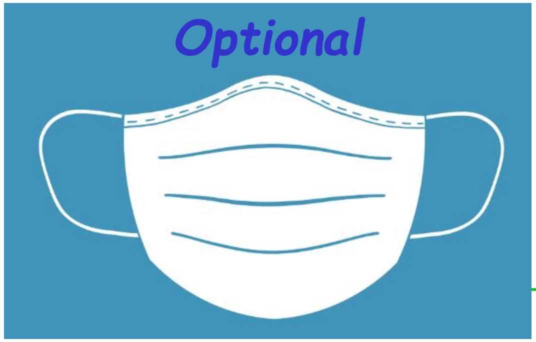
WORCESTER POLYTECHNIC INSTITUTE
MECHANICAL ENGINEERING DEPARTMENT

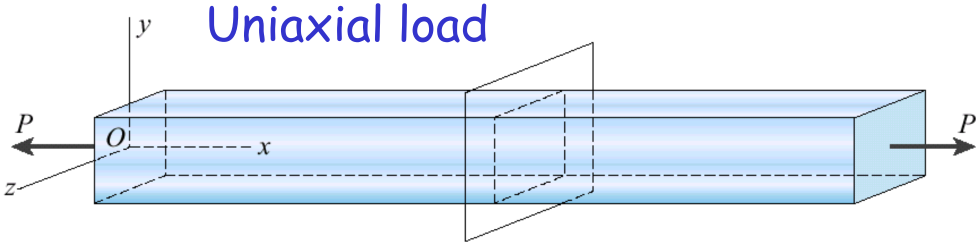
DESIGN OF MACHINE ELEMENTS
ME-3320, B'2024

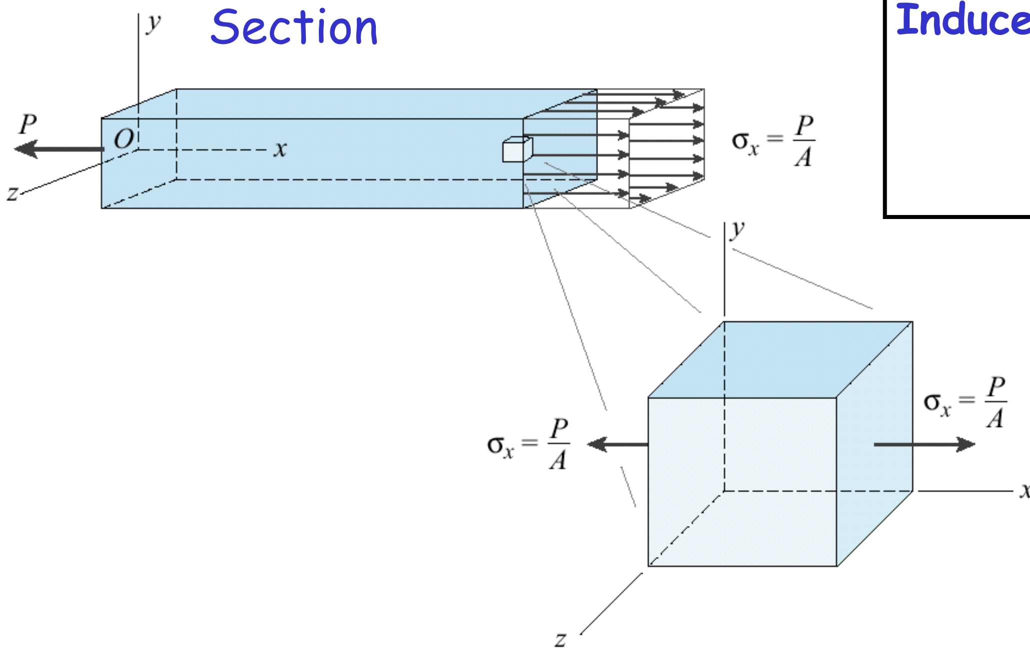
Lecture 06

October 2024



Stress distribution in cross-sections
Uniaxial load





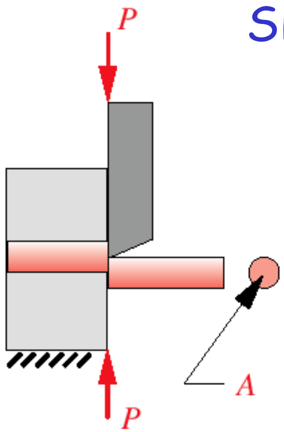
Induced change in length:

$$\Delta s = \frac{P\,l}{A\,E}$$

Stress cube for uniaxial stress loading

Stress distribution in cross-sections

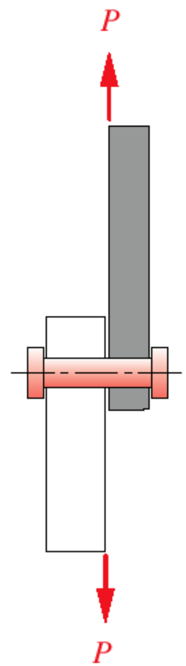
Direct shear



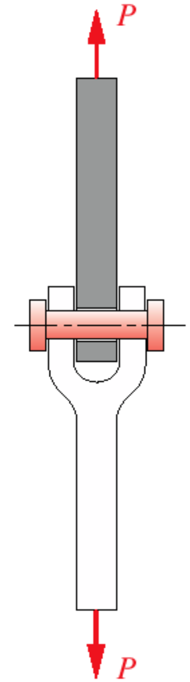
(a) Direct shear

Single shear stress:

$$\tau_{xy} = \frac{P}{A_{shear}}$$



(a) Pivot in single shear



(b) Pivot in double shear

Double shear stress:

$$\tau_{xy} = \frac{P}{2A_{shear}}$$

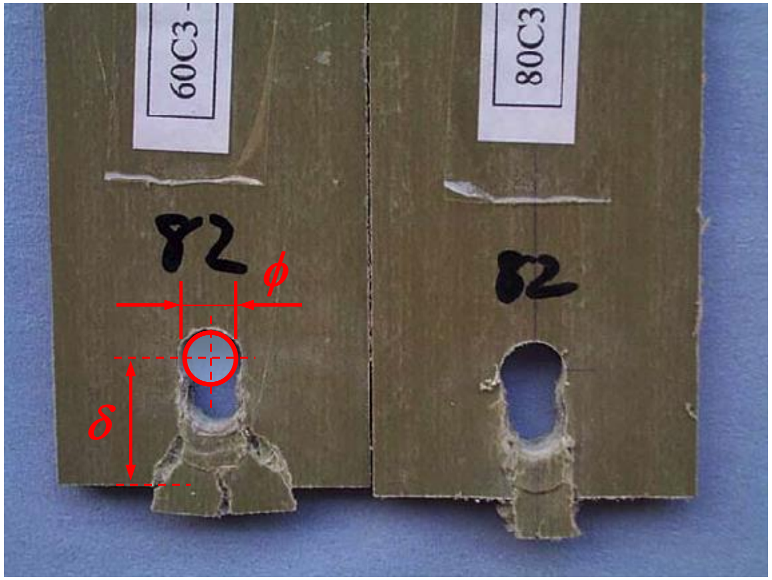
Stress distribution in cross-sections

Bearing and tearout: modes of failure

Bearing load

$$\sigma = \frac{P}{A_{bearing}}, \qquad A_{bearing} = \phi \cdot T$$

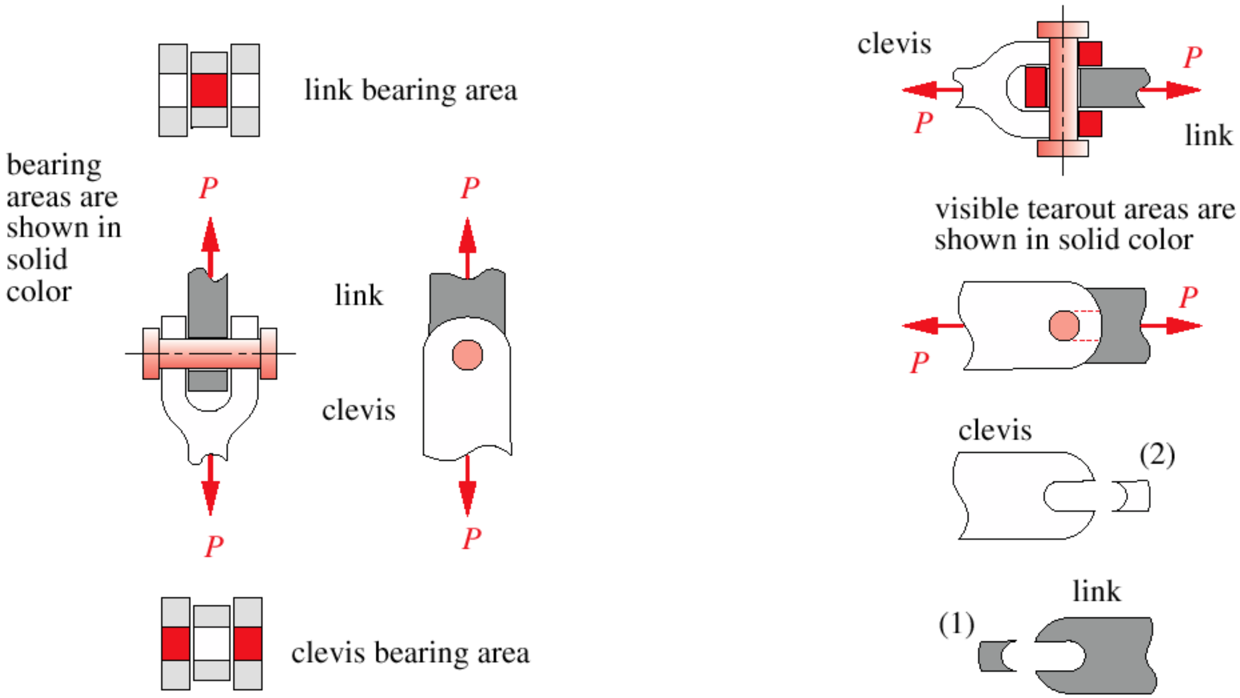
Tearout load



$$\sigma = \frac{P}{A_{tearout}}, \qquad A_{tearout} = 2 \cdot \delta \cdot T$$

T = Thickness of the plate

Stress distribution in cross-sections
Bearing and tearout



(a) Bearing-stress areas

(b) Tearout failure

$$\sigma = -\frac{P}{A_{bearing}}, \qquad A_{bearing} = l\,d$$

(*Areas colored in* ***solid red***)

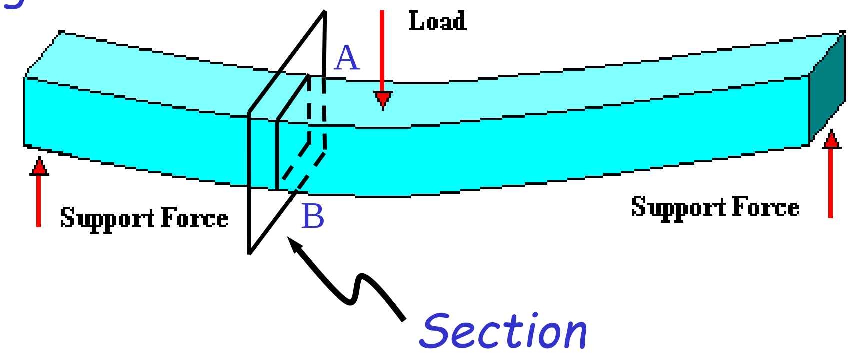
$$\sigma = \frac{P}{A_{tearout}}, \qquad A_{tearout}$$

(*Areas colored in* ***solid red***)

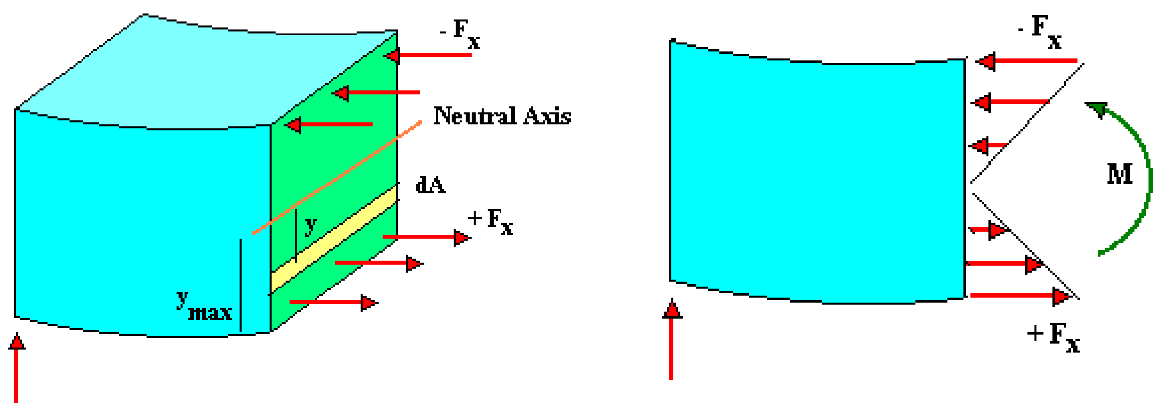
Stress distribution in cross-sections

Bending stress

Bending load



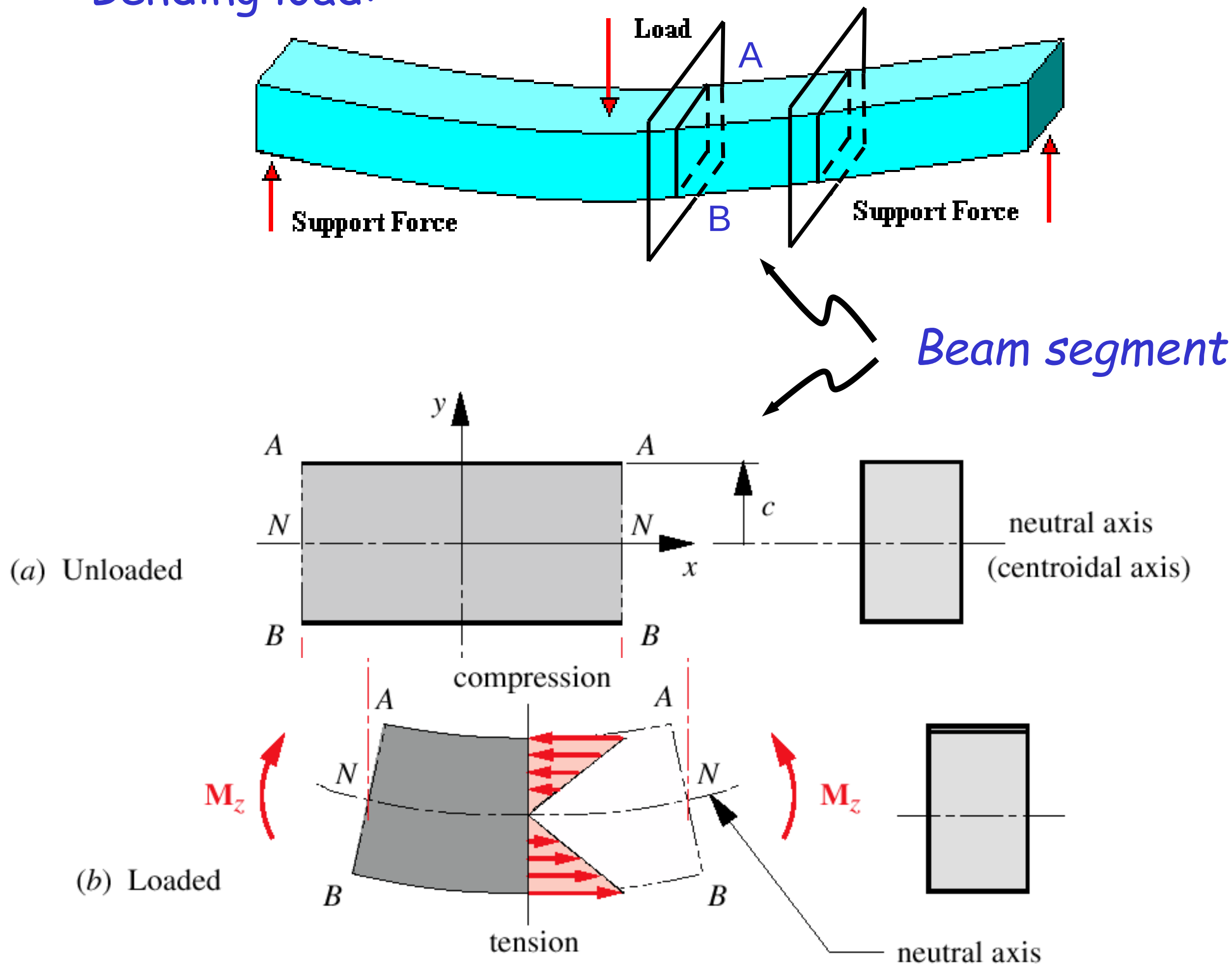
Internal distribution of bending forces



Stress distribution in cross-sections

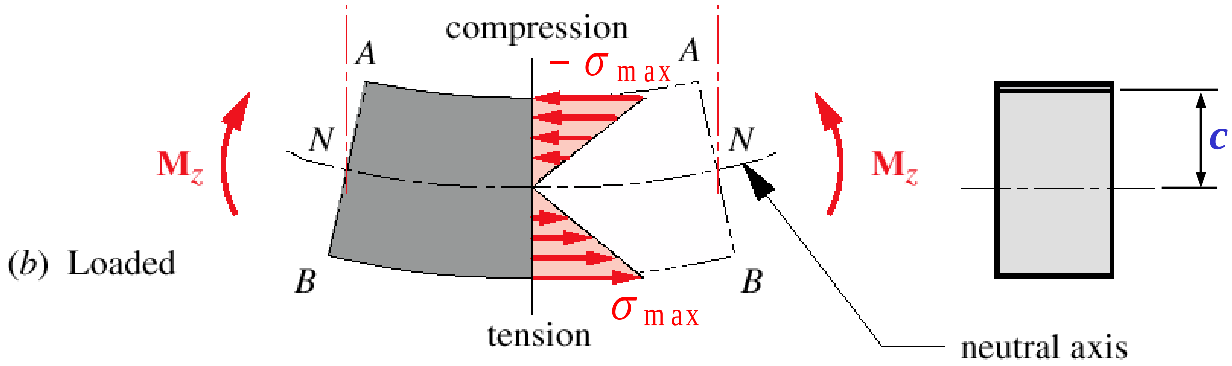
Bending stress

Bending load:



Stress distribution in cross-sections

Bending stresses



Bending stresses:

$$\sigma_x(x, y) = -\frac{My}{I}$$

Recall that $M = M(x)$

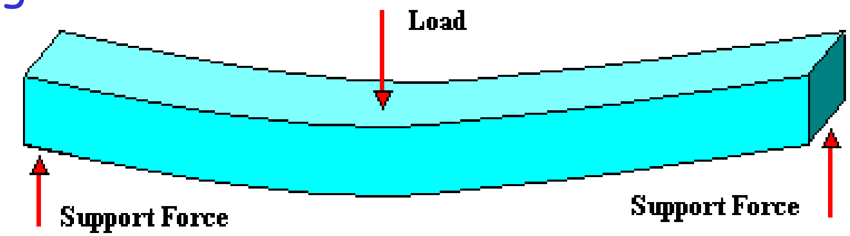
Maximum bending stress:

$$\sigma_{\max}(x) = \frac{Mc}{I}$$

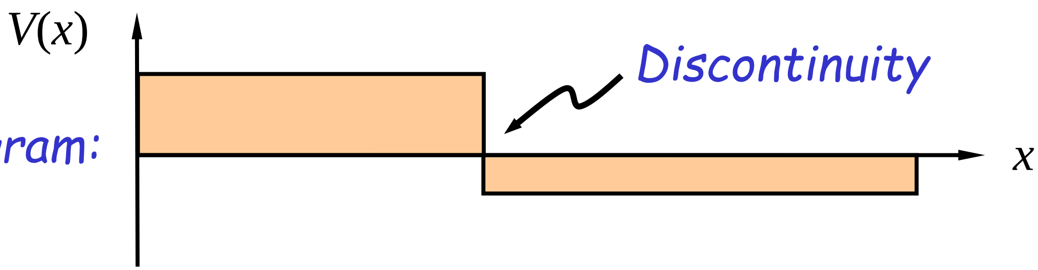
Stress distribution in cross-sections

Bending stresses

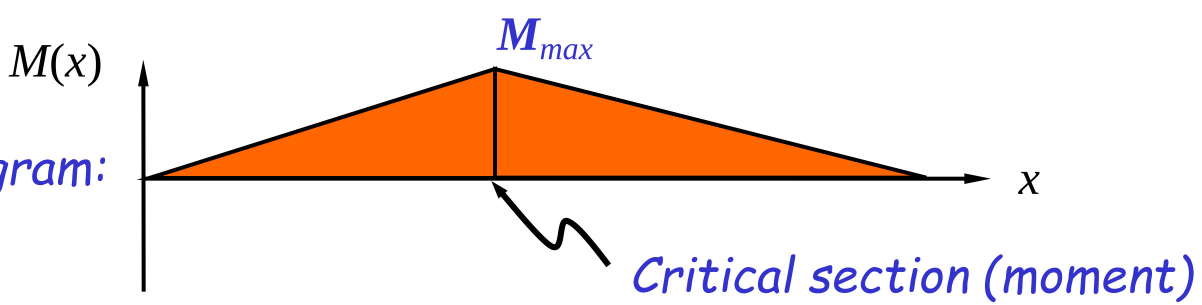
Bending load:



Shear diagram:



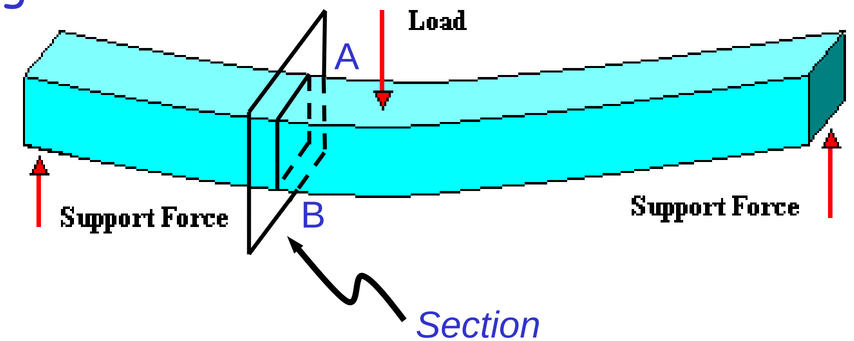
Moments diagram:



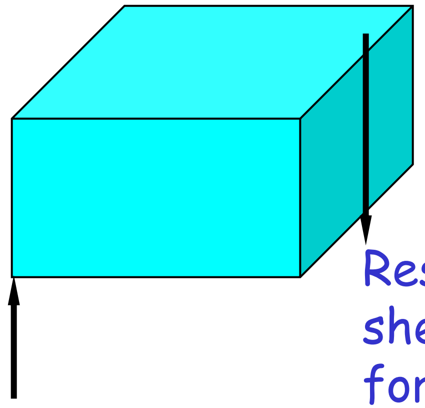
Stress distribution in cross-sections

Transverse shear stress

Bending load:

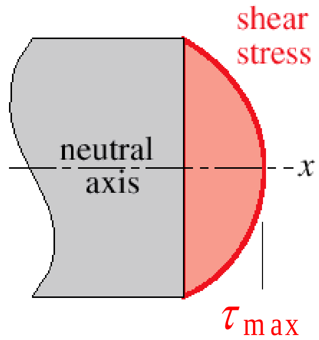


Internal distribution of transversal shear forces



Stress distribution in cross-sections

Transverse shear stress

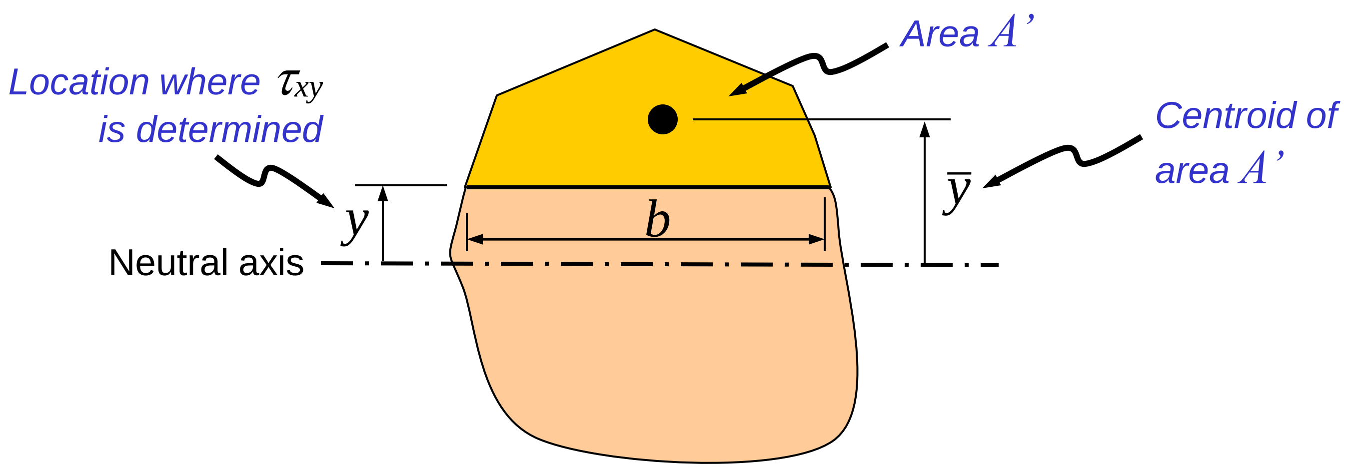


Transverse shear stress:

$$\tau_{xy} = \frac{V\,Q}{I\,b} \quad \textbf{with} \quad Q = \bar{y}\,A'$$

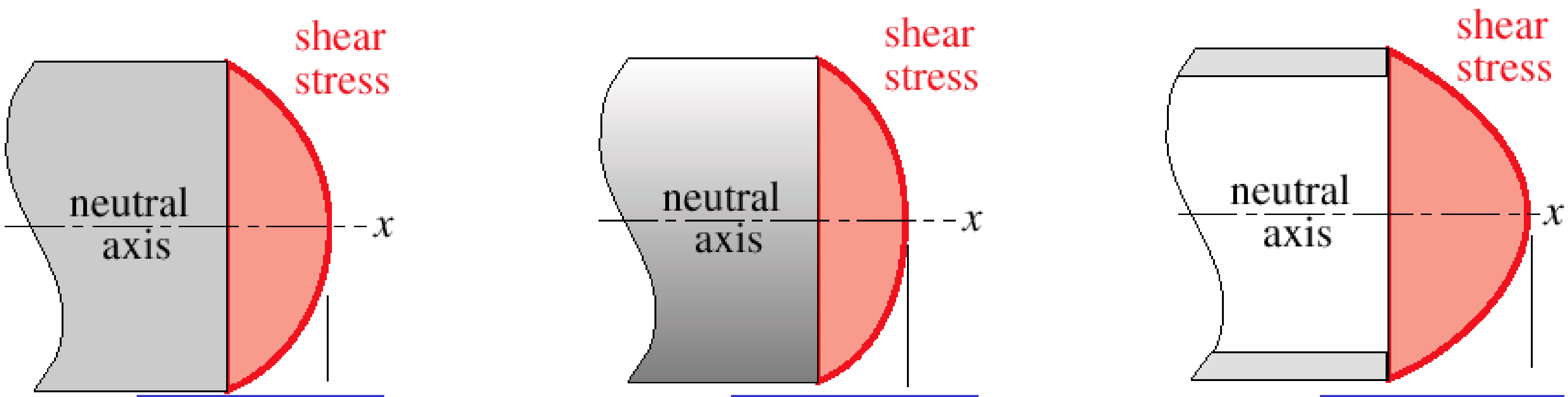
Recall that $V = V(x)$

Generic cross-section:



Stress distribution in cross-sections

Transverse shear stress: distribution in common sections



$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

(*a*) rectangular beam

$$\tau_{max} = \frac{4}{3}\frac{V}{A}$$

(*b*) solid round beam

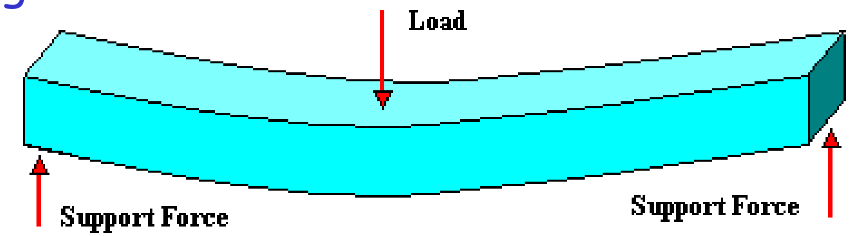
$$\tau_{max} \cong \frac{2V}{A}$$

(*c*) hollow round beam

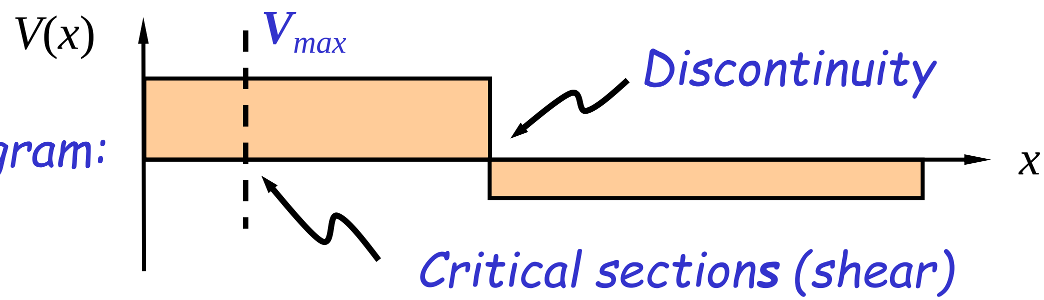
Stress distribution in cross-sections

Transverse shear stress

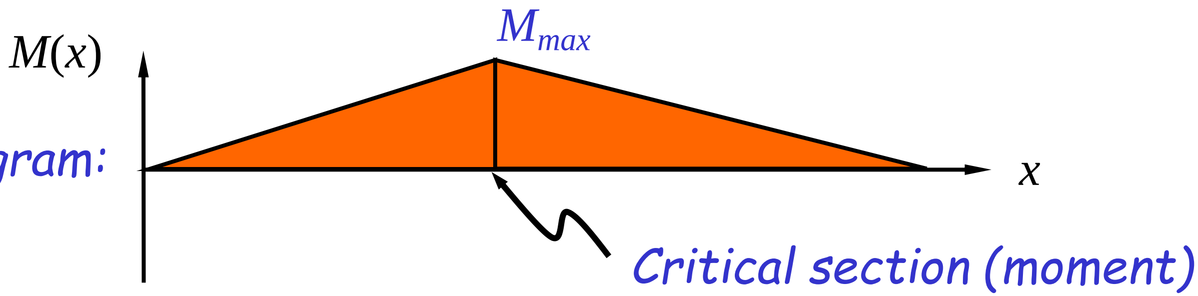
Bending load:



Shear diagram:

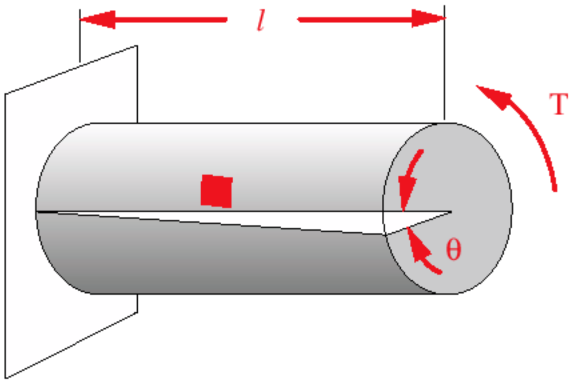


Moments diagram:



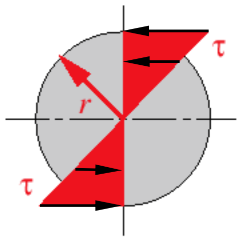
Stress distribution in cross-sections

Torsional stress



(a) Deflection θ

Pure shear stress



(b) Shear-Stress Distribution τ

Shear stress distribution:

$$\tau = \frac{T\,\rho}{J} \;, \qquad \text{recall that} \quad J = I_x + I_y$$

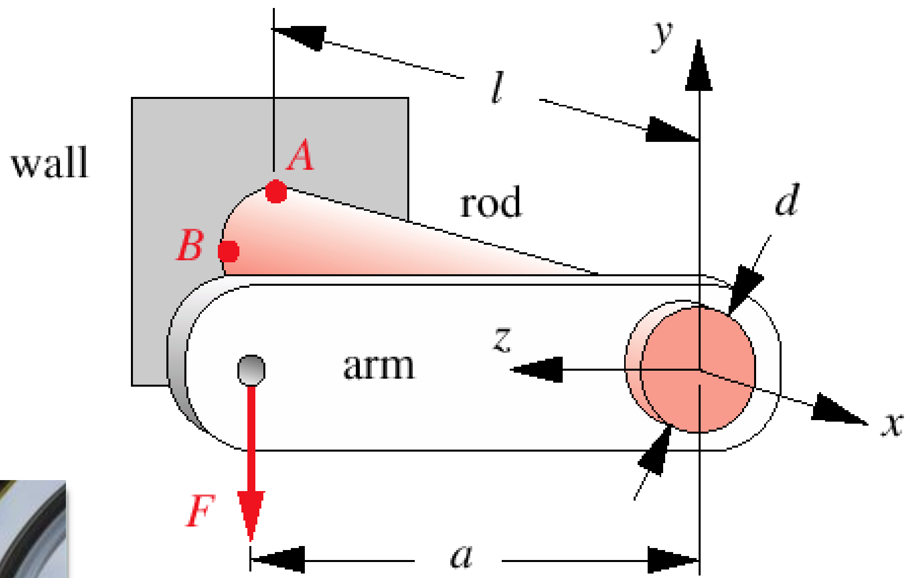
Angular deflection:

$$\theta = \frac{T\,l}{K\,G} \;, \qquad \text{recall that the shear modulus is} \quad G = \frac{E}{2(1+\nu)}$$

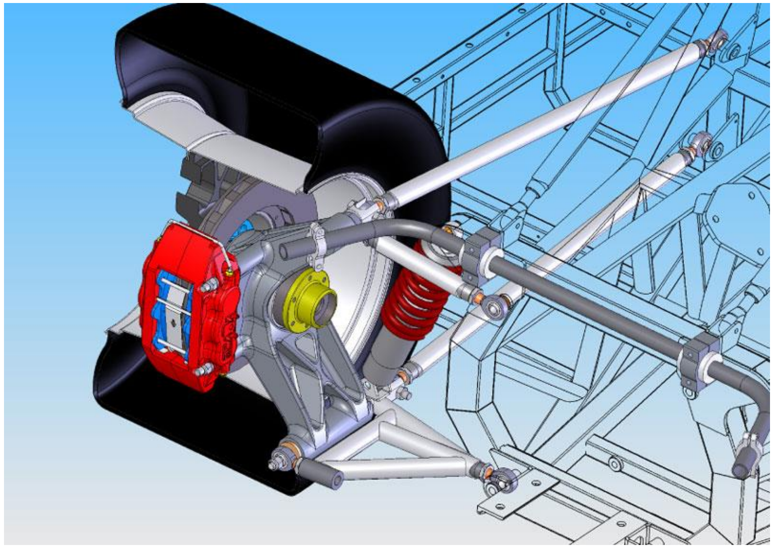
(K is a geometric factor -- see Table 4-3, p.211)

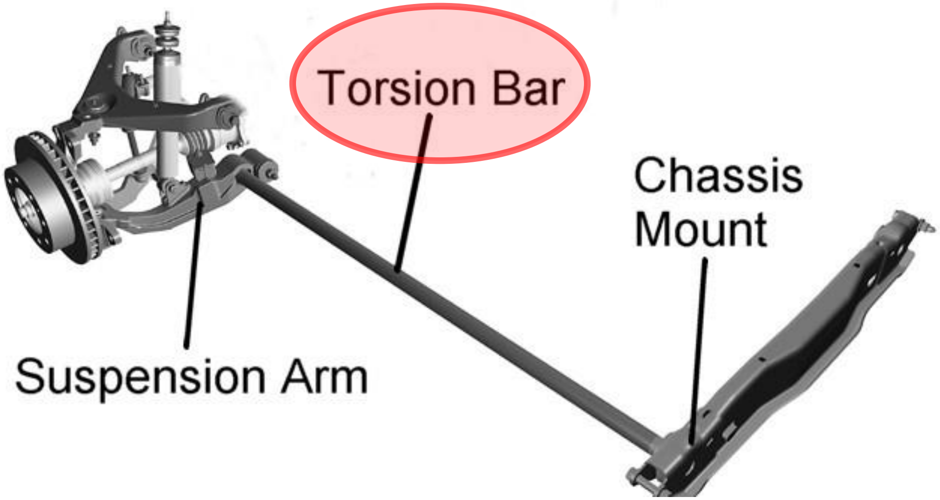
Stress distribution in cross-sections
Examples

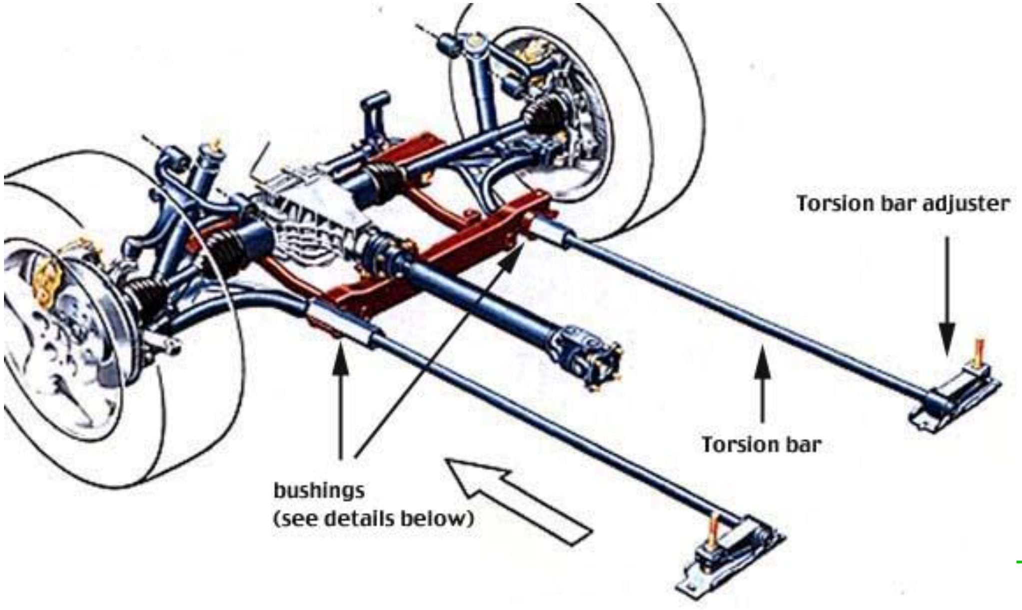
Find the most highly stressed locations on the bracket model shown. Define and draw stress elements (cubes) at points A and B , which are two critical points on the rod's critical section.



Uses of the bracket model configuration: suspension system

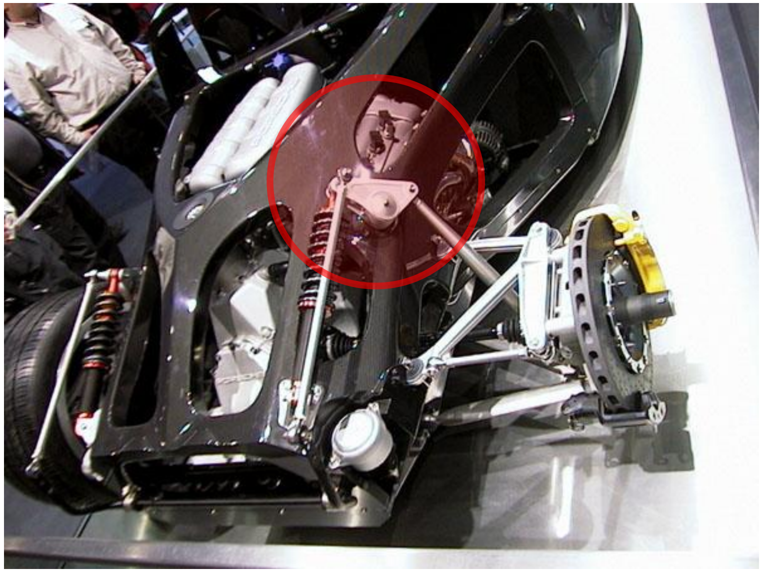


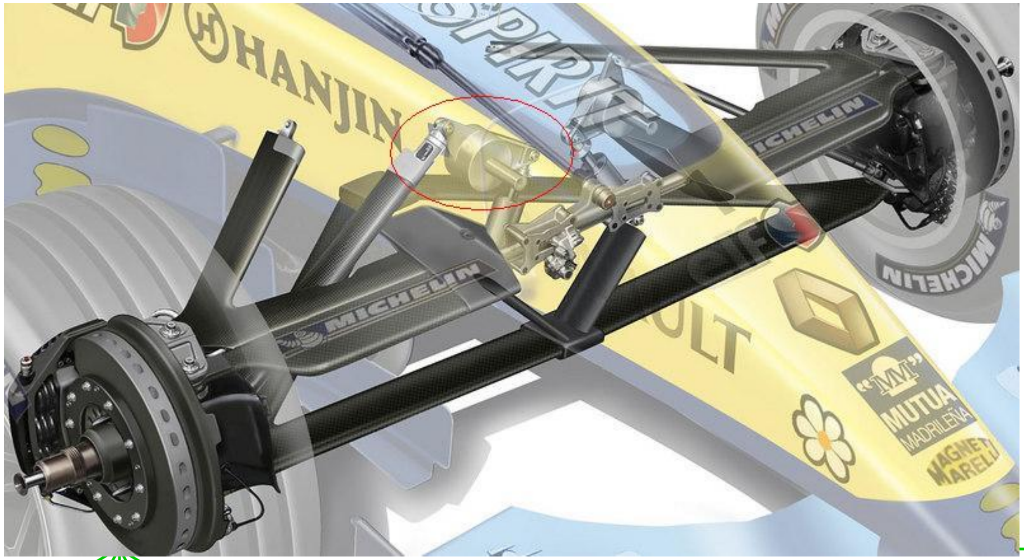




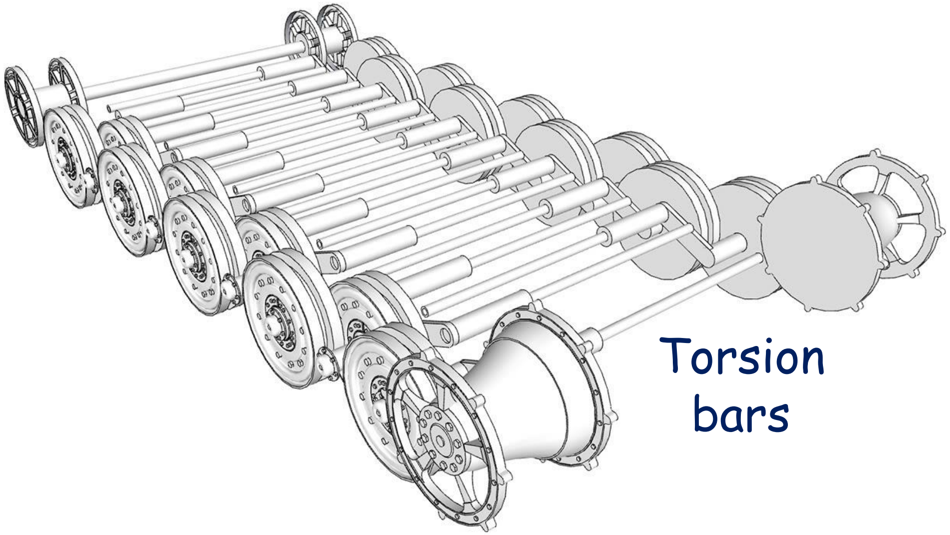
Uses of the <u>bracket model</u> configuration: suspension system

Torsion bar

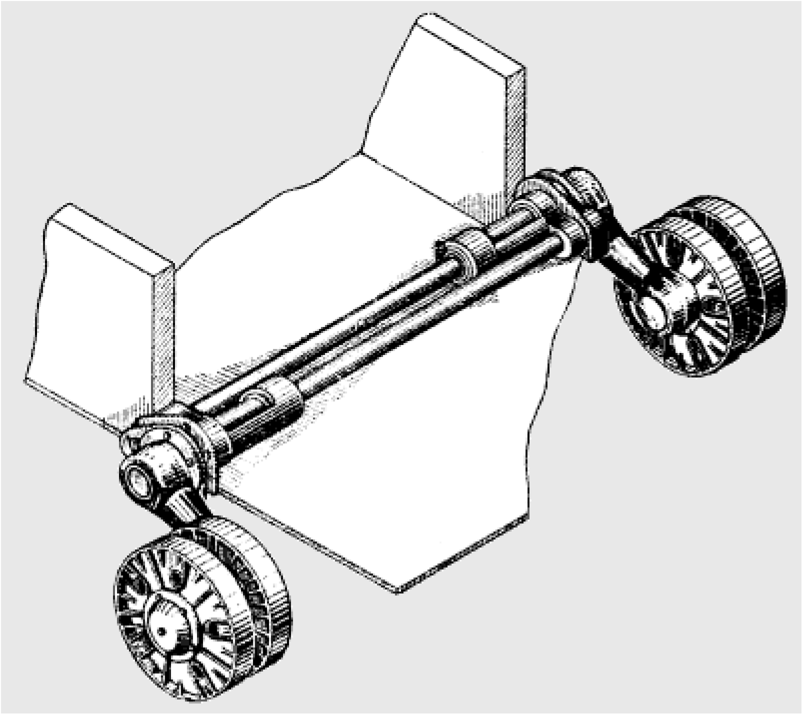




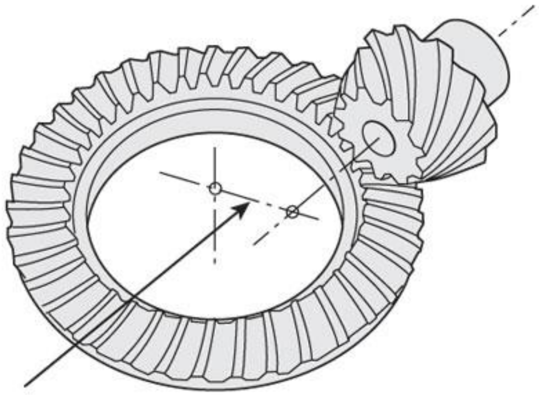
Uses of the bracket model configuration: suspension system



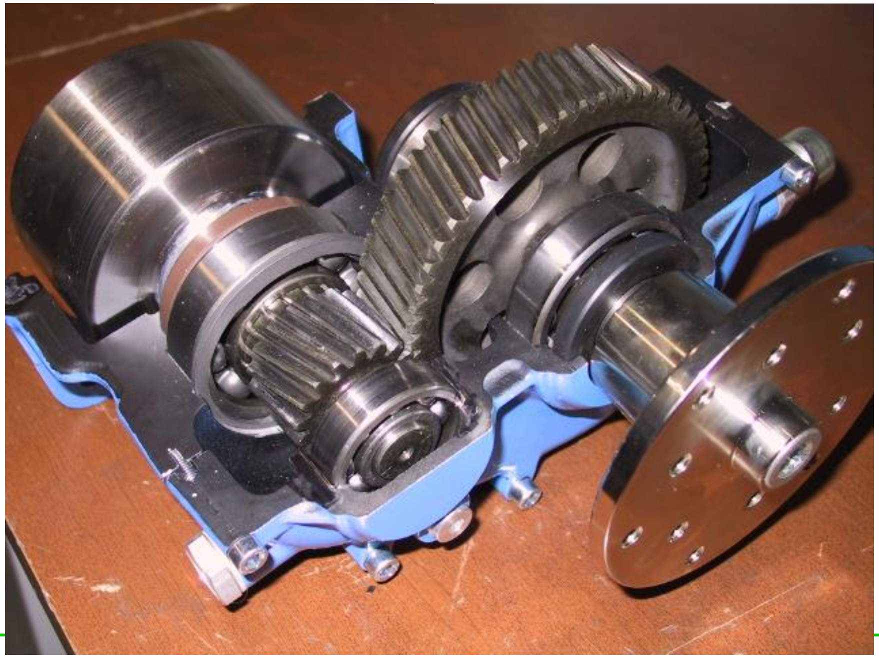
Torsion bars

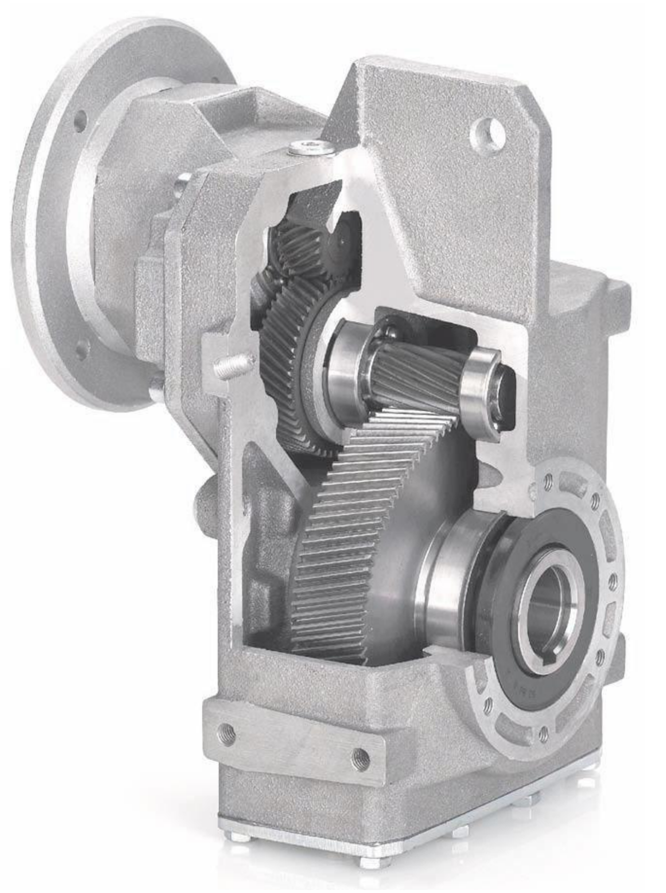


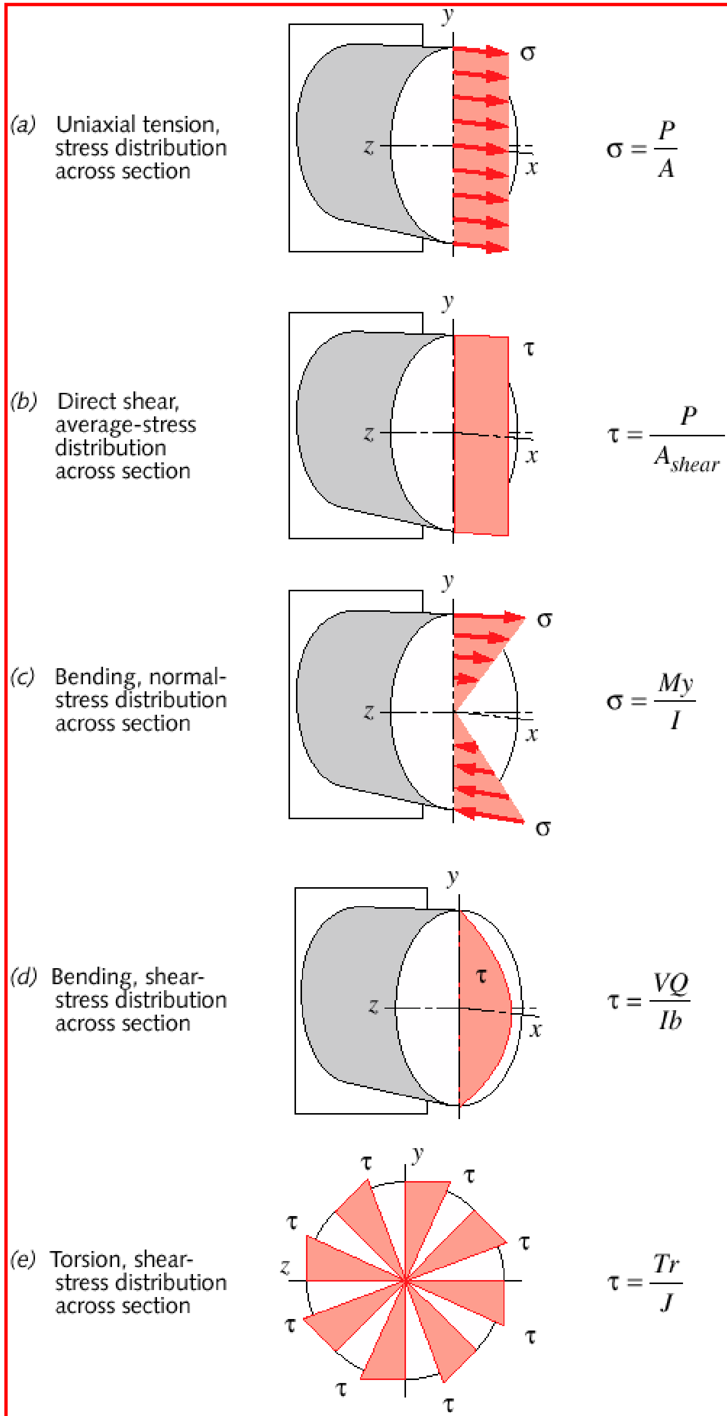
Uses of the bracket model configuration: transmissions



Hypoid Gear



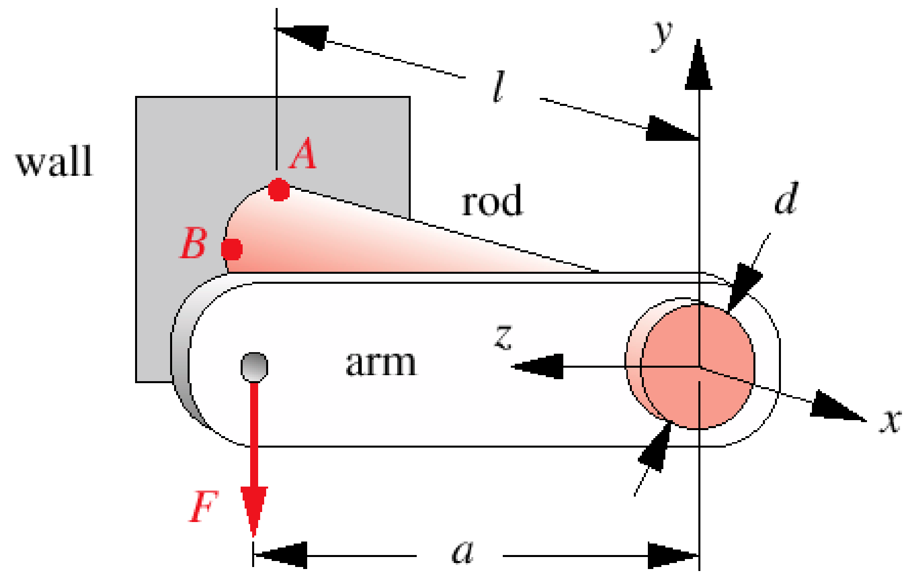




Stress distribution in cross-sections

Examples

Find the most highly stressed locations on the bracket shown. Draw stress elements (cubes) at points A and B



Deflection in beams

Example E1 - in class

Recall:

$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$ Load function - deflection

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$ Shear function - deflection

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$ Moment function - deflection

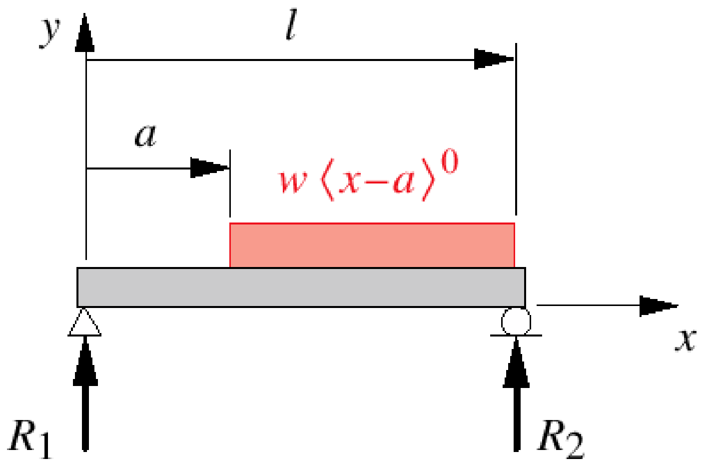
$$\theta = \frac{dy}{dx}$$ Slope - deflection

$$y = f(x)$$ Deflection

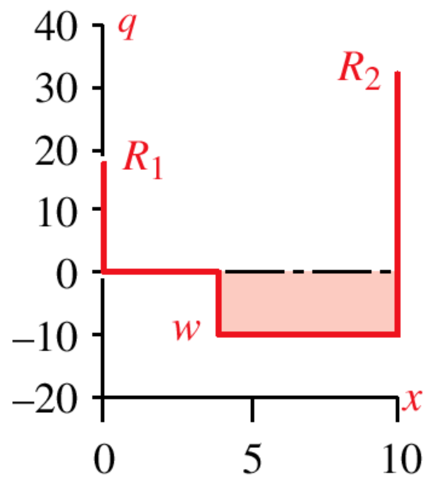
Use of singularity functions: slope & deformations

Example E1 (based on Norton's examples 3-2B and 4-4)

Determine: shear, moments, slope, and deflection functions for the simply supported beam shown:



(*a*) Simply supported beam with uniformly distributed loading



Reading assignment

- Chapters 4 of textbook: Sections 4.7 to 4.11
- Review notes and text: ES2501, ES2502

Homework assignment

- Author's: see website of our course for details
- Solve: see website of our course for details