torsion} \\ 0.7 & \text{Axial Loading, e.g., tension/compression} \end{cases}$$

Fatigue failure

Correcting theoretical fatigue data

- **Size effects (correction; cylindrical parts):**

$$C_{size} = \begin{cases} 1.0 & d \le 0.3\,\text{in (8mm)} \\ 0.869d^{-0.097} & 0.3\,\text{in} \le d \le 10\,\text{in} \\ 1.189d^{-0.097} & 8.0\,\text{mm} \le d \le 250\,\text{mm} \\ \text{Larger sizes use } 0.6 & \end{cases} \Leftarrow \text{cylindrical parts}$$

- **Size effects (correction; non-cylindrical parts):**

$$d = \left\{ d_{equiv}; \quad d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}} \right\} \Leftarrow \text{non-cylindrical parts}$$

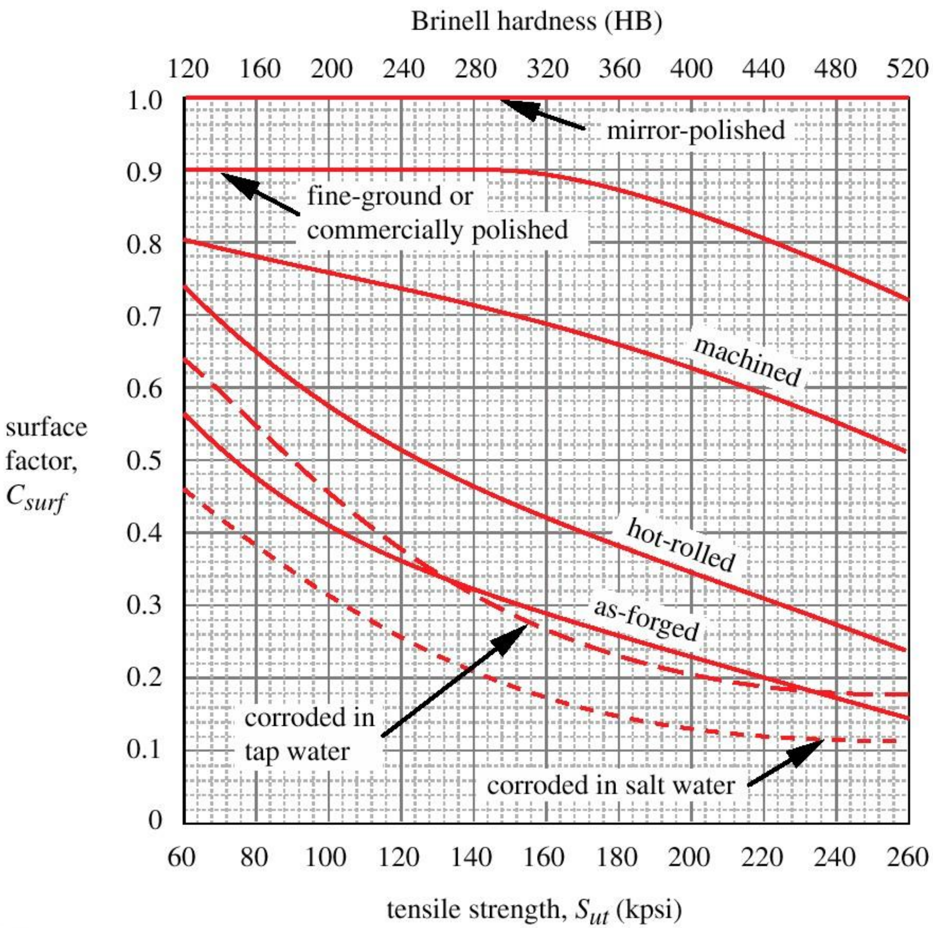
(See page 363 of Norton's)

$$A_{95} = \left\{ \begin{array}{l} \text{portion of the cross-sectional area of a} \\ \text{nonround part that is stressed between} \\ \text{95\% and 100\% of its max. stress} \end{array} \right\}$$

Fatigue failure

Correcting theoretical fatigue data

- **Surface effects (correction):** $C_{surface} = \{\text{See Figure 6-26}\}$



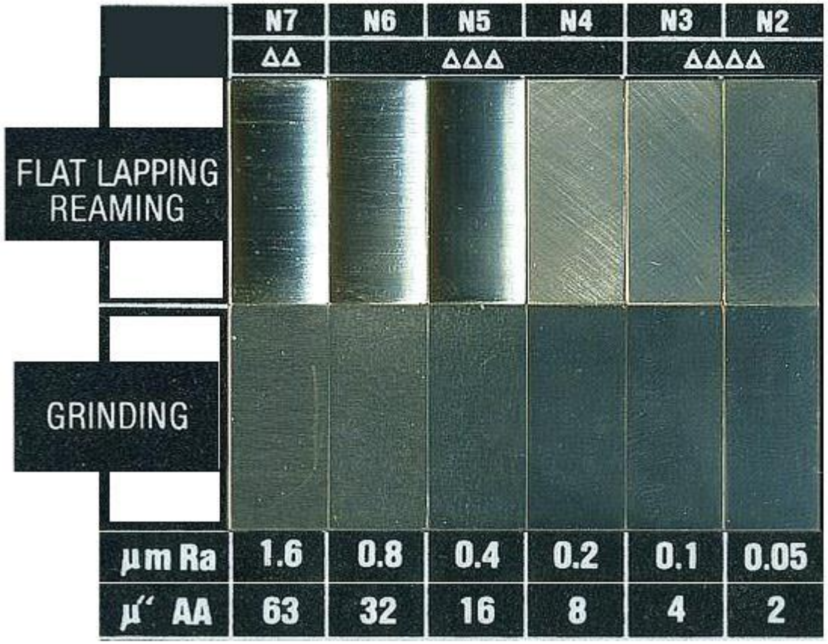


FIGURE 6-26

Surface Factors for Various Finishes on Steel (From Fig. 12.6, p. 234, R. C. Juvinall, *Stress, Strain, and Strength*, McGraw-Hill, New York, 1967, with permission)

Fatigue failure
Correcting theoretical fatigue data

- **Surface effects (correction; Shigley and Mischke):**

$$C_{surf} = \{ A \left(S_{ut}\right)^{b} \quad \text{if } C_{surf} > 1 \text{ then } C_{surf} = 1$$

(Use this model in the required units, as indicated in Table 6-3)

Table 6-3 Coefficients for the Surface-Factor Equation

Source: Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989, p. 283 with permission

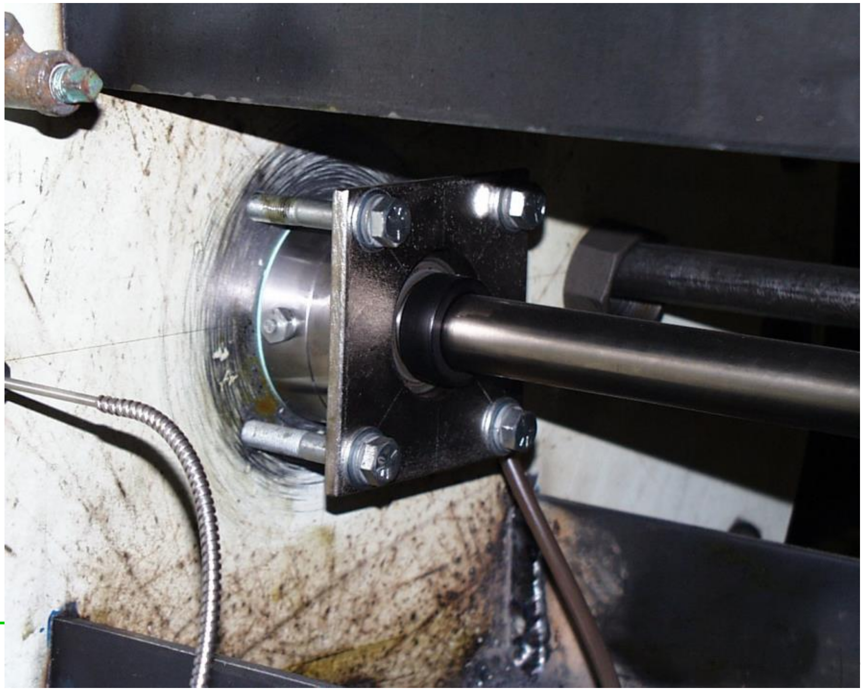
	MPa		kpsi	
Surface Finish	***A***	***b***	***A***	***b***
Ground	1.58	–0.085	1.34	–0.085
Machined or cold-drawn	4.51	–0.265	2.7	–0.265
Hot-rolled	57.7	–0.718	14.4	–0.718
As-forged	272	–0.995	39.9	–0.995

Fatigue failure

Correcting theoretical fatigue data

- **Temperature effects (correction):**

$$C_{temp} = \begin{cases} 1.0 & T \le 450\,°C\ (840°F) \\ 1-0.0058(T-450) & 450\,°C < T \le 550\,°C \\ 1-0.0032(T-840) & 840\,°F < T \le 1020\,°F \end{cases}$$



Fatigue failure

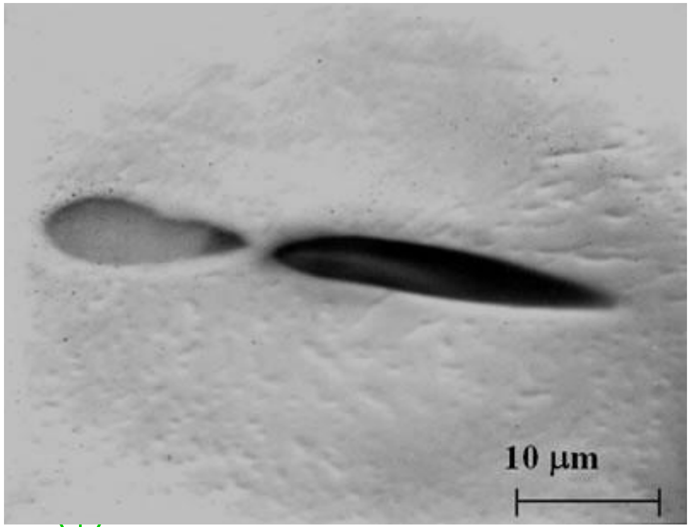
Correcting theoretical fatigue data

- **Reliability effects (correction):** $C_{reliab} = \{\text{See Table 6-4}\}$

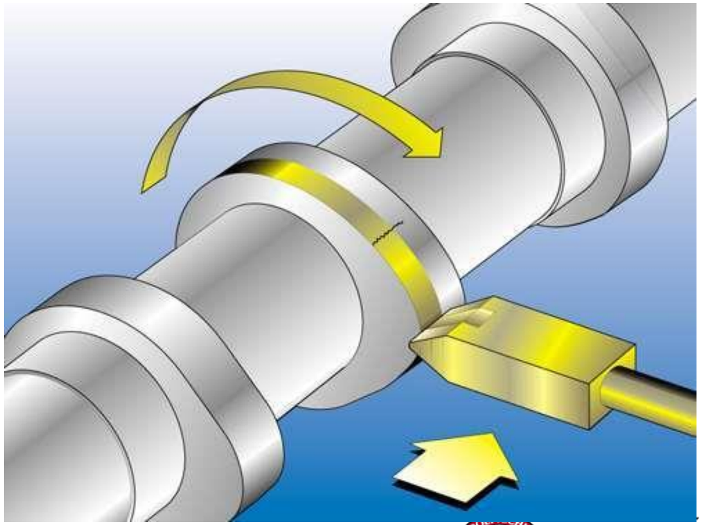
Table 6-4
Reliability Factors
for $S_d = 0.08\ \mu$ ← Standard deviation

Reliability %	C_{reliab}
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Within materials



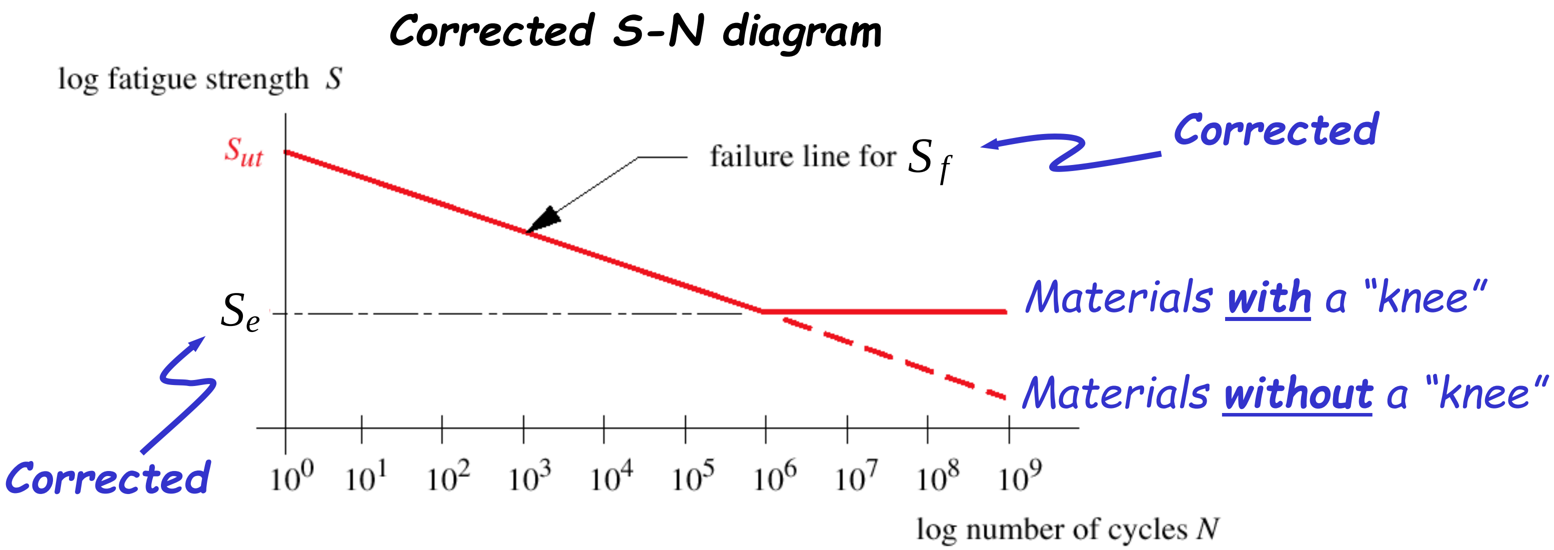
Manufacturing



Fatigue failure

Correcting theoretical fatigue data

- **Once fatigue strength/endurance limit has been corrected… <u>construct estimated (corrected) S-N diagram</u>**



Fatigue failure

Creation of estimated S-N diagrams

- **Fatigue strength at 10^3 cycles:** S_m

$$S_m = \begin{cases} 0.90 S_{ut} & \text{Bending} \\ 0.75 S_{ut} & \text{Axial Loading} \end{cases} \Leftarrow \text{at } N = 10^3 \text{ cycles}$$

Fatigue failure

Creation of estimated S-N diagrams

- **Curve fitting of model (HCF-high cycle fatigue):** $S(N) = a N^b$

Use corrected values for S_f or S_e

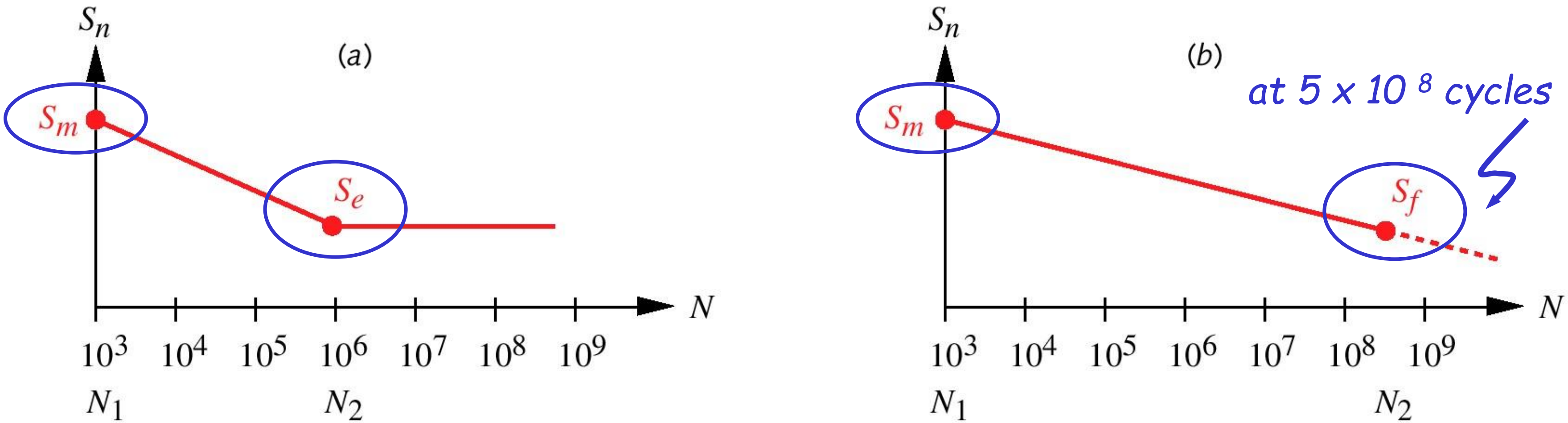


FIGURE 6-33

Estimated *S-N* Curves for (*a*) Materials with Knee, (*b*) Materials Without Knee

Fatigue failure

- Review Example 6-1: Ferrous materials (with a "knee")

EXAMPLE 6-1

Determining Estimated S-N Diagrams for Ferrous Materials

Problem Create an estimated *S-N* diagram for a bar and define its equations. How many cycles of life can be expected if the alternating stress is 100 MPa?

Units $MPa := 10^6 \cdot Pa$ $\quad C := 1$

Given The bar is square and has a hot-rolled finish. The loading is fully reversed bending.

Tensile strength	$S_{ut} := 600 \cdot MPa$
Maximum temperature	$T_{max} := 500 \cdot C$
Bar side dimension	$b := 150 \cdot mm$
Alternating stress	$\sigma_a := 100 \cdot MPa$
Reliability	$R := 0.999$

Assumptions Infinite life is required and is obtainable since this ductile steel will have an endurance limit. A reliability factor of 99.9% will be used.

Fatigue failure

- **Review Example 6-2: Nonferrous materials (without a "knee")**

EXAMPLE 6-2

Determining Estimated S-N Diagrams for Nonferrous Materials

Problem — **Create an estimated *S-N* diagram for an aluminum bar and define its equations. What is the corrected fatigue strength at 2E7 cycles?**

Units — $ksi := 10^3 \cdot psi \qquad F := 1$

Given — **The forged 6061-T6 bar is round. The loading is fully reversed torsion.**

Tensile strength	$S_{ut} := 45 \cdot ksi$
Maximum temperature	$T_{max} := 300 \cdot F$
Bar diameter	$d := 1.5 \cdot in$
Reliability	$R := 0.990$

Assumptions — **A reliability factor of 99.0% will be used. The uncorrected fatigue strength will be taken at 5E8 cycles.**

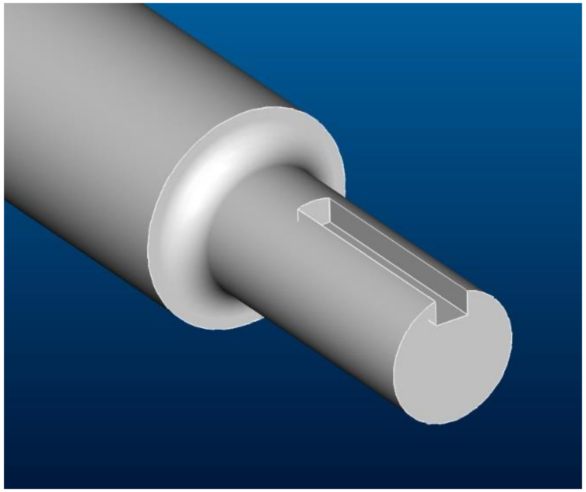
Fatigue Stress Concentration Factors (FSCF)

Fatigue failure theories:

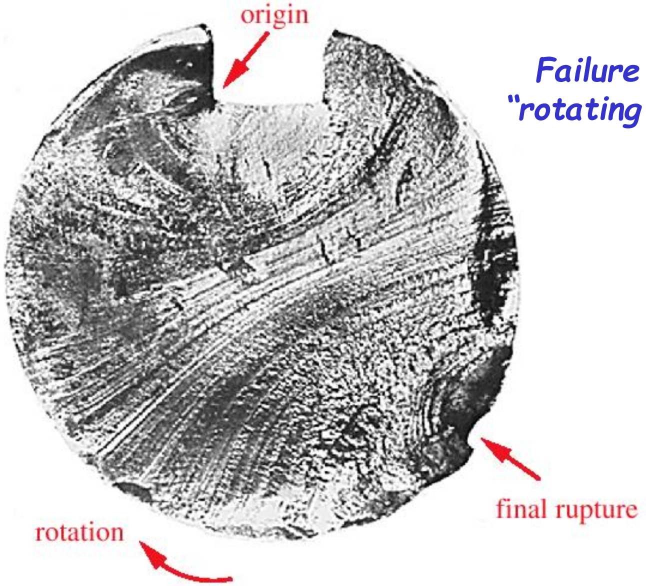
Surface defects & stress concentrations

- **Fatigue failures always begin at a crack**
  - Cracks may be present in raw material used for fabrication (crystallographic defects; inclusions; etc.)
  - Cracks may be introduced during fabrication
  - Cracks develop over time due to cyclic loading (& corrosion)
  - Cracks develop around stress concentrations

Shaft with keyway



Shaft failed in fatigue.
Crack initiated at keyway



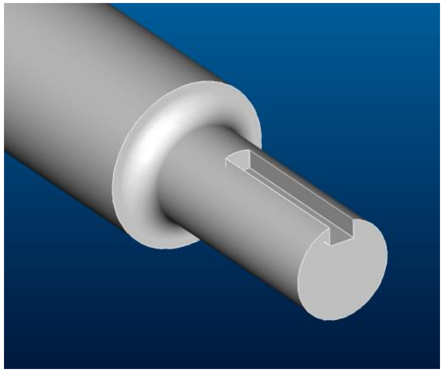
Failure due to "rotating bending"

Fatigue failure

Notches and stress concentrations

- **Notches introduce stress-concentrations.** See lectures 07-08 and 13

Shaft with keyway



- **Correcting for stress-concentrations.**
  Stress concentration factors in fatigue: K_f, K_{fs}

- **Use of stress concentration factors in fatigue:**

$$\sigma = K_f \ \sigma_{\text{nominal}}$$

$$\tau = K_{fs} \ \tau_{\text{nominal}}$$

Fatigue failure

Notches and stress concentrations

- **Stress concentration factors in fatigue:**

$$K_f = 1 + q(K_t - 1)$$

  - **Theoretical (static) stress-concentration factor:** K_t

  - **Notch sensitivity factor:** $q = \dfrac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}}$

    $\sqrt{a}$ = *Neuber's constant*

  - **Neuber's constant (depends on the value of the ultimate tensile strength of the material used). See, for example, Tables 6-6, 6-7, and 6-8**

Fatigue failure: Neuber's constant

Notches and stress concentrations

Table 6-6
Neuber's Constant for Steels

S_{ut} (ksi)	$\sqrt{a}$ ($in^{0.5}$)
50	0.130
55	0.118
60	0.108
70	0.093
80	0.080
90	0.070
100	0.062
110	0.055
120	0.049
130	0.044
140	0.039
160	0.031
180	0.024
200	0.018
220	0.013
240	0.009

Table 6-7
Neuber's Constant for Annealed Aluminum

S_{ut} (kpsi)	$\sqrt{a}$ ($in^{0.5}$)
10	0.500
15	0.341
20	0.264
25	0.217
30	0.180
35	0.152
40	0.126
45	0.111

Table 6-8
Neuber's Constant for Hardened Aluminum

S_{ut} (kpsi)	$\sqrt{a}$ ($in^{0.5}$)
15	0.475
20	0.380
30	0.278
40	0.219
50	0.186
60	0.162
70	0.144
80	0.131
90	0.122

May need to do curve fitting in order to determine Neuber's constant functions:

$$y = f(x)$$

$$y = \text{Neuber's constant} = \sqrt{a}$$

$$x = S_{ut}$$

Fatigue failure: Neuber's constant

Notches and stress concentrations

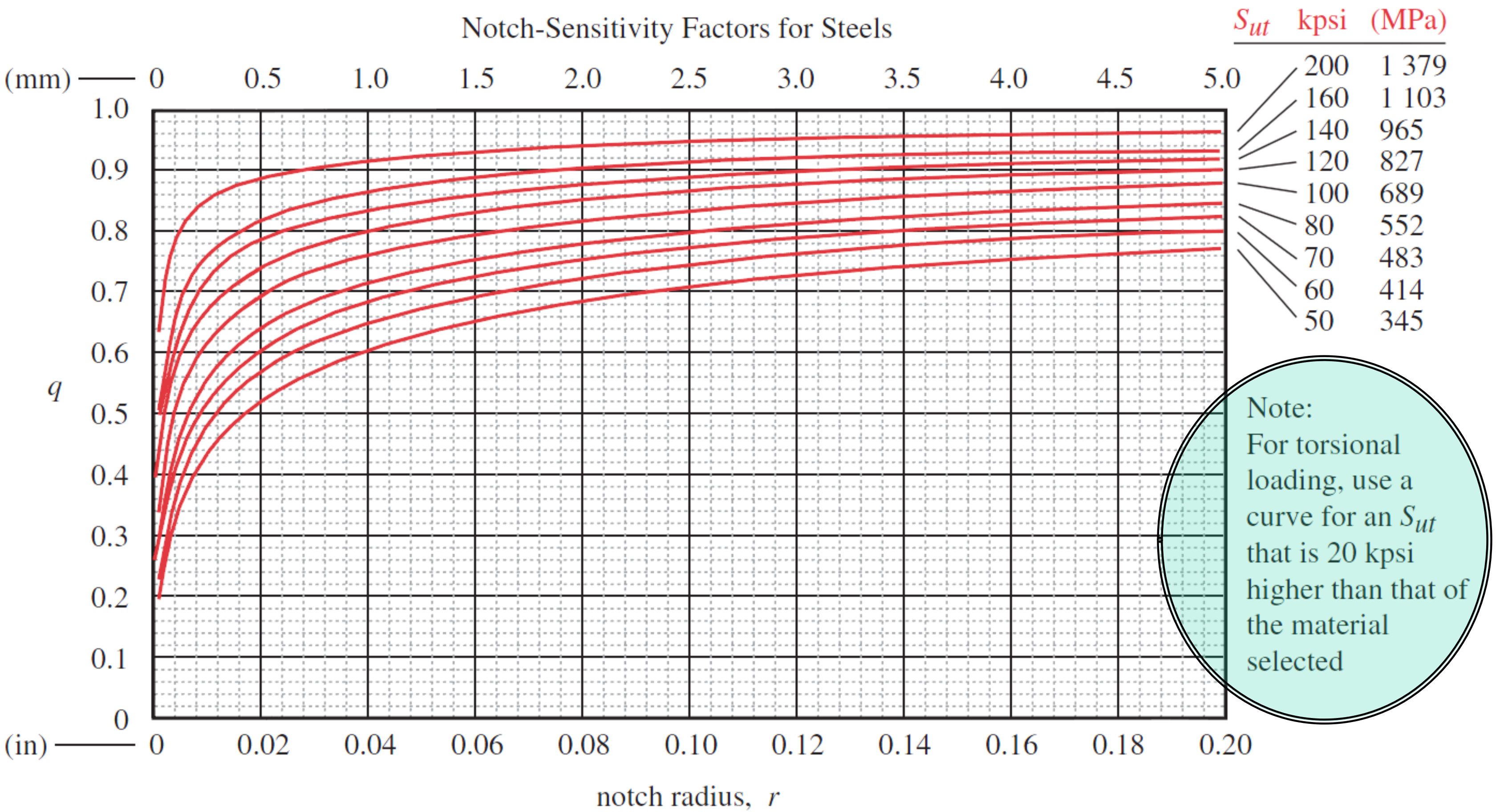


FIGURE 6-36 Part 1

Notch-Sensitivity Curves for Steels Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in *Metal Fatigue* by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

Fatigue failure: Neuber's constant

Notches and stress concentrations

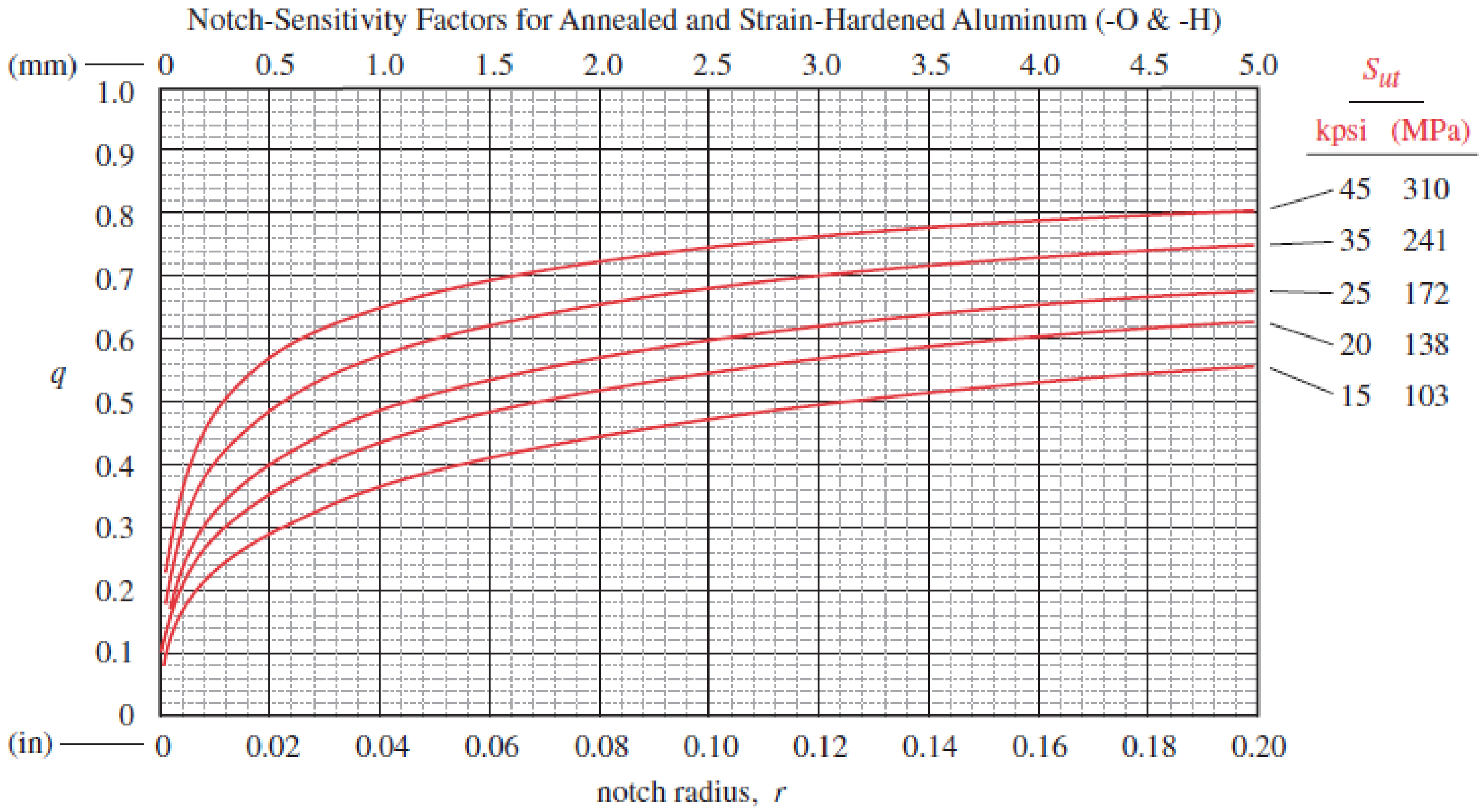


FIGURE 6-36 Part 2

Notch-Sensitivity Curves for Aluminums Calculated from Equation 6.13 Using Data from Figure 6-35 as Originally Proposed by R. E. Peterson in "Notch Sensitivity," Chapter 13 in *Metal Fatigue* by G. Sines and J. Waisman, McGraw-Hill, New York, 1959.

Fatigue failure

- **Review Example 6-3: *determining fatigue stress-concentration factors***

EXAMPLE 6-3

Determining Fatigue Stress Concentration Factors

Problem
A rectangular, stepped bar similar to that shown in Figure 4-36 is to be loaded in bending. Determine the fatigue stress-concentration factor for the given dimensions.

Units
$ksi := 10^3 \cdot psi$

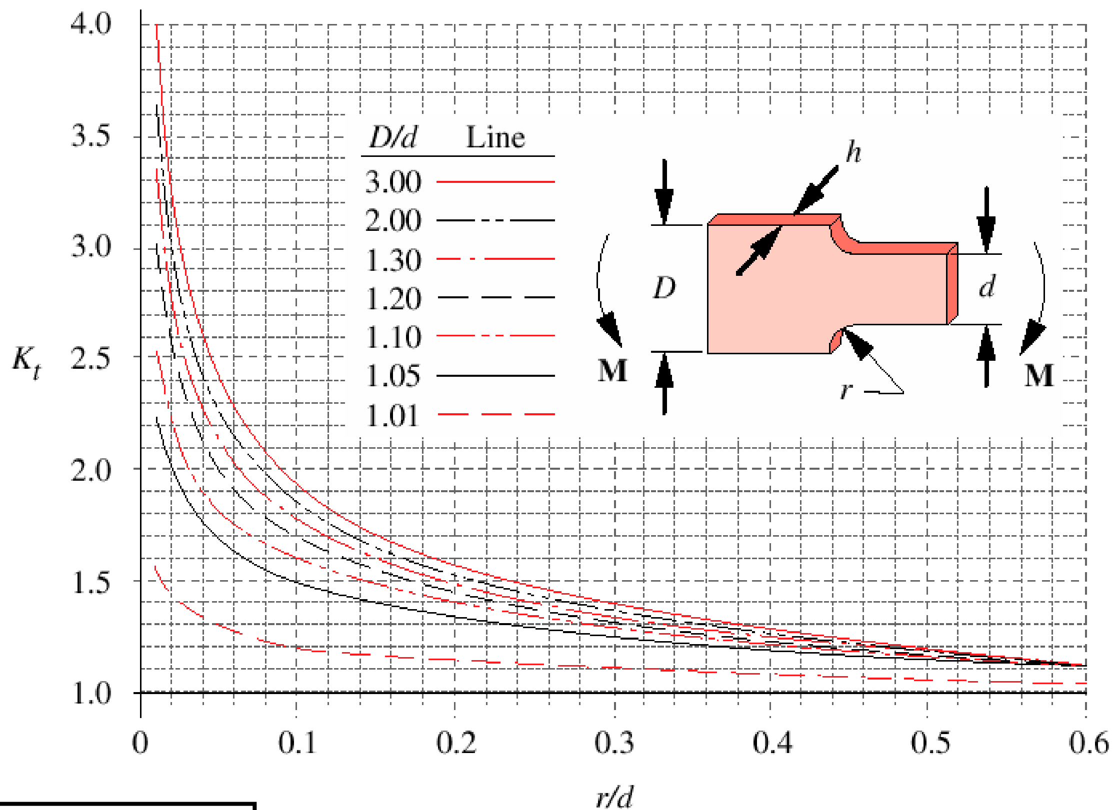
Given
Using the nomenclature in Figure 4-36:

Tensile strength $S_{ut} := 100 \cdot ksi$

Dimensions $D := 2 \cdot in$ $d := 1.8 \cdot in$ $r := 0.25 \cdot in$

Fatigue failure

- **Review Example 6-3: *determining fatigue stress-concentration factors***



$$\sigma_{nom} = \frac{Mc}{I} = 6\frac{M}{hd^2}$$

$$\sigma_{max} = K_t \sigma_{nom}$$

and :

$$K_t = A\left(\frac{r}{d}\right)^b$$

where :

D / d	*A*	*b*
3.00	0.907 20	–0.333 33
2.00	0.932 32	–0.303 04
1.30	0.958 80	–0.272 69
1.20	0.995 90	–0.238 29
1.10	1.016 50	–0.215 48
1.05	1.022 60	–0.191 56
1.01	0.966 89	–0.154 17

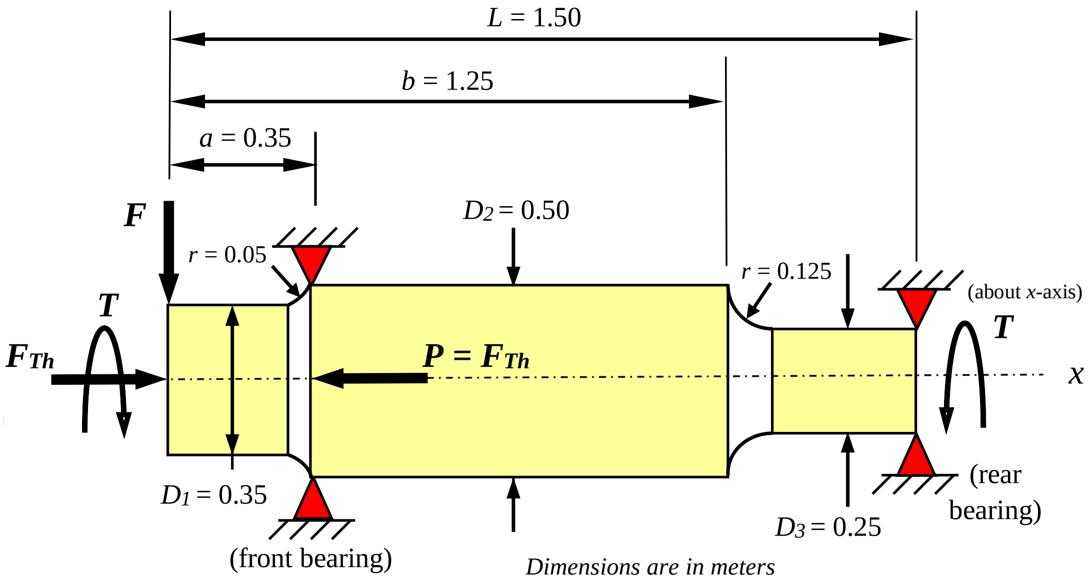
FIGURE 4-36

Geometric Stress-Concentration Factors and Functions for a Stepped Flat Bar in Bending - Also see the File APP_E-10
Source: Fig. 73, p. 98, R. E. Peterson, Stress Concentration Factors, John Wiley & Sons, 1975, with the publisher's permission

"Representative example: stress concentrations

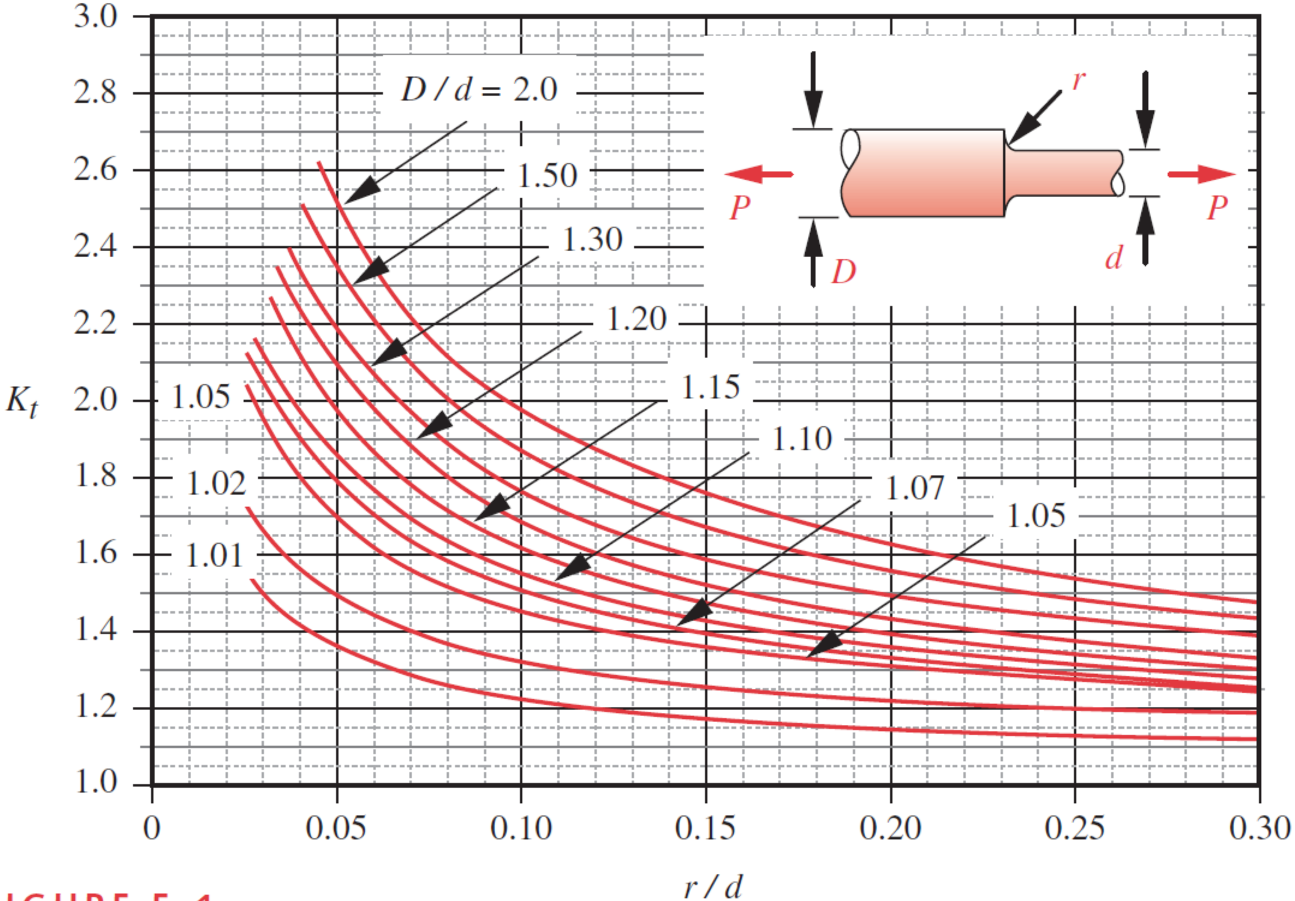
Class discussions

EXAMPLE: The main shaft of an 850 kW wind turbine is being redesigned. Designers realized that it is necessary to do stress analysis on the shaft *while accounting for*: (a) weight of the shaft; (b) weight *F* of the hub-blades assembly, which has a *mass* of 10 metric tons, (c) torque *T* produced at the rotational speed of 10 RPM at full power, and (d) *thrust load P of 36 kN*, (e) *fatigue* stress concentrations.



Representative example: stress concentrations

- Class discussions



$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

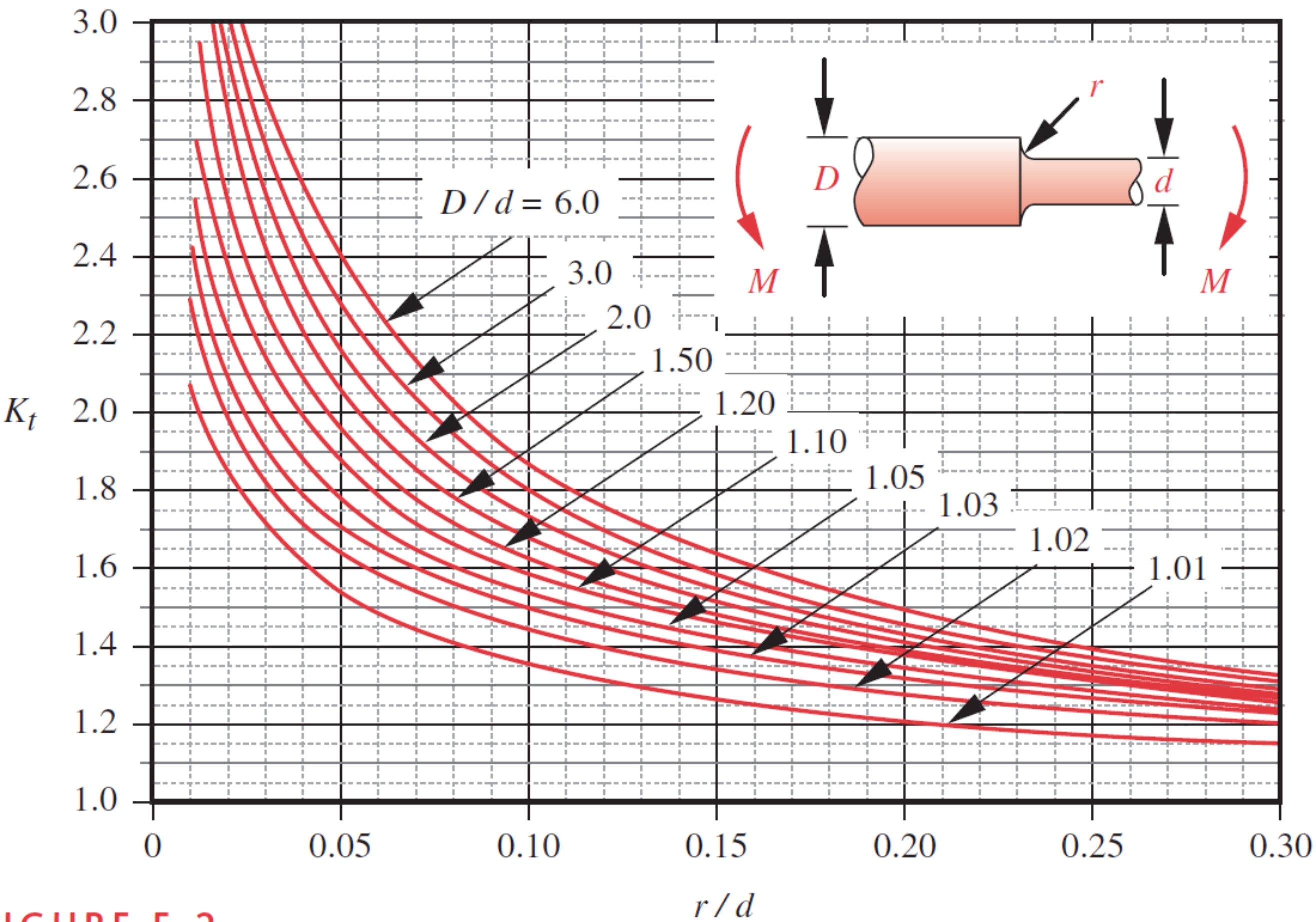
D / d	A	b
2.00	1.014 70	–0.300 35
1.50	0.999 57	–0.282 21
1.30	0.996 82	–0.257 51
1.20	0.962 72	–0.255 27
1.15	0.980 84	–0.224 85
1.10	0.984 50	–0.208 18
1.07	0.984 98	–0.195 48
1.05	1.004 80	–0.170 76
1.02	1.012 20	–0.124 74
1.01	0.984 13	–0.104 74

FIGURE E-1

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Axial tension

Representative example: stress concentrations

- Class discussions



$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

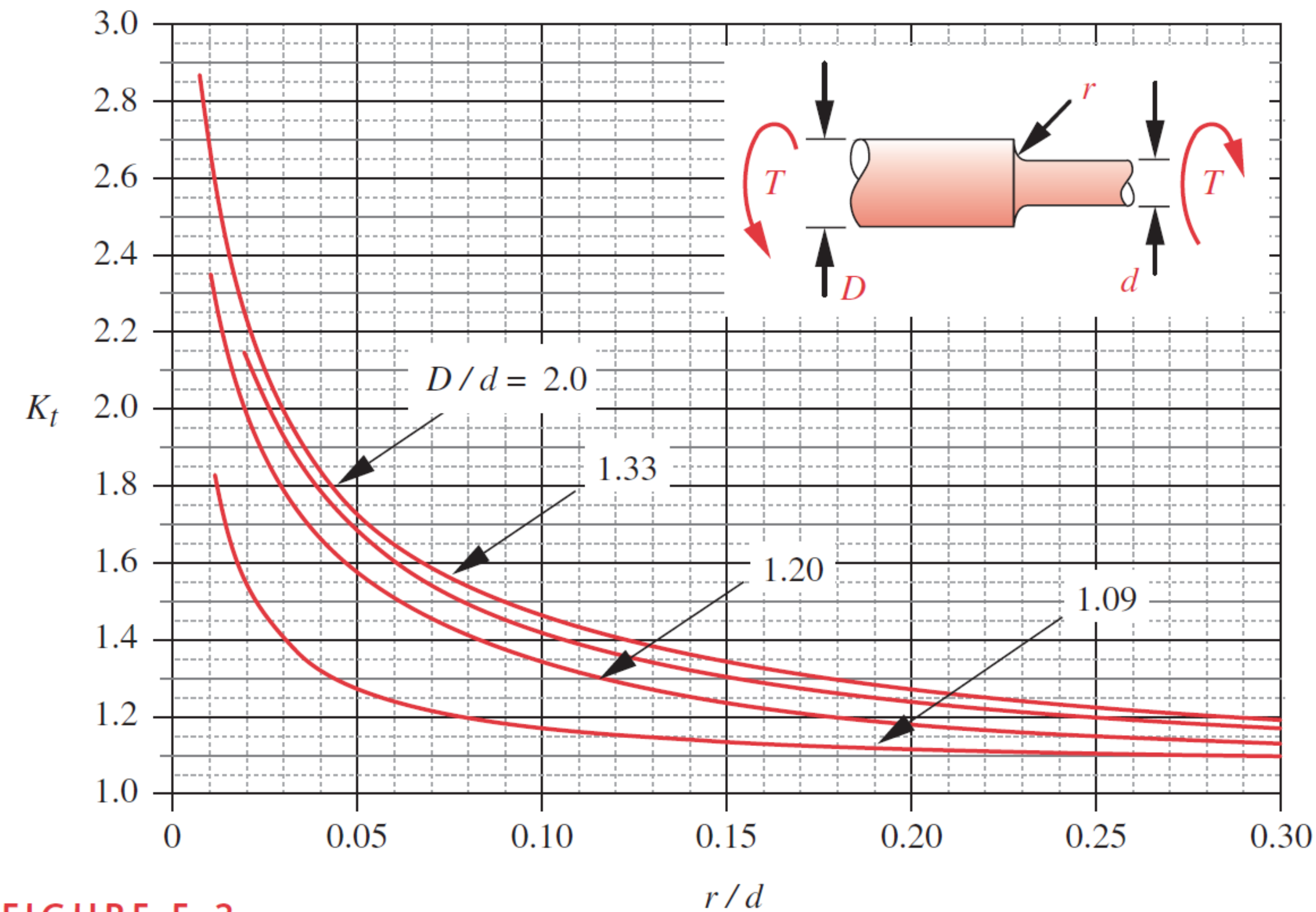
D / d	*A*	*b*
6.00	0.878 68	–0.332 43
3.00	0.893 34	–0.308 60
2.00	0.908 79	–0.285 98
1.50	0.938 36	–0.257 59
1.20	0.970 98	–0.217 96
1.10	0.951 20	–0.237 57
1.07	0.975 27	–0.209 58
1.05	0.981 37	–0.196 53
1.03	0.980 61	–0.183 81
1.02	0.960 48	–0.177 11
1.01	0.919 38	–0.170 32

FIGURE E-2

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Bending

Representative example: stress concentrations

- **Class discussions**



$$K_t \cong A\left(\frac{r}{d}\right)^b$$

where :

D/d	A	b
2.00	0.863 31	–0.238 65
1.33	0.848 97	–0.231 61
1.20	0.834 25	–0.216 49
1.09	0.903 37	–0.126 92

FIGURE E-3

Geometric Stress-Concentration Factor K_t for a Shaft with a Shoulder Fillet in Torsion

Reading assignment

- **Chapters 6 of textbook: Sections 6.0 to 6.5**
- **Review notes and text: ES2001, ES2501, ES2502**

Homework assignment

- **Author's:** as indicated in website of our course
- **Solve:** as indicated in website of our course