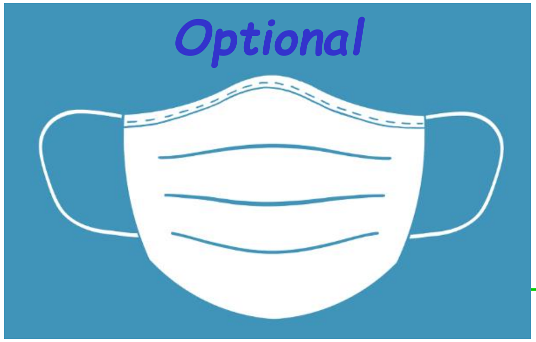
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

DESIGN OF MACHINE ELEMENTS
ME-3320, B'2024

Lecture 07-08

November 2024



Deflection in beams
Example E1 -- in class

Recall:

$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$ Load function - deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function - deflection

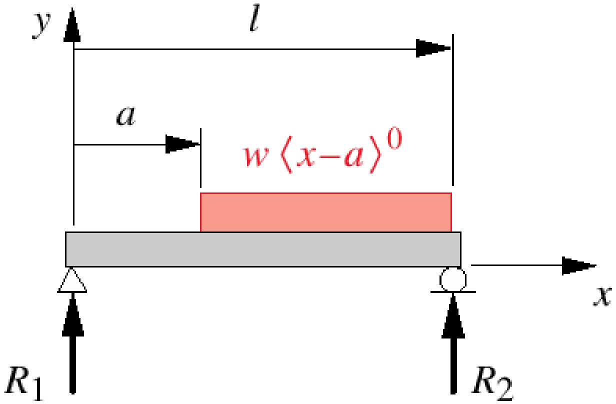
$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function - deflection

$$\theta = \frac{dy}{dx}$$ Slope - deflection

$$y = f(x)$$ Deflection

Shear and bending-moment, slope-deflection diagrams

Singularity functions: in-class examples (loading functions)

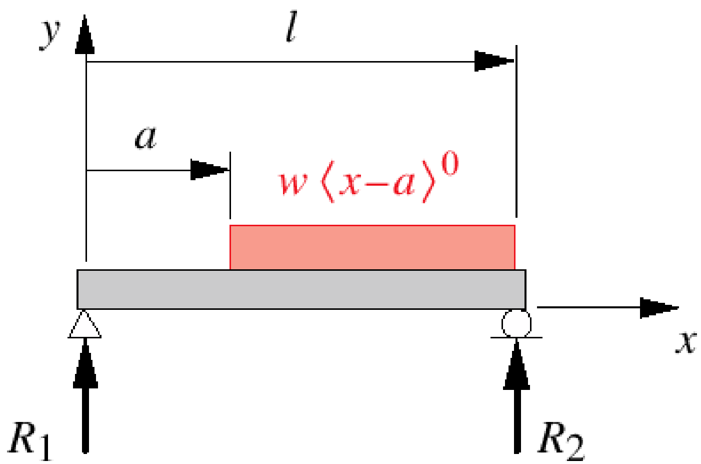


(a) Simply supported beam with uniformly distributed loading

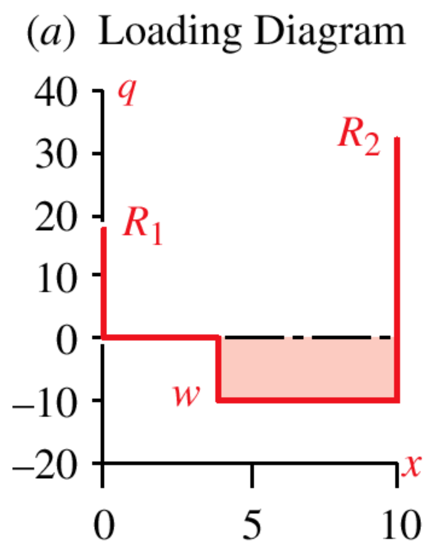
Deflection in beams

Example E1 (based on Norton's example 3-2B)

Determine and plot the shear, moment, slope, and deflection functions for the simply supported beam shown:



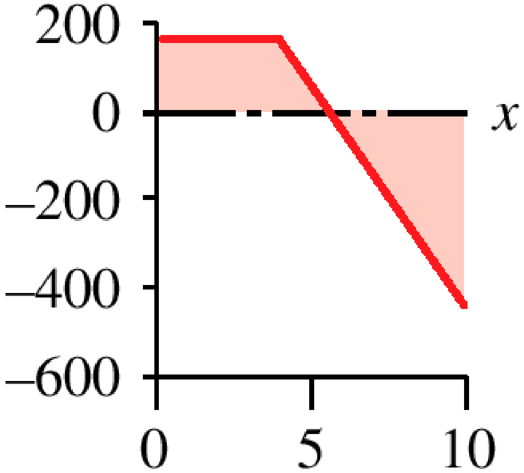
(a) Simply supported beam with uniformly distributed loading



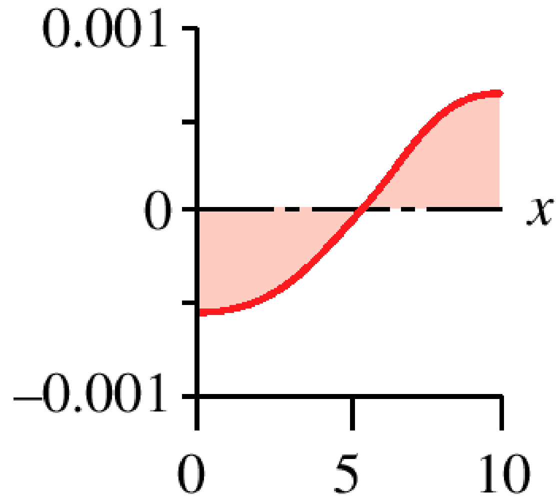
Deflection in beams

Example E1 -- in class

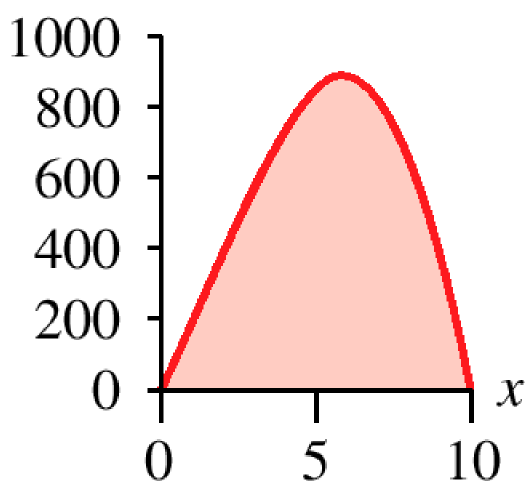
Shear Diagram (lb)



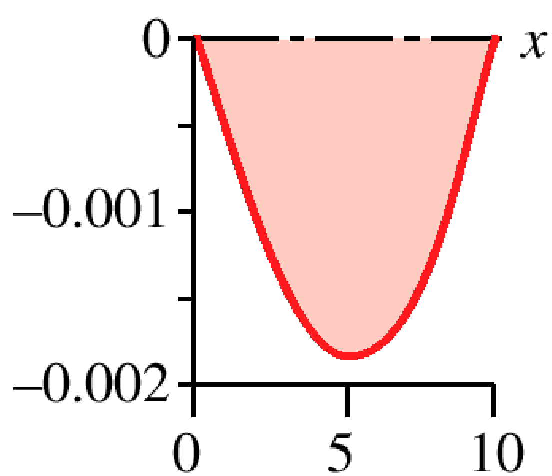
Slope Diagram (rad)



Moment Diagram (lb-in)



Deflection Diagram (in)



Plotting singularity functions in MathCad

Example E1 (Based on Norton's example 3-2B)

To generate the shear and moment functions over the length of the beam, equations (*b*) and (*c*) must be evaluated for a range of values of x from 0 to l, after substituting the above values of C_1, C_2, R_1, and R_2 in them. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

Range of x	$x := 0 \cdot in, 0.01 \cdot l .. l$
Unit step function	$S(x,z) := if(x \ge z, 1, 0)$

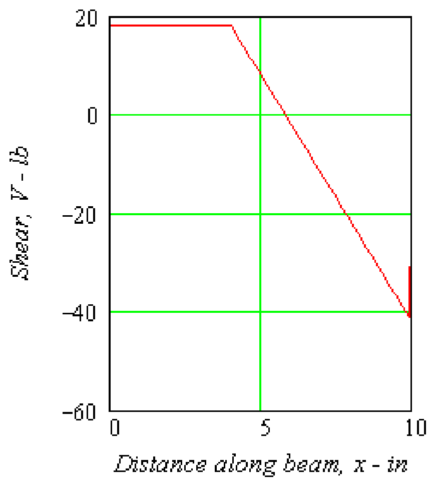
Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^0 - w \cdot S(x,a) \cdot (x-a)^1 + R_2 \cdot S(x,l) \cdot (x-l)^0$$

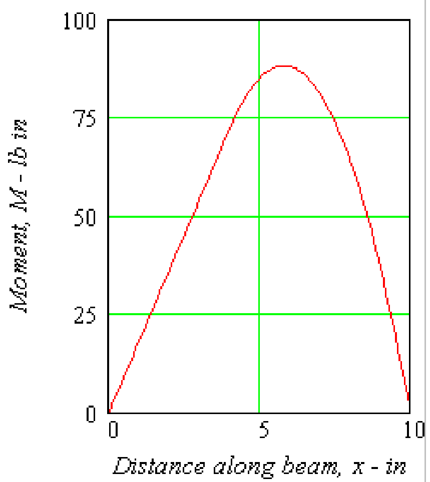
$$M(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^1 - \frac{w}{2} \cdot S(x,a) \cdot (x-a)^2 + R_2 \cdot S(x,l) \cdot (x-l)^1$$

Plot the shear and moment diagrams.

(*b*) Shear Diagram



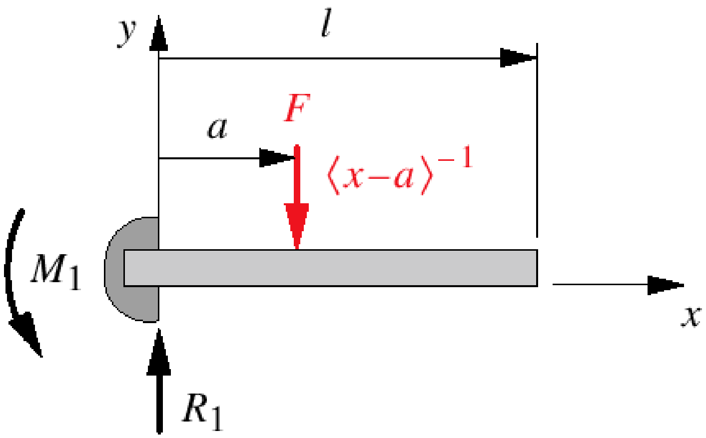
(*c*) Moment Diagram



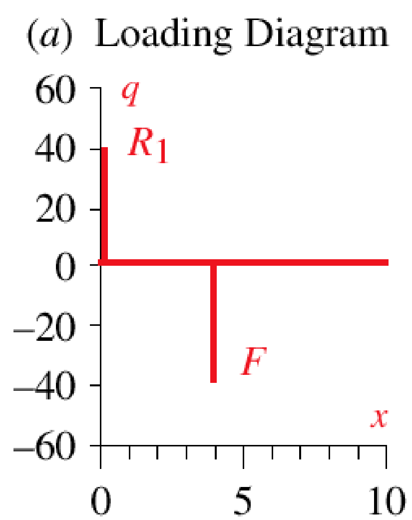
Deflection in beams

Example E2 (Based on Norton's example 3-3B)

Determine and plot the shear, moment, slope, and deflection functions for the cantilever beam shown:



(b) Cantilever beam with concentrated loading



Deflection in beams

Example E2 -- in class

Recall:

$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$ Load function - deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function - deflection

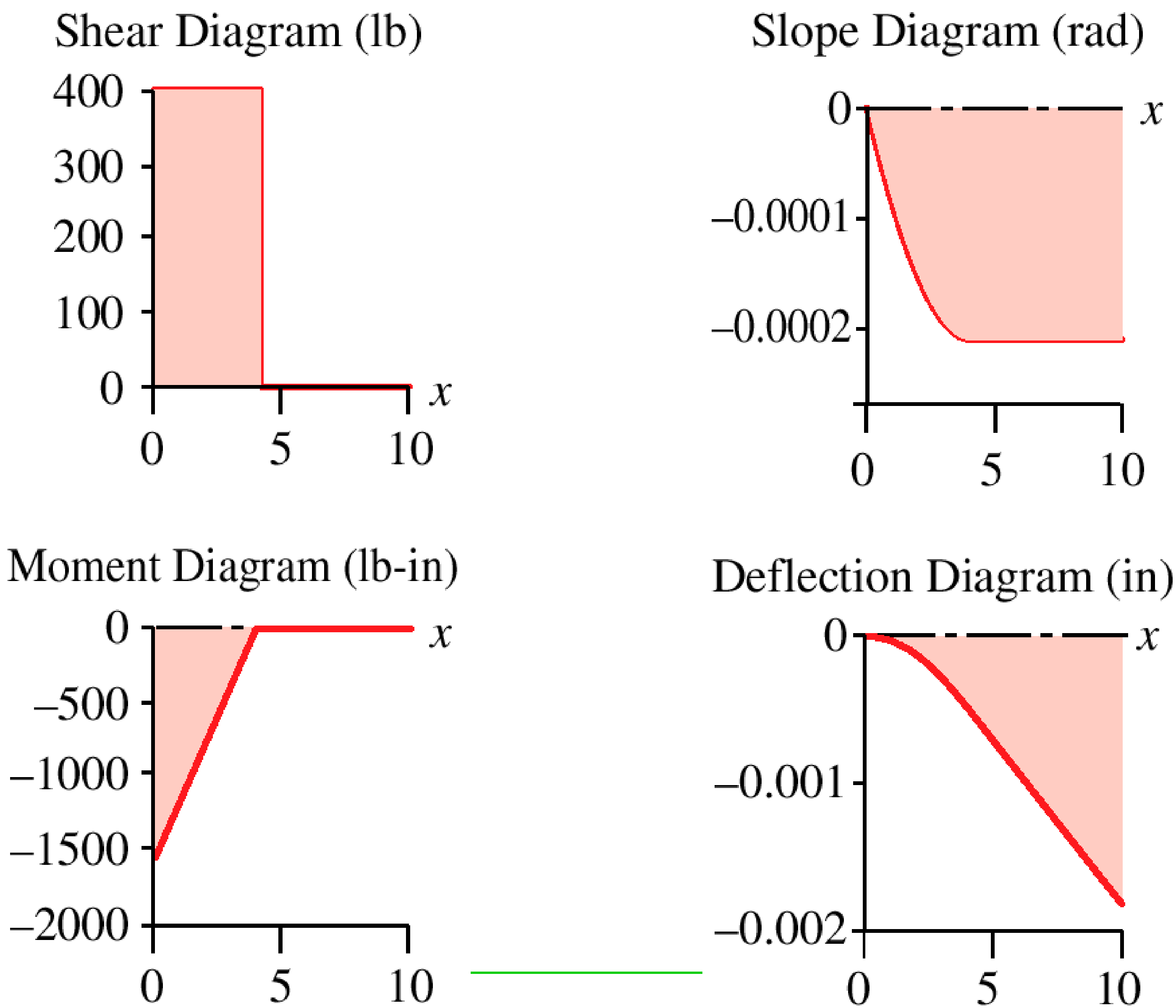
$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function - deflection

$$\theta = \frac{dy}{dx}$$ Slope - deflection

$$y = f(x)$$ Deflection

Deflection in beams

Example E2 -- in class



Plotting singularity functions in MathCad

Example E2 (Based on Norton's example 3-3B)

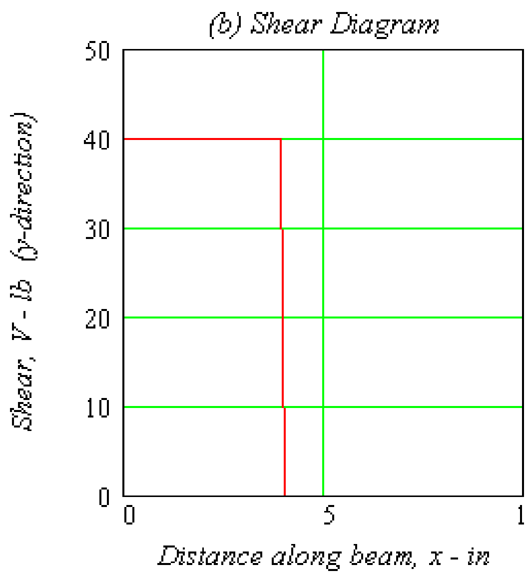
To generate the shear and moment functions over the length of the beam, equations (*b*) and (*c*) must be evaluated for a range of values of x from 0 to l, after substituting the above values of C_1, C_2, R_1, and M_1 in them. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than the dummy variable z, and a value of one when it is greater than or equal to z. It will have the same effect as the singularity function.

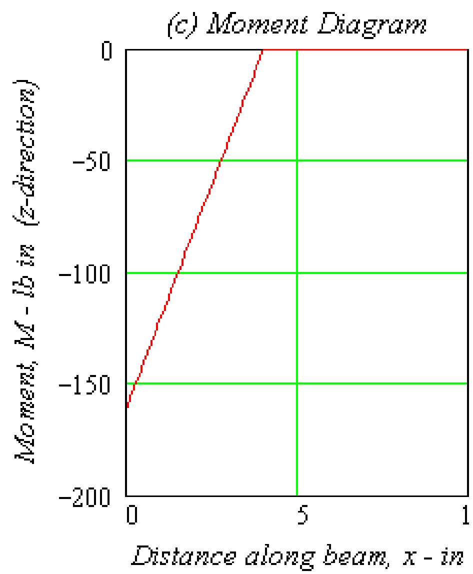
Range of x	$x := 0 \cdot in, 0.01 \cdot l .. l$
Unit step function	$S(x,z) := if(x \ge z, 1, 0)$

Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^0 - F \cdot S(x,a) \cdot (x-a)^0$$

$$M(x) := -M_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^0 + R_1 \cdot S(x, 0 \cdot in) \cdot (x-0)^1 - F \cdot S(x,a) \cdot (x-a)^1$$

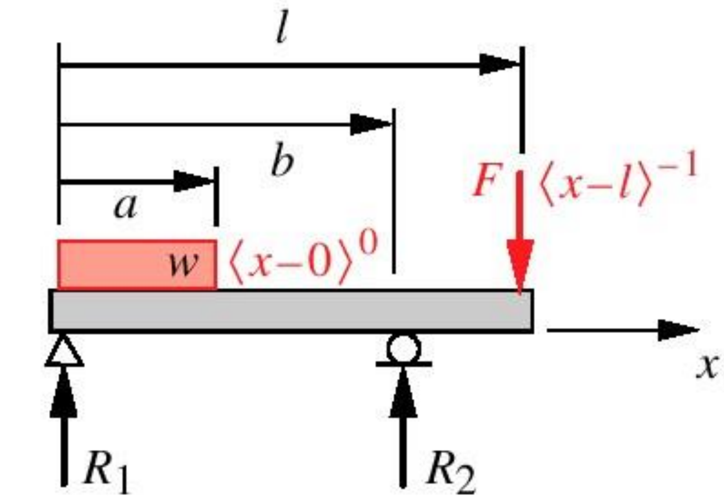




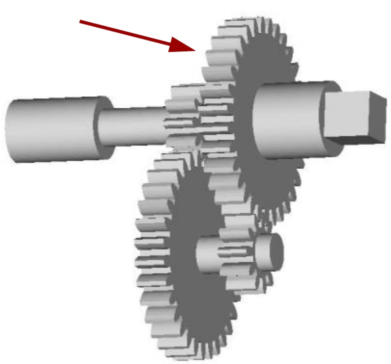
Deflection in beams: solve in class, if time permits

Determine and plot the shear, moment, slope, and deflection functions for the beams shown:

Example E3
(Based on Norton's Example 4-6)

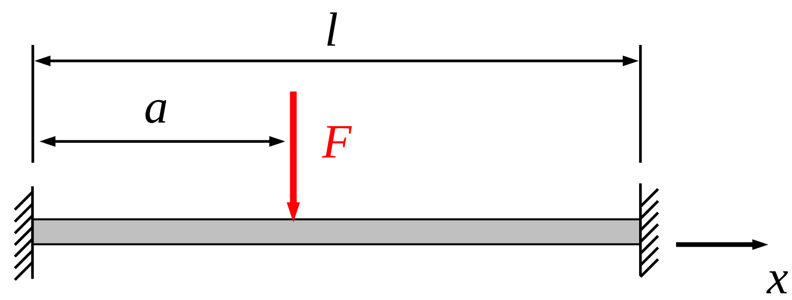


(c) Overhung beam with concentrated force and uniformly distributed loading



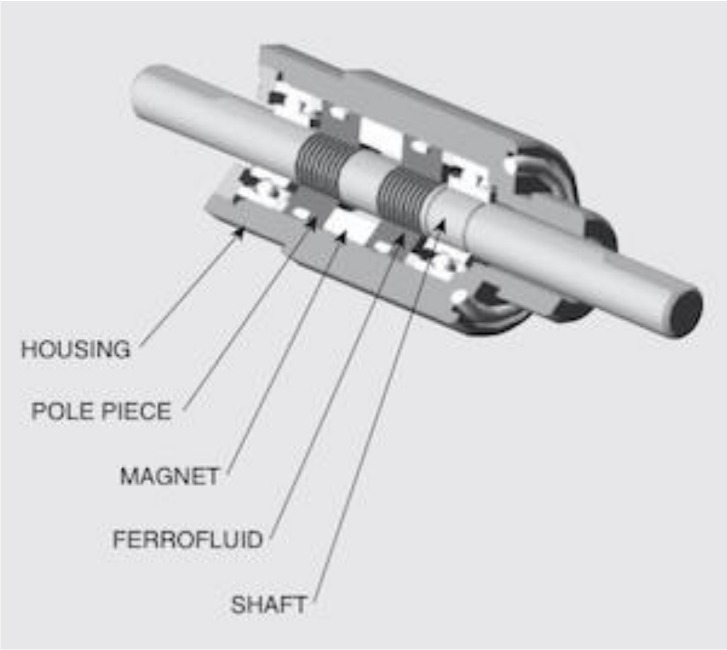
(can be solved with the method of superposition)

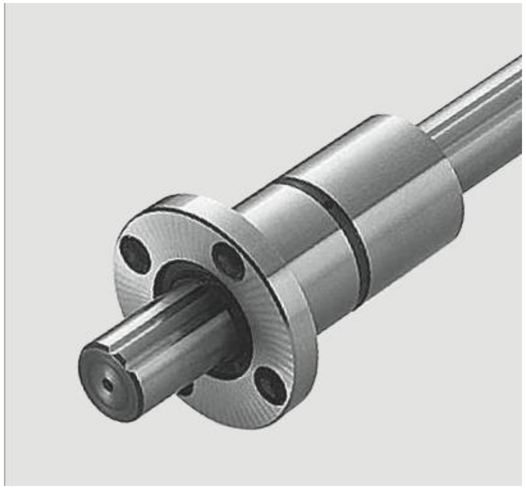
Example E4
(Statically indeterminate)

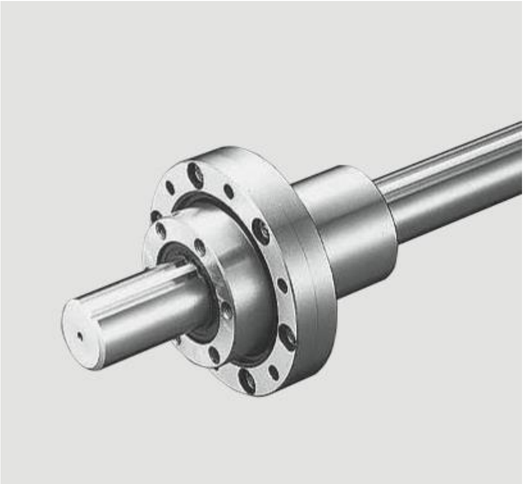


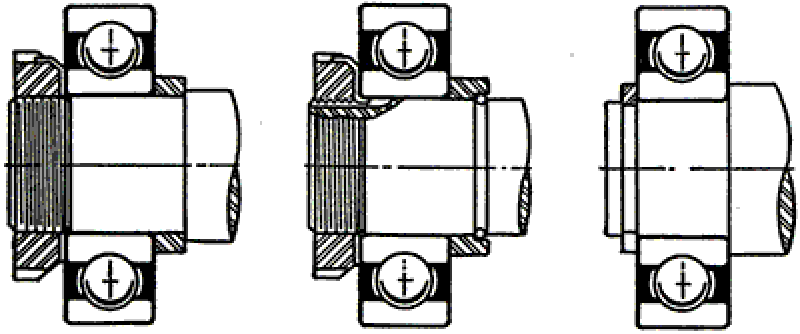
Fully constrained beam with concentrated load

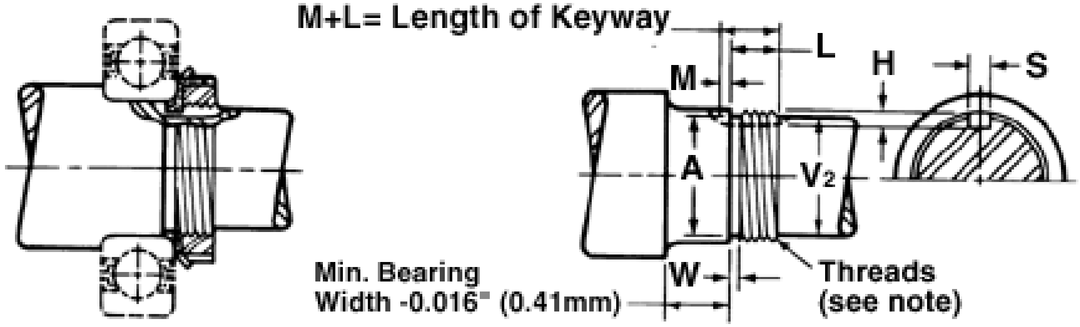
Design components to minimize stress concentrations





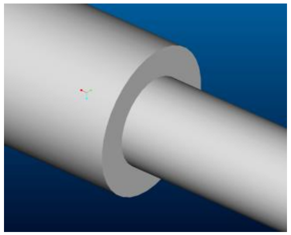


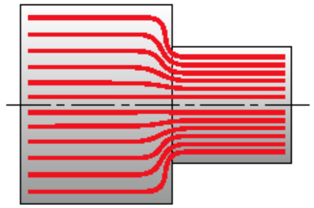




Designing to minimize stress concentrations

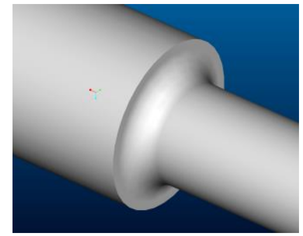
Initial design

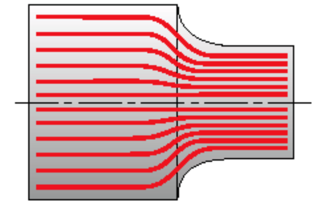




(*a*) Force flow around a sharp corner

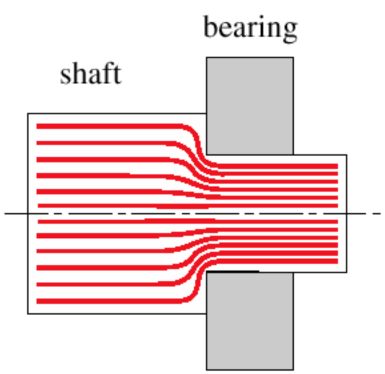
Improved design



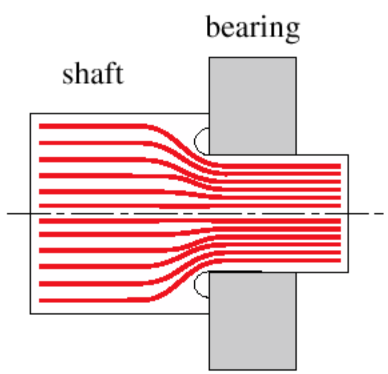


(*b*) Force flow around a radiused corner

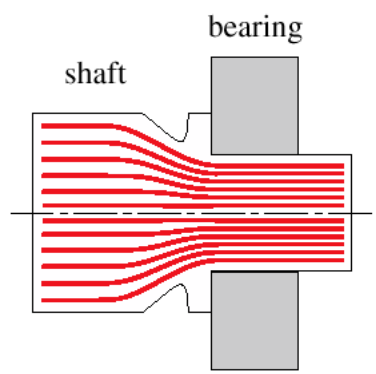
Modifications to reduce stress concentrations at a sharp corner



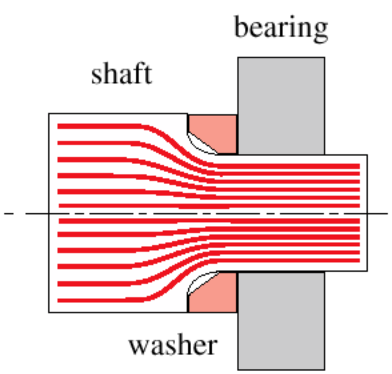
(*a*) Stress concentration at a sharp corner



(*b*) Stress concentration reduced with radius



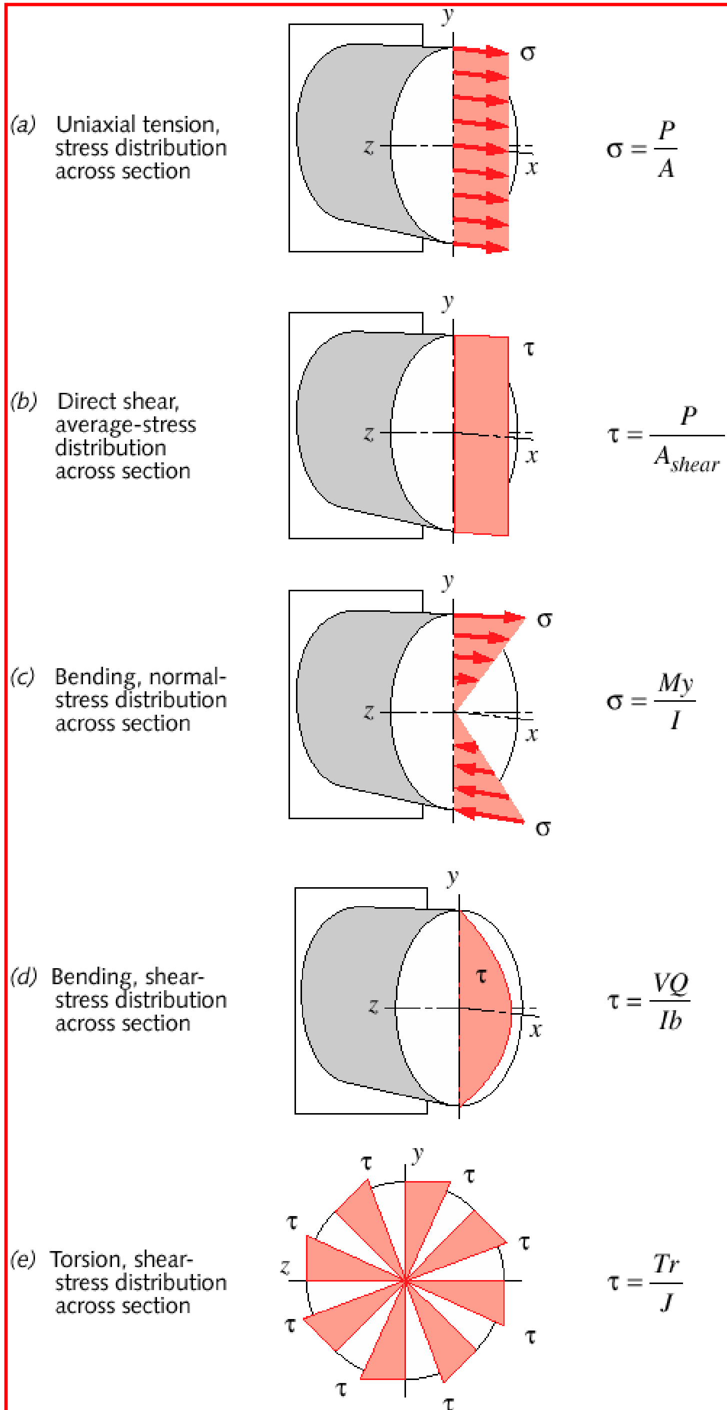
(*c*) Stress concentration reduced with groove



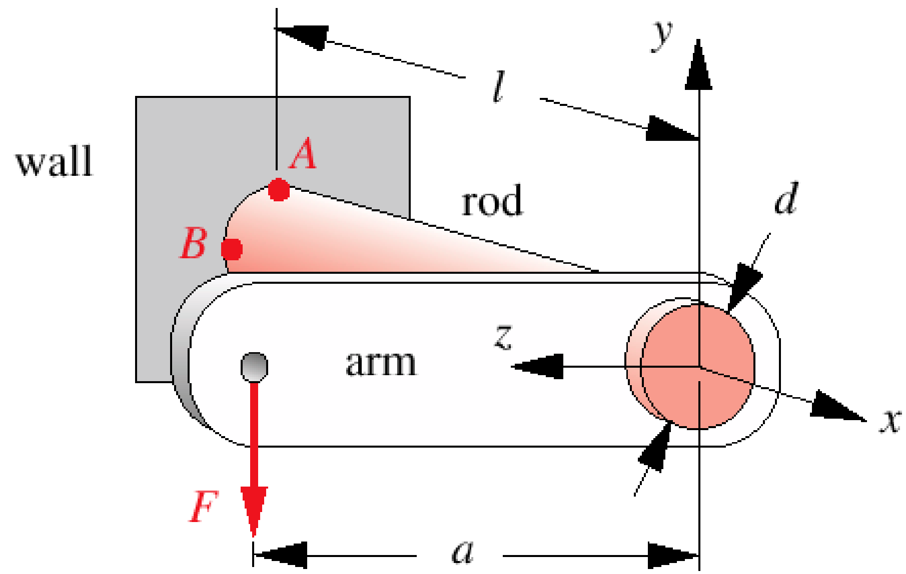
(*d*) Stress concentration reduced with washer

Stress distribution in cross-sections

Examples



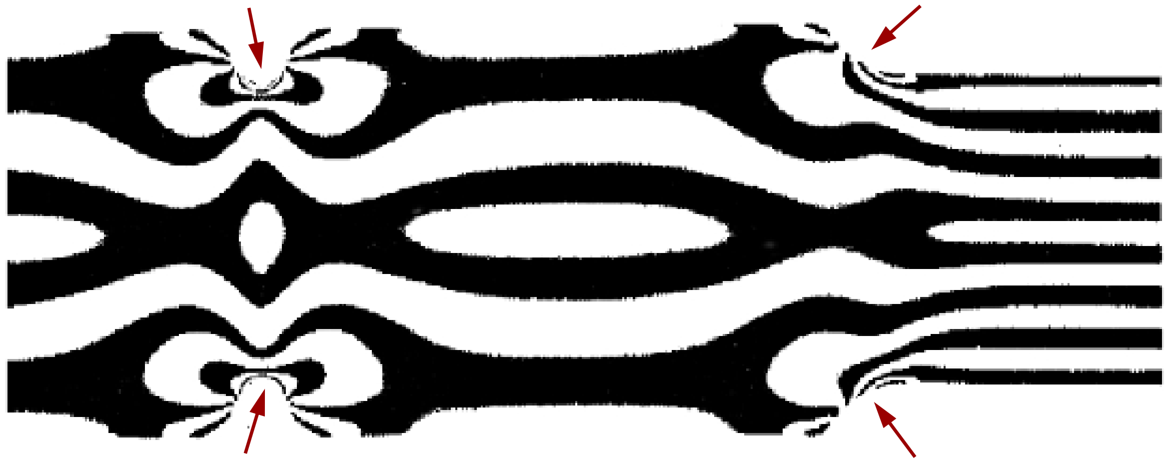
Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points *A* and *B*

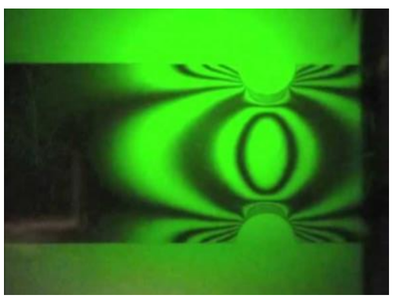


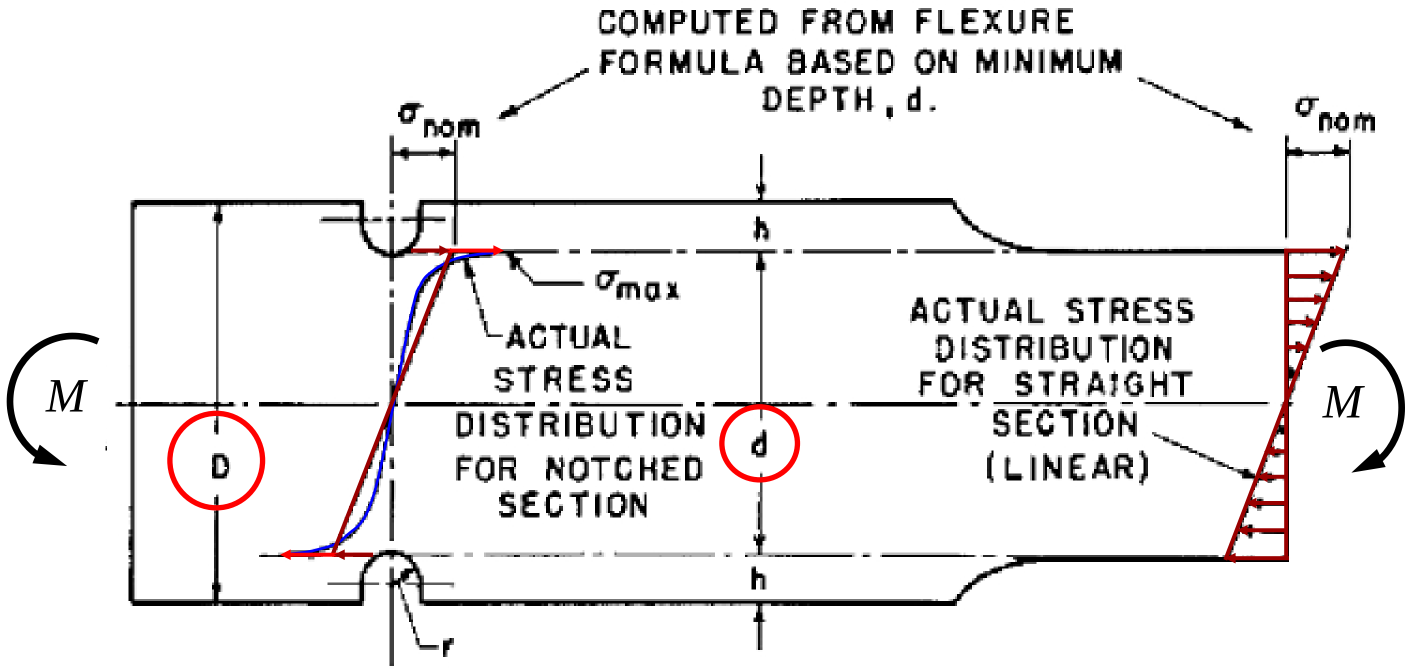
Stress concentrations: demo during class lecture

Experimentally obtained fringe patterns using photoelasticity: patterns reveal distribution of internal stresses.

Note locations subjected to stress concentrations.







Stress concentrations

Bending

Nominal bending stress:

$$\sigma_{nom} = \frac{Md}{I}$$

$$\sigma_{\max} = K_t \frac{Md}{I}$$

K_t is the geometric stress concentration factor -- normal stress

Shear

Nominal shear stress:

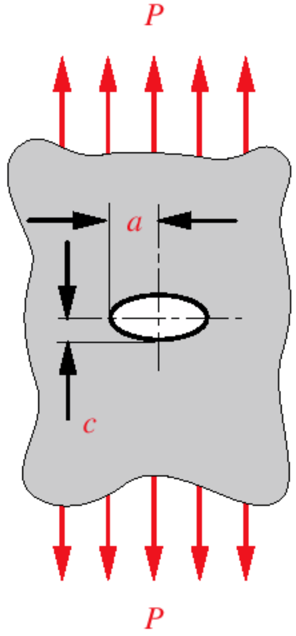
$$\tau_{nom}$$

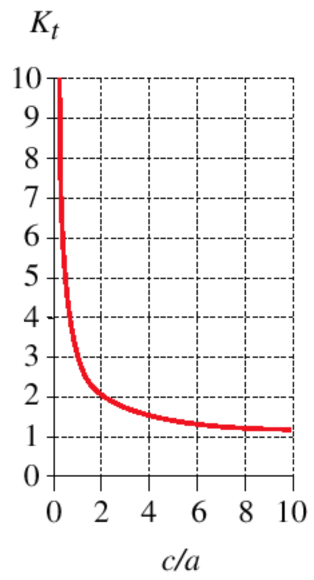
$$\tau_{\max} = K_{ts}\,\tau_{nom}$$

K_{ts} is the geometric stress concentration factor -- shear stress

Stress concentration factors

Stress concentration at the edge of an elliptical hole in a plate (axial load)

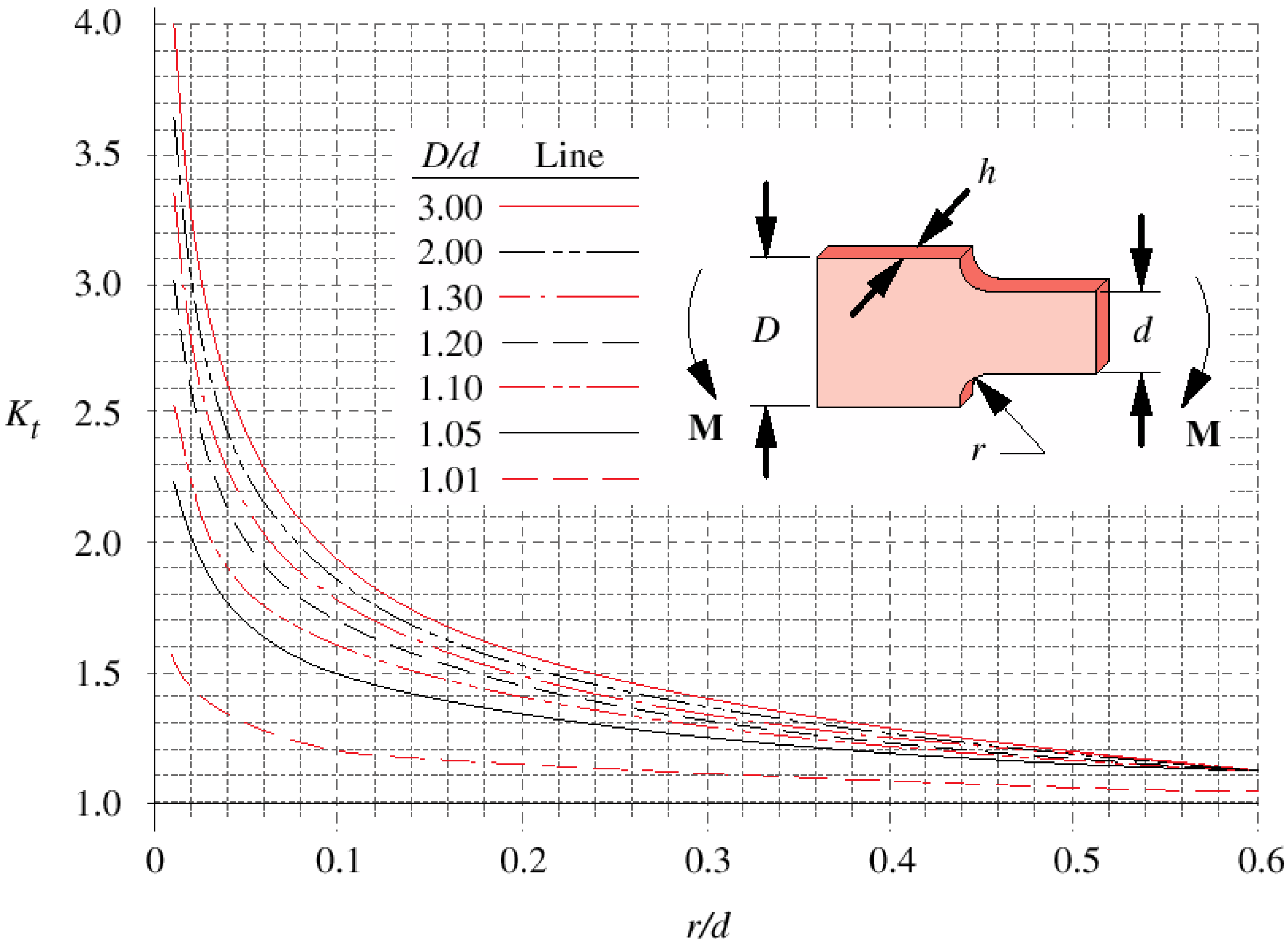




$$K_t = 1 + 2\left(\frac{a}{c}\right)$$

Stress concentration factors

Stress concentration in a stepped flat bar subjected to bending



$$\sigma_{nom} = \frac{Mc}{I} = 6\frac{M}{hd^2}$$

$$\sigma_{max} = K_t \sigma_{nom}$$

and :

$$K_t = A\left(\frac{r}{d}\right)^b$$

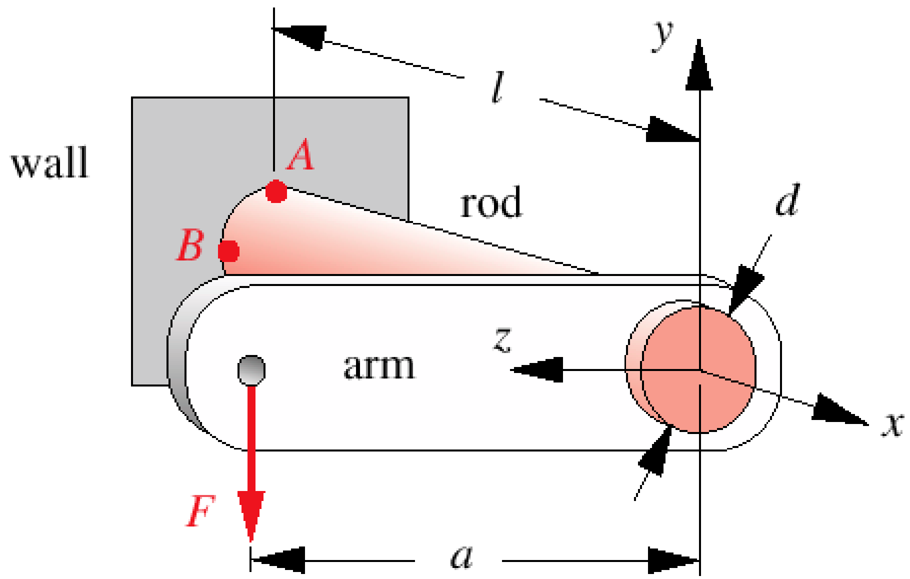
where :

D / d	A	b
3.00	0.907 20	–0.333 33
2.00	0.932 32	–0.303 04
1.30	0.958 80	–0.272 69
1.20	0.995 90	–0.238 29
1.10	1.016 50	–0.215 48
1.05	1.022 60	–0.191 56
1.01	0.966 89	–0.154 17

Stress distribution in cross-sections

Example: geometric stress concentration factors

Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points A and B (also C and D, opposite to A and B, respectively). *Assume a stress concentration factor of 2.5 in both bending and torsion.*



Reading

- Chapter 9: design case studies
- Chapters 4 of textbook: Sections 4.12 to 4.19
- Review notes and text: ES-2501, ES-2502

Homework assignment

- **Author's:** consult website of our course
- **Solve:** consult website of our course